Uncertainty: Inquiry without quantification

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June 16, 2026

Abstract

A recent proposal for the epistemology of measurement maintains that uncertainty estimates are not mere auxiliaries to but integral to measurement. Yet many measurement results do not include estimates of uncertainty. We defend uncertainty assessment as crucial for measurement by clarifying the epistemological work done by uncertainty inquiries, which need not result in a quantified uncertainty. We illustrate our argument with cases from the history of measurements of the speed of light.

1 Introduction

The *usefulness problem* for measurement is the problem of how measurement results produced in one particular context and set of conditions can be made useful in other contexts and other conditions (Beauchemin and Staley, 2025, 2026). The problem of

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usefulness serves as a pragmatic formulation of the question of how measurement produces scientific knowledge. In our treatment of measurement as inquiry, we have argued that *uncertainty estimation* has a crucial role to play in solving the usefulness problem, and that *underdetermination*, through its relationship to uncertainty estimation, contributes in a positive way to solving the usefulness problem for measurement. We argue that uncertainty estimates are not mere auxiliaries to measurement results, but are the products of a process integral to measurement.

This latter claim faces a challenge to its adequacy within the domain to which we have previously applied it: Even within physics, not every measurement includes an explicit quantitative estimate of uncertainty. How, then, can it be that procedures of estimating uncertainty are integral to measurement?

In this paper, we argue that uncertainty *assessment* is equally crucial in measurement in the physical sciences, with or without a quantitative estimate of uncertainty. Uncertainty assessment, as we use the term, includes explicit quantification of uncertainty (uncertainty estimation), but also incorporates conclusions regarding the uncertainty of a measurement result couched in qualitative terms, including implicit characterizations. We propose to illustrate our argument by contrasting examples from physics in which measurements are conducted, but uncertainty estimates on the results are not provided, with examples of measurements of the same quantity for which an uncertainty estimate is explicitly provided. Our analysis of these cases will exemplify an epistemological strategy for recognizing the contribution of uncertainty assessment to measurements lacking quantified uncertainty estimates.

An uncertainty assessment is part of a broader set of scientific strategies and practices to assess the suitability of a given measurement result for its epistemic and

pragmatic objectives, and as such plays a critical role in warranting the usefulness of the result for other inquiries. [1]

The key to our argument lies in, first, treating measurement as inquiry, and second, a pragmatic epistemological account of scientific inquiry. In that framework, inquiry is an activity involving an interplay between a use mode and a critical mode. This interplay is crucial to our approach to the problem of usefulness, in which uncertainty is assessed through a sub-inquiry in the critical mode into the underdetermination of measurement procedures, enabling a positive epistemic use for that underdetermination. We argue in this paper that this interplay of use and critical mode activities of inquiry allows extending our previous argument to inquiries that produce no estimate of uncertainty. A pragmatic framework enables the articulation of such a solution, and in the process reveals previously unappreciated aspects of how measurement produces knowledge.

2 Framing the problem

Quantitative estimation of uncertainty is pervasive in physical science measurement, but has not always been so. Even if one thinks of the rise of this practice as one aspect of the process by which statistical thinking came to permeate modern life (Porter, 2020), that does not suffice to account for specific ways in which measurement uncertainty is evaluated and used by scientists. What does the estimation of uncertainty contribute to the epistemic achievement of measuring things? What does estimating uncertainty

[1] We postulate that such an epistemological strategy may be generalized to investigate the significance of uncertainty in social scientific measurements, although we defer the implementation of this project to future work.

accomplish? Are their other ways to accomplish that, available to and used by both scientists before the development of quantitative techniques and scientists today?

Philosophical approaches to measurement have tended to treat uncertainty estimation as ancillary to the measurement result, serving to quantify the quality of the measurement (International Bureau of Weights and Measures (BIPM), 1993, 25). More recent views recognize the central role played by uncertainties in measuring and communicating measurement results. Eran Tal notes that uncertainty is not merely "extrinsic" to measurement or a reflection of its "imperfections" (Tal, 2012, 28). One also finds precedent in the writings of metrologists for the idea that a reported measurement uncertainty provides a "constitutive component" of the measurement result (Mari et al., 2023, 67).

In recent studies focused on quantitative estimates of measurement uncertainty in the physical sciences, we have defended a view that makes uncertainty estimates crucial to producing useful measurement results, on the grounds that uncertainties enable investigators to solve the problem of measurement usefulness (Beauchemin and Staley, 2025, 2026). This problem arises from a tension between two features of scientific measurement: (1) In measuring, inquirers aim to achieve a result that enjoys evidential support. (2) The results of measurement are meant to be useful as evidential support in subsequent inquiries. Achieving the first aim requires relating the measurement result closely to the conditions in which it was produced; the result is warranted under those conditions. Achieving the second aim requires the ability for inquirers in other conditions to rely on the result as a resource for their own inquiries, conducted under different conditions. One might pose the problem of usefulness thus: How do specific and concrete measurement procedures executed in one context produce results that can be

used in the conduct of scientific inquiry in a broader range of contexts?

To understand how scientists address the problem of usefulness, we adopt a view of measurement as inquiry: (1) Measurement is inquiry directed at the aim (among others) of learning what value(s) one may attribute to something treated as evaluable by a community of scientists and subject to normative constraints. (2) Every measurement result includes, implicitly or explicitly, an evaluation of uncertainty. (3) The uses of a result in subsequent inquiries are guided by its sensitivity to the targets of such inquiries. Each of these claims plays an integral role in allowing measurement to produce useful results; here we focus on claim (2).

The integral role of uncertainty in measurement arises from underdetermination in measurement. Measurement inquiry involves resources, tasks, and objectives. These are related to one another through relations of interdependence and feedback. Measurement objectives do not determine the resources used and tasks performed in a given measurement. Thus, deciding how to conduct a given measurement constitutes a coordination problem involving choices regarding all three elements. Different choices might lead to different results. Such variability threatens to render a measurement result unwarranted, or useless in a context that differs from that in which it was produced.

Uncertainty estimation responds to the resulting problem of usefulness: On the basis of a critical mode inquiry into the potential variability of measurement results arising from the aforementioned underdetermination, investigators can secure their result (Staley, 2020), while also communicating useful information to potential users of measurement results, by incorporating an estimate of such variability into the measurement result itself.

In what follows we will consider three measurements of the speed of light, from 1728

to 1876, that address the same problem through critical mode scrutiny. The cases vary in the way they report their results; only one includes a quantified estimate of uncertainty in the result. None of them treat uncertainty in the way that would be expected of a similar measurement today. We will show how all three incorporate a critical inquiry into the performance of measurement that serves the same ends, irrespective of whether an uncertainty estimate is included in the result. These uncertainty inquiries play equally central roles in securing the usefulness of their respective measurement results.

3 Summary of cases

The speed of light c, a fundamental parameter of physics, is considered constant in the vacuum, and universal for any observer in any inertial frame. While, today, the value of the speed of light is taken to be exact, and is used to define the meter standard, its value of $299,792,458\ m/s$ was fixed in 1983 based on the most precise measurement at the time ($c = 299,792,457.4 \pm 1.1\ m/s$ (Evenson et al., 1972)), and its agreement with measurement results obtained in other laboratories (Blaney et al., 1974). These values for the speed of light constitute the culmination of a centuries-long sequence of measurements, of increasing precision and accuracy, using various techniques and principles.

James Bradley set out to measure stellar parallax in 1728 (Bradley, 1729). He wound up discovering a phenomenon that disclosed the need to account for the speed of light in making precise astronomical observations, but his method for determining that speed relied on assumptions drawn from astronomy. The first laboratory measurement of c that did not rely on astronomical parameters was performed by Hippolyte Fizeau in

1849 (Fizeau, 1849a). None of the three reports of Fizeau's measurement results (Fizeau, 1849a,b, 1851) include an uncertainty estimate on c. In contrast, Marie Alfred Cornu, who replicated Fizeau's approach 27 years later, devoted a large fraction of his 300-page report to uncertainty, leading him include an estimate of it in his result: $c = 300,400 \pm 300\ km/s$ (Cornu, 1876). The absence of uncertainty estimates in Fizeau, in contrast to Cornu's intense focus on uncertainty as a relevant quantity, reflects well the concern we expressed in our introduction.

Our discussion will proceed in reverse chronological order, beginning with Cornu's detailed and thorough report.

In the Part I of his paper, Cornu discusses the principles of the measurement process he borrowed from Fizeau and improved to get a value for the speed of light. This method consists of sending a light beam through the indentations of a rotating cogwheel to a distant mirror that reflects the beam back to the observer, located at the source. Depending on the rotational velocity of the cogwheel, that beam will be blocked or passed through the next indentation. Calculating ratios of angular velocities for sequences of bright and dark spots observed at the terminus allows Cornu, like Fizeau before him, to extract a value for the speed of light. Cornu acknowledges the large underdetermination of the measurement results using Fizeau's method, and even argues that this allows the elimination of human errors inherent to any method of observation. Consequently, Cornu explains very carefully how various sources of error related to the measurement method can prevent realizing the theoretical conditions of the measurement model deployed in the inquiry, as well as how these errors could be mitigated, helping achieve the targeted level of precision (0.1%). No less than 75 pages are dedicated to such an error discussion, including a discussion of errors due to

conditions of observation of the light signal, as well as errors due to inhomogeneities in the main apparatus parameters (e.g. size of the cogwheel, and its rotational velocity). Yet, this discussion does not lead to an estimate of the uncertainty related to the individual sources considered. Instead, Cornu discusses the insignificance of the errors discussed, the robustness of the measurement approach, the justification of the assumptions used despite the source of error considered, or the techniques or models used to assess and mitigate sources of error. For instance, Cornu made claims like "the small accidental inequalities in the dents of the wheel have no influence on the measurement result when the method of double observation is used" (Cornu, 1876, 42).

The second part of the paper discusses in detail Cornu's experimentation, including ancillary measurements of input parameters used in calculating c. For many sources of error, Cornu explains how including a correction factor mitigates the error, how he models that correction factor, and how the resulting uncertainty after correction remains insignificant. For example, in his discussion on the estimate of the distance between the two stations used as the source and reflection points of the light signal, Cornu explains carefully his conclusion that various errors affecting the determination of that distance largely offset each other, leaving a residual uncertainty negligible for the targeted precision of the complete experiment (Cornu, 1876, 121).

Throughout the paper, Cornu discusses experimental tests he performed to verify a given uncertainty estimate (e.g., his discussion of the regularity of the chronograph, pp. 211-216), and cross-checks to verify, often in multiple ways, the measurement models he relies on (e.g., the geometric calculation of distance between the ocular and objective of the microscope used). However, his uncertainty estimate strategy does not consist in estimating the contribution to the total uncertainty of each source considered and then

adding them up. Instead, Cornu argues that each source of error considered is sufficiently bounded to be consistent with his overall objective of measuring the speed of light at a precision of 0.1%. The uncertainty value quoted in the result is simply the precision he targeted when he set out to measure, the upper bound on all the investigations he carried out in a critical mode of inquiry.

As mentioned above, Fizeau does not quote any uncertainty on his measurement result. The text that first reports his measurement is very succinct (Fizeau, 1849a). Yet, there are hints that mitigation of sources of error, variability of the results due to conditions of experimentation, and assessment of the performance of the measurement in relation to its objectives are central concerns for which Fizeau has accounted. Fizeau highlights the fact that he obtains consistent results by varying parameters of his experiment, such as the size of the optical aperture, the distance between the source and the mirror, the brightness and darkness of the observed light spot, and the speed of the wheel. Most importantly, Fizeau concludes by promising a more detailed report of all the circumstances he studies in his experiment for the members of the Académie des Sciences to scrutinize, implicitly acknowledging that such details are essential to the process. A report to the Revue Scientifique et Industrielle, the same year, adds a few more details about Fizeau's verbal report at the Academie des Sciences (Fizeau, 1849b). For example, it clarifies Fizeau's experimental goals of producing a measurement of the speed of light that does not depend on assumptions about astrophysical systems, which are hard to validate, but to rely on an instrumental setup fully accessible to the experimenters. In a nutshell, Fizeau sought to avoid the largest systematic effects possibly affecting results obtained by Bradley (1728), and Rømer (1676) before him. In addition, that report insists on Fizeau's concern to precisely control the input value of

the distance between the optical devices used in the experiment, and eliminate as much as possible confounding factors potentially affecting the observation of bright and dark spots when the speed of the wheel is changed. Finally, the report mentions that Fizeau repeated his measurement 28 times and took the average of these results, a typical method used to mitigate random errors on a measurement. Despite being short and scarce in details, these reports indicate that his process for measuring the speed of light includes considerations similar to those that guided Cornu in his inclusion of an uncertainty estimate in his result, and similar to Cornu's warranting that the measurement met its targeted objective.

It is finally interesting to note that such a practice of assessing elements of underdetermination of the measurement result and of assessing their impact on that result can be found much earlier in the history of science. In Bradley's report on his measurement of the speed of light in 1728 (Bradley, 1729), we can see similar considerations, including something closer to an uncertainty estimate than what Fizeau provided. First, as with Cornu and Fizeau, Bradley specified that his measurement is seeking precision. Bradley's intially targeted quantity, though, was a measurement of parallax rather than the speed of light. He states his initial objectives as "verifying and confirming those [observations], that Dr. Hook formerly communicated to the publick, which seemed to be attended with Circumstances that promised greater Exactness in them, than could be expected in any other, that had been made and published on the same Account" (Bradley, 1729, 637). After explaining the context motivating the measurement he performed, he presents the principles of his measurement (ibid., pp. 646-648). The idea was to measure the angular difference in the observed positions of fixed stars at different moments of the year to extract a measurement of the speed of

light in terms of Earth's velocity around the Sun.[2] He also details how his measurement was meant to address its scientific objectives, and what were the conditions in which he performed his measurement. Bradley discusses different sources of variation possibly affecting his results, and how these are either mitigated or do not impact his observational conclusions. For example, he notes that his calculation assumes a circular orbit for Earth rather than an elliptical one "to avoid too perplexed a Calculus", but notes that the result "would not sensibly differ" (ibid., p. 651). Bradley then discusses a cross-check using stars of different Magnitudes in his calculation of the speed of light, and concludes that he found "the same Velocity of Light" from his Observations (ibid., p. 654). Finally, Bradley explicitly discusses the uncertainty stemming from the instrument he used in his observation (ibid., p. 661) and the conditions in which they have been used (e.g., apparent variations of the observed stars' position due to the "tremulous or undulating Motion of the Air," ibid., p. 657) and quotes an uncertainty of 2" in the observed position of the stars used in his calculation of the speed of light. Bradley does not propagate this uncertainty estimate to his measurement result for the speed of light, but clearly demonstrates that he engaged in the same critical mode of inquiry as Cornu (who did use his systematic studies to quote a final uncertainty on his measurement of c), and as contemporary physicists do in producing uncertainty estimates.

What the cases of Cornu, Fizeau, and Bradley illustrate well is that measurement

[2]The light from a distant source appears to be from a different location for a moving telescope due to the finite speed of light. The greater precision of Bradley's observations enabled him both to recognize the phenomenon – the aberration of light – and exploit it for his measurement (Fisher, 2010).

inquiries are concerned with the sources of variability that can lead to different attributed values to the measurand. Such considerations are critical to the acceptability of a result to the relevant community of inquirers. In many of the examples of underdetermination considerations presented above, the inquirers perform ancillary investigations on the source of variability to assess their possible impact on the measurement result at stake. Often, they develop models to quantify how these variations could change the measurement results, to assess their significance. These physicists also pursue validating and stress-testing the assumptions at the core of their measurement design, and testing the precision of their instrument and of the parameter values they used as input to their measurement process. They assess the quality of their own measurement with respect to the goals of their inquiry. They even investigate the assessment processes they deploy to warrant their conclusions. Finally, reporting these investigations and considerations, and submitting them to the judgment of their peers was critical in all of the cases discussed above, whether or not an uncertainty was quoted on the final result. The main differences in uncertainty reporting in these cases is that Fizeau did not quote any uncertainty, but promised to come back later to do a better job, Bradley estimated an upper limit on the uncertainty coming from different sources but did not include them in his final result for the speed of light, and Cornu quoted an uncertainty of 0.1% on c corresponding to the upper limit he tolerated for each source of error possibly affecting the result.[3] In the end, the work done by the three physicists was

[3]Cornu's approach to the uncertainty on his measurement relies on an argument about the comparatively small – though not explicitly quantified – contributions of numerous sources. In this respect, his approach differs significantly from the recommendations of metrologists today Joint Committee for Guides in Metrology Working Group I (2008).

methodologically comparable, with the decision whether to estimate uncertainty in the final result conditioned by the objectives of each inquiry. These objective-oriented decisions are also governed by contingent norms of the research traditions within which investigators operate, that evolve over time. This entire process is integral to measurement, and plays an essential role in making measurement results useful, even in situations in which uncertainties are not estimated.

4 Measurement uncertainty without quantification

Our purpose in presenting the three cases discussed is twofold: First, we use these cases to motivate and illustrate our analysis of the epistemological contribution of uncertainty assessment to measurement, and our contention that such assessment is crucial to measurement in the physical sciences. Second, these cases demonstrate how an uncertainty inquiry can make such an epistemological contribution even when the stated measurement result does not include an estimate of uncertainty (Bradley, Fizeau), and when it does include such an estimate, but that estimate has been produced in a way that, by current metrological standards, might be judged inadequate (Cornu).

By examining cases such as this through the lens of the problem of usefulness, certain epistemological features come into focus that allow us to clarify our treatment of measurement as inquiry.

First, we can now clarify what we mean by the terms *uncertainty inquiry*, *uncertainty assessment*, and *uncertainty estimate*.

An *uncertainty inquiry* is an inquiry, constituting a part of a measurement inquiry,

Yet it does real epistemological work for him.

conducted in the critical mode. An uncertainty inquiry investigates sources of variability of the measurement result, including those arising from underdetermination of the measurement procedure. Resources used in such an inquiry may include (among other things) prior empirical knowledge – whether homespun (the susceptibility of human observers to error) or sophisticated (Cornu's detailed mathematical modeling of the dentation in his rotating wheel) – as well as ancillary experimentation and measurement (Cornu's scrutiny of and reliance on the distance measurement between the source and reflection point) and explicit variation of tasks or resources (Fizeau's variation of numerous experimental parameters; Bradley's cross-check of his calculation using stars with different magnitudes). The objective of an uncertainty inquiry is to enhance the usefulness of the measurement, both insofar as the measurement result is rendered relevant across a broader range of contexts of use and insofar as the report of the measurement inquiry, by incorporating a report of the uncertainty inquiry, *warrants* to the user that the measurement result can be relied upon for such use.

An *uncertainty assessment* is a report of the outcome of such an uncertainty inquiry. An *uncertainty estimate* is an uncertainty assessment that is expressed quantitatively.[4]

With this terminology in place, looking at uncertainty through the lens of the problem of usefulness, we see how uncertainty inquiries incorporate a variety of strategies and practices that are critical for the usefulness of a measurement result because they address the underdetermination of the measurement procedure and provide resources for those who may seek to use the measurement result. By carrying out and documenting

[4]Because the term 'assessment' can be used to refer to both a process and the product of a process, one may – as we sometimes do – use the same term to refer to both the uncertainty inquiry and the report of its outcome. Similar comments apply to 'estimate.'

the extent and the limitations of the measurer's efforts to secure their result, measurers enable potential users to judge the suitability of the result for their purposes. As the cases discussed here document, these objectives can be met in the absence of an uncertainty estimate, or by uncertainty estimates produced by procedures that fall short of metrologists' contemporary standards of rigor.

How an investigator chooses to conduct an uncertainty inquiry will depend to a significant extent upon the objectives of the measurement, as those relate both to the potential contexts of use of a result and to the demands of warranting of the result with regard to the expectations of the audience. Bradley's stated objective consists initially of "verifying and confirming" the results of Hooke's 1669 attempt to measure stellar parallax,[5] but the course of his inquiry leads to the undoing of Hooke's result, which failed to account for the finite velocity of light. For Fizeau, it is important to measure c in a manner that improves upon Bradley's measurement by being independent of inadequately supported astronomical assumptions, thus enabling the use of the result to test various theories of the ether (Fizeau, 1851). Cornu's objective was to improve upon Fizeau's measurement enough to reach a level of precision of 0.1%.

Furthermore, these three measurements all illustrate how historically situated norms make a difference to the conduct of an uncertainty inquiry.

[5]Fisher (2010) casts significant doubt on Bradley's claimed objective, arguing for a more complex picture of what Bradley was seeking to accomplish.

5 Conclusion

We have argued that uncertainty assessments are crucial for measurement in the physical sciences. Such assessments are the outcome of uncertainty inquiries, which constitute a critical mode component of measurement inquiries, serving to warrant the usefulness of measurement results. How exactly an uncertainty inquiry is conducted will depend on both the objectives of the inquiry and the norms that govern practices of inquiry in the relevant community of inquirers. Uncertainty assessments communicate important and relevant information regarding measurements to potential users, and they can serve this function without taking the quantitative form of an uncertainty estimate.

Our argument does not deny that uncertainty estimation can produce epistemic value beyond what can be achieved with a strictly qualitative discussion of uncertainty. A quantitative estimate enables a much finer discrimination regarding the *sensitivity* of a measurement result, such as would help in determining its potential use for testing among competing hypotheses. It is hard to imagine how one could even consider deciding to fix the speed of light as a constant to be used in defining the system of units for measuring other quantities, were it not for the history of measuring c with ever-smaller uncertainties that led up to that decision. In the absence of such quantification, fixing c as a constant would amount to a decision to off-load unquantified and uncontrolled variabilities into the measurement of any other quantity relying on that definition.

We have relied on examples from the history of physical science to illustrate our claims about non-quantitative uncertainty assessment. But one can easily find measurements in the physical sciences today that do not include uncertainty estimates. Our approach provides an epistemological strategy for the analysis of such cases.

References

Beauchemin, P.-H. and K. W. Staley (2025). Making measurement useful: integrating measurement, uncertainty, and sensitivity. *Synthese 206*(3), 110.

Beauchemin, P.-H. and K. W. Staley (2026). How uncertainty and underdetermination allow measurement to produce useful results. *Measurement 267*, 120330.

Blaney, T., C. Bradley, G. Edwards, B. Jolliffe, D. Knight, W. R. Rowley, K. Shotton, and P. Woods (1974). Measurement of the speed of light. *Nature 251*, 46.

Bradley, J. (1729). Account of a new discovered Motion of the Fix'd Stars. *Philosophical Transactions 35*, 637–660.

Cornu, M. A. (1876). *Détermination de la vitesse de la lumière: d'après des expériences exécutées en 1874 entre l'Observatoire et Montlhéry*. Gauthier-Villars.

Evenson, K., J. Wells, F. Petersen, B. Danielson, G. Day, B. R.L., and J. Hall (1972). Speed of Light from Direct Frequency and Wavelength Measurements of the Methane-Stabilized Laser. *Phys. Rev. Lett. 29*(19), 1346.

Fisher, J. (2010). Conjectures and reputations: The composition and reception of James Bradley's paper on the aberration of light with some reference to a third unpublished version. *The British Journal for the History of Science 43*(1), 19–48.

Fizeau, H. (1849a). Sur une expérience relative à la vitesse de propagation de la lumière. *Comptes rendus hebdomadaires des séances de l'Académie des sciences (Paris) 29*(4), 90–92.

Fizeau, H. (1849b). Sur une expérience relative à la vitesse de propagation de la lumière. *Revue scientifique et industrielle 5*(3), 393–397.

Fizeau, H. (1851). Sur les hypothèses relatives à l'éther lumineux, et sur une expérience qui paraît démontrer que le mouvement des corps change la vitesse avec laquelle la lumière se propage dans leur intérieur. *Comptes rendus hebdomadaires des séances de l'Académie des sciences (Paris) 23*(13), 349–355.

International Bureau of Weights and Measures (BIPM) (1993). *International Vocabulary of Basic and General Terms in Metrology*. International Bureau of Weights and Measures.

Joint Committee for Guides in Metrology Working Group I (2008). *Evaluation of Measurement Data – Guide to the Expression of Uncertainty in Measurement*. Joint Committee for Guides in Metrology.

Mari, L., M. Wilson, and A. Maul (2023). *Measurement across the Sciences: Developing a Shared Concept System for Measurement* (2nd ed. 2021. ed.). Springer Series in Measurement Science and Technology. Cham: Springer International Publishing.

Porter, T. (2020). *The Rise of Statistical Thinking* (New ed.). Princeton, NJ: Princeton University Press.

Staley, K. W. (2020). Securing the empirical value of measurement results. *British Journal for the Philosophy of Science 71*(1), 87–113.

Tal, E. (2012). *The Epistemology of Measurement: A Model-Based Account*. Ph. D. thesis, University of Toronto, Toronto.