On the Multiple Deaths of Whitehead's Theory of Gravity

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ABSTRACT

Whitehead's 1922 theory of gravitation continues to attract the attention of philosophers, despite evidence presented in 1971 that it violates experiment. We demonstrate that the theory strongly fails five quite different experimental tests, and conclude that, notwithstanding its meritorious philosophical underpinnings, Whitehead's theory is truly dead.

1. Introduction and summary

In 1922, the distinguished mathematician and philosopher Alfred North Whitehead (1861-1947) , then in his 60th year, published a relativistic theory of gravity with the property, which it shares with Einstein's theory, of containing no arbitrary parameters. Furthermore, when suitably interpreted, it yields the same predictions as General Relativity (GR), not only for the three classic tests of light bending, gravitational redshift and the precession of the perihelion of Mercury, but also for the Shapiro time delay effect (Shapiro 1964), recently confirmed to one part in 10^5 (Bertotti *et al.* 2003).

The reason for this coincidence was realized early on by Eddington (1922). In the case of vanishing cosmological constant the Schwarzschild solution is not only an exact solution of Einstein's theory, it is an exact solution of Whitehead's theory as well. Thus it gives the same predictions for the parametrized post-Newtonian (PPN) parameters $\gamma = \beta = 1$. Eddington's remark nicely explained an observation of Temple (1924) that the predictions of the precession of the perihelion for the two theories agree exactly, and gave rise to the (incorrect) idea that it is indistinguishable

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from GR, a point refuted by Harvey (1964) by the observation that Birkhoff's theorem fails for Whitehead's theory: the field outside a spherically symmetric source is not just given by the Schwarzschild solution but in general contains an additional constant of integration which is in principle measurable.

In fact an even stronger statement can be made. This remarkable correspondence of exact solutions extends to the Kerr solution (Russell & Wasserman 1987) and thus to the corresponding Lense-Thirring or frame dragging effects (Rayner 1955a). Thus experiments such as that involving the LAGEOS satellites (Ciufolini & Pavlis 2004) which have verified the effect at the 10-15% level and the ongoing NASA-Stanford Gravity Probe B superconducting gyroscope experiment, which aspires to an accuracy of 1%, cannot distinguish Whitehead's from Einstein's theory on the basis of frame dragging (we will see below that LAGEOS actually tests Whitehead because of the failure Birkhoff's theorem).

The mathematical explanation for this striking, but accidental, coincidence is that both the Schwarzschild solution and the Kerr solution may be cast in Kerr-Schild form (Kerr & Schild 1965). That is, coordinates exist for which

$$g_{\mu\nu} = \eta_{\mu\nu} + l_\mu l_\nu, \tag{1}$$

where

$$\eta^{\mu\nu} l_\mu l_\nu = g^{\mu\nu} l_\mu l_\nu = 0\,. \tag{2}$$

and l_μ is tangent to a null geodesic congruence,

$$l_{[\mu;\nu} l^\nu l_{\lambda]} = 0\,, \tag{3}$$

where l^μ is obtained from l_μ by index raising using either the metric $\eta_{\mu\nu}$ or the metric $g_{\mu\nu}$. It follows (Kerr & Schild 1965) that

$$h_{\mu\nu} = l_\mu l_\nu \tag{4}$$

satisfies the *linearized* Einstein equations. If this can be chosen to agree with Whitehead's retarded solution, then his metric and that of Einstein will agree exactly.

Thus for a single particle at rest at the origin, in spherical polar Minkowski coordinates t, r, θ, ϕ, Whitehead's metric is

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) + \frac{2M}{r}(dt - dr)^2. \tag{5}$$

On the other hand, the Schwarzschild metric, in standard Schwarzschild coordinates (T, r, θ, ϕ) is

$$ds^2 = -(1 - \frac{2M}{r})dT^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\,. \tag{6}$$

If we set

$$t = T - M\ln(\frac{r}{2M} - 1)\,, \tag{7}$$

the two metrics are seen to coincide, which is Eddington's observation (Eddington 1922). If we define $u = t - r$, then the coordinates u, r, θ, ϕ are nowadays referred to as outgoing Eddington-Finkelstein coordinates. Thus Whitehead's spacetime manifold is geodesically incomplete with respect to his curved metric because outgoing Eddington-Finkelstein coordinates cover only the lower half of the full Kruskal manifold. The surface $r = 2M$ is the past event horizon, and Whitehead's particle is naked and corresponds to what is now called a White Hole (Harvey 1964), the time reverse of a Black Hole. The Kerr solution is also of Kerr-Schild form and is also an exact solution of Whitehead's metric (Russell & Wasserman 1987) when expressed in terms of advanced null coordinates. It corresponds therefore to a naked rotating White Hole. For strong fields therefore, even if a single object is considered, the two theories would be expected to make very different predictions. Note also that this exact correspondence between solutions of Whitehead's theory and solutions of Einstein's theory holds only for a special class of solutions. Not every solution $h_{\mu\nu}$ of linear theory may be cast in the form (4) such that (2) and (3) hold, i.e to be of Kerr-Schild form. Moreover not every solution in the Kerr-Schild class need be of the retarded form specified by Whitehead. Thus there is no general agreement between the predictions of Einstein and those of Whitehead.

In any case, for many years the two theories were considered to be experimentally indistinguishable, and this gave rise to much philosophical discussion as to whether additional criteria, for example aesthetic considerations or philosophical preconceptions, were needed in order to reject or accept one of them. This is brought out in Broad's review of Whitehead's book *The Principle of Relativity* (Broad 1923), and a particularly clear discussion indicative of the mood in the late 1950's is that of Bonnor (1958).

From today's perspective, one can say that the principal difference between Einstein and Whitehead is the latter's insistence on fixed *a priori* spatio-temporal relations, which in practice meant the adoption of a fixed, and in particular unobservable, background Minkowski spacetime. This is stated with admirable clarity by the philosopher John Bain (1998), who provides a valuable account of how Whitehead's ideas about relativity were embedded in his overall philosophy of nature (see also Tanaka (1987)).

In fact, by the late 1960's the promise of new technology had led to a more optimistic, empirical viewpoint, and rival theories of gravity were carefully scrutinised both for internal consistency and for testable predictions additional to the three classic tests. An important milestone was Shapiro's time delay prediction (Shapiro 1964). An outcome of this line of research was the discrediting by Will (1971b) of Whitehead's theory.

However, Whitehead's philosophical ideas continue to attract widespread attention, often under the rubric of Process Philosophy, and perhaps because of his formidable achievements in the foundations of mathematics and logic. He was after all co-author with Bertrand Arthur William Russell (1872-1970) of the epoch making *Principia Mathematica*. As a result, many of his followers have been reluctant to abandon his theory of gravity despite the growing observational evidence against it.

Will's original disproof of Whitehead's theory was based on the fact that Whitehead's theory predicts an anisotropy in the "locally measured" Newton's constant due to distant matter. Thus a mass M at a distance r from the Earth produces an effective Newton's contant

$$G_{eff} = G\left(1 + \frac{2GM}{rc^2} + \frac{GM}{rc^2}\cos^2\theta\right) . \tag{8}$$

where θ is the angle between the Earth's radial direction and the distant gravitating body. This would produce anomalous Earth tides that would show up in gravimeter experiments, yet there was no experiment evidence for such effects (Warburton & Goodkind 1976). As a critique of Will's argument, it was pointed out that the resultant Earth tides depend on the distribution of extra-solar system matter (Mentock 1996) whose distribution is uncertain, and so a cancellation might take place. However, as we shall show below, allowing for these uncertainties will not change the predicted effect sufficiently to invalidate Will's argument. Another attempt to avoid Will's argument was to change the interpretation (Reinhardt & Rosenblum 1974; Hyman 1989). In Chiang & Hamity (1975) it was shown that the re-interpretation of Reinhardt & Rosenblum (1974) would not achieve this goal, and they obtained the same result for the anisotropy of Newton's constant (8) as did Will. These general conclusions, while accepted by Bain (1998), were rejected by Fowler (1974) and the latter's remarks were reiterated by Tanaka (1987). Similar reservations have been expressed by Russell & Wasserman (1987).

In fact, normally in science a single incorrect prediction is regarded as sufficient grounds for rejecting a theory; hence the well known dictum of "Darwin's Bulldog"

> The great tragedy of Science - the slaying of a beautiful hypothesis by an ugly fact.
>
> Thomas H. Huxley (1825 - 1895)

By contrast, as we are reminded by Popper (1959), the confirmation of a theory is never complete. The best one can do is to subject it to increasingly precise and exacting tests covering a wider and wider range of phenomenona and circumstances.

It turns out that Whitehead's theory is definitely excluded by several modern experiments, and our aim in this article and the reason for our title is to point out that any one of them is sufficient for rejection. In other words judged by modern scientific and technological standards, Whitehead's theory, beautiful as it may seem in the eyes of many of its beholders, is truly dead. By contrast, Einstein's theory passes all of these tests with flying colors.

Specifically, Whitehead's theory fails five tests, most of them by many orders of magnitude

1. *Anisotropy in G.* We have reexamined Will's 1971 derivation, incorporating a model for the mass distribution of the galaxy that includes a dark matter halo. The predicted effect is still at least 100 times larger than the experimental bound.

2. *Nordtvedt effect and lunar laser ranging.* Whitehead's theory predicts that massive, self-gravitating bodies violate the weak equivalence principle in that their acceleration in an external gravitational field depends on their gravitational binding energy (Nordtvedt effect). The predicted size is 400 times larger than that permitted by lunar laser ranging.

3. *Gravitational radiation reaction and the binary pulsar.* The theory predicts anti-damping of binary orbits due to gravitational radiation reaction at a level $(v/c)^3$ beyond Newtonian gravity, in contrast to the $(v/c)^5$ damping effect in GR. Thus it strongly violates binary pulsar data by about four orders of magnitude, and with the wrong sign.

4. *Violation of Birkhoff's theorem, and LAGEOS satellites.* The static, spherically symmetric solution of the theory for finite sized bodies has an additional contribution dependent on the body's size (Harvey 1964; Rayner 1954; Synge 1952). This produces an additional advance of the perigee of the LAGEOS II satellite, in disagreement with observations by a factor of 10.

5. *Momentum conservation and the binary pulsar.* Whitehead's theory predicts an acceleration of the center of mass of a binary system, a violation of momentum conservation (Clark 1954). Precise timing of the pulsar B1913+16 in the Hulse-Taylor binary pulsar rules out this effect by a factor of a million.

Any of these tests alone would have been enough to kill Whitehead's theory, so collectively they amount to overkill. On the other hand they illustrate both the precision and depth that modern technology has brought to the problem of testing gravity, and serve as a warning to any would-be inventor of an alternative gravity theory, or to anyone who might hope that a suitably modified or reinterpreted Whiteheadian theory would pass muster (Schild 1956; Hyman 1989; Reinhardt & Rosenblum 1974). It is not sufficient to check the "classic tests" of light bending, perihelion advance of Mercury, and gravitational redshift. There is now an exhaustive battery of empirical checks that must be done.

The remainder of this paper provides some technical details to support these conclusions. Throughout, we adopt the "canonical" version of Whitehead's theory, specified as follows. One first assumes the presence of a flat background metric $\eta_{\mu\nu}$, whose Riemann tensor vanishes everywhere. This background metric defines null cones for any chosen spacetime event x^μ, given by points x'^μ satisfying

$$\eta_{\mu\nu} y^\mu y^\nu = 0\,, \quad y^\mu = x^\mu - x'^\mu\,. \tag{9}$$

The physical metric $g_{\mu\nu}$ is then given by (henceforth we use units in which $G = c = 1$)

$$\begin{aligned}
g_{\mu\nu}(x^\alpha) &\equiv \eta_{\mu\nu} - 2\sum_a m_a \frac{(y_a^-)_\mu (y_a^-)_\nu}{(w_a^-)^3}\,, \\
(y_a^-)^\mu &= x^\mu - (x_a^-)^\mu\,, \\
\eta_{\mu\nu}(y_a^-)^\mu (y_a^-)^\nu &= 0\,, \\
w_a^- &= \eta_{\mu\nu}(y_a^-)^\mu (dx_a^\nu/d\sigma)^-\,,
\end{aligned}$$

$$d\sigma^2 \;=\; \eta_{\mu\nu}dx^\mu dx^\nu \,, \tag{10}$$

where the sum is over over all particles, with rest mass m_a. Indices on $(y_a)^\mu$ are raised and lowered using $\eta_{\mu\nu}$. The quantities $(x_a^-)^\mu$, $(dx_a^\nu/d\sigma)^-$ are to be evaluated along the past flat null cone of the field point x^μ.

Following Synge (1952), we assume that matter fields couple *only* to the physical metric $g_{\mu\nu}$. This makes Whitehead's theory a "metric theory" of gravity (see Will (1993) for discussion). As such, it automatically satisfies the Einstein Equivalence Principle (EEP), which has been verified to extremely high precision using laboratory Eötvös-type experiments (parts in 10^{13}) and gravitational redshift experiments (parts in 10^4), among others. The background metric then has no further direct physical consequences, apart from its role in defining the physical metric. This point mass expression can be generalized to continuous fluids in a straightforward way (Will 1993).

2. Post-Newtonian limit and gravitational radiation reaction in Whitehead's theory

2.1. Solution of Whitehead's theory to 1.5 post-Newtonian order

We wish to evaluate the Whitehead metric within the near-zone of a slow-motion gravitating system, in order to derive the equations of motion. This corresponds to field points such that $|\mathbf{x}| \ll \lambda \sim R/v$, where λ is roughly a gravitational wavelength, R and $v \ll 1$ are the characteristic size and internal velocity of the system. Accordingly, we want to evaluate $g_{\mu\nu}$ at $(t, \mathbf{x})$ in terms of source varables $\mathbf{x}_a$ evaluated at the *same* time t. We make the standard assumption of post-Newtonian theory that $v^2 \sim m/r \sim \epsilon$, where ϵ is a small parameter used for bookeeping purposes. Our goal is to determine the metric through 1.5 post-Newtonian order, or to order $\epsilon^{3/2}$ beyond Newtonian gravity; this involves evaluating g_{00} through $O(\epsilon^{5/2})$, g_{0j} through $O(\epsilon^2)$, and g_{ij} through $O(\epsilon^{3/2})$. This will include the usual post-Newtonian terms relevant for solar-system tests, as well as, it will turn out, the leading effects of gravitational radiation reaction in this theory.

We expand the *retarded* position of the a-th particle by

$$\begin{aligned} \mathbf{x}_a^- \;&\equiv\; \mathbf{x}_a(t - |\mathbf{x} - \mathbf{x}_a^-|) \\ &\approx\; \mathbf{x}_a - \mathbf{v}_a|\mathbf{x} - \mathbf{x}_a^-| + \frac{1}{2}\mathbf{a}_a|\mathbf{x} - \mathbf{x}_a^-|^2 + \ldots \,, \end{aligned} \tag{11}$$

where $\mathbf{x}_a$, $\mathbf{v}_a$ and $\mathbf{a}_a$ are the position, velocity and acceleration of the a-th particle at the field-point time t. We can then expand the spatial component $(y_a^-)^i = (x - x_a^-)^i$ in terms of the instantaneous difference $z_a^i \equiv (x - x_a)^i$ according to

$$(y_a^-)^i = z_a^i + \epsilon^{1/2} v_a^i y - \frac{1}{2}\epsilon a_a^i y^2 + \frac{1}{6}\epsilon^{3/2}\dot{a}_a^i y^3 + O(\epsilon^2)\,, \tag{12}$$

where $y \equiv |\mathbf{y}_a^-|$. We also expand the retarded velocity component $(v_a^-)^i \equiv (dx_a^i/dt)^-$ according to

$$(v_a^-)^i = v_a^i - \epsilon^{1/2} a_a^i y + \frac{1}{2}\epsilon \dot{a}_a^i y^2 + O(\epsilon^{3/2})\,. \tag{13}$$

Note that, because the quantity $(y_a^-)^\mu$ is null with respect to the flat metric, $(y_a^-)^0 = y$, and thus

$$w_a^- = (dt/d\sigma)^-(-y + \epsilon^{1/2}\mathbf{y}_a^- \cdot \mathbf{v}_a^-)\,. \tag{14}$$

The foregoing expressions can then be iterated to the required order in ϵ to convert all expressions into functions of $\mathbf{v}_a$, $\mathbf{a}_a$, $\dot{\mathbf{a}}_a$, $\mathbf{z}_a$, and $z_a = |\mathbf{z_a}|$. The result is

$$g_{\mu\nu} = \eta_{\mu\nu} + \epsilon \sum_a m_a (h_a)_{\mu\nu}\,, \tag{15}$$

where

$$\begin{aligned}
(h_a)_{00} &= \frac{2}{z_a} + \epsilon^{1/2}\frac{4\mathbf{v}_a\cdot\mathbf{z}_a}{z_a^2} + \epsilon\left[2\frac{v_a^2}{z_a} - 5\frac{\mathbf{a}_a\cdot\mathbf{z}_a}{z_a} + \frac{(\mathbf{v}_a\cdot\mathbf{z}_a)^2}{z_a^3}\right] \\
&\quad + \epsilon^{3/2}\left[\frac{8}{3}\dot{\mathbf{a}}_a\cdot\mathbf{z}_a - 2\mathbf{v}_a\cdot\mathbf{a}_a + 6\frac{v_a^2\mathbf{v}_a\cdot\mathbf{z}_a}{z_a^2} - 12\frac{\mathbf{v}_a\cdot\mathbf{z}_a\mathbf{a}_a\cdot\mathbf{z}_a}{z_a^2} - 4\frac{(\mathbf{v}_a\cdot\mathbf{z}_a)^3}{z_a^4}\right] + O(\epsilon^2)\,, \\
(h_a)_{0j} &= -\frac{2z_a^j}{z_a^2} - \epsilon^{1/2}\left[2\frac{v_a^j}{z_a} + 2\frac{\mathbf{v}_a\cdot\mathbf{z}_a z_a^j}{z_a^3}\right] \\
&\quad + \epsilon\left[a_a^j + 4\frac{\mathbf{a}_a\cdot\mathbf{z}_a z_a^j}{z_a^2} - \frac{v_a^2 z_a^j}{z_a^2} + 2\frac{(\mathbf{v}_a\cdot\mathbf{z}_a)^2 z_a^j}{z_a^4} - 4\frac{\mathbf{v}_a\cdot\mathbf{z}_a v_a^j}{z_a^2}\right] + O(\epsilon^{3/2})\,, \\
(h_a)_{ij} &= \frac{2z_a^i z_a^j}{z_a^3} + \epsilon^{1/2}\frac{4z_a^{(i}v_a^{j)}}{z_a^2} + O(\epsilon)\,.
\end{aligned} \tag{16}$$

Indices on spatial vectors are raised and lowered using the Cartesian metric; parentheses around indices denote symmetrization, while square brackets denote antisymmetrization.

The first term in $(h_a)_{00}$ can be recognized as yielding the normal Newtonian potential U, given by

$$U(t,\mathbf{x}) = \sum_a \frac{m_a}{z_a} = \sum_a \frac{m_a}{|\mathbf{x}-\mathbf{x}_a|}\,. \tag{17}$$

Note the presence of 0.5PN terms in the metric; these are terms of order $\epsilon^{1/2}$ in $(h_a)_{00}$, and ϵ^0 in $(h_a)_{0j}$. Because of general covariance, we are free to change coordinates to manipulate the form of the physical metric. In particular, we can remove these 0.5PN terms, can manipulate the PN terms to put them into a form to make comparisons with the standard parametrized post-Newtonian (PPN) framework (Will 1993), and can simplify the 1.5PN terms. Even though the background metric $\eta_{\mu\nu}$ will change its form under such coordinate transformations, this will have no physical consequences, since only $g_{\mu\nu}$ couples to matter.

The following coordinate transformation kills the 0.5PN terms in the physical metric, puts the PN terms into the standard PPN gauge, and also kills the 1.5PN terms in $(h_a)_{ij}$:

$$\begin{aligned}
t &= \bar{t} - 2\epsilon\bar{L}^0 + \frac{5}{2}\epsilon^{3/2}\bar{X}_{,\bar{0}} + O(\epsilon^2)\,, \\
x^i &= \bar{x}^i + \epsilon\bar{X}_{,\bar{j}} - 2\epsilon^{3/2}\bar{L}^j + O(\epsilon^2)\,,
\end{aligned} \tag{18}$$

where commas denote partial derivatives, and where

$$
\begin{aligned}
\bar{L}^0 &= \sum_a m_a \ln \bar{z}_a \,, \\
\bar{L}^j &= \sum_a m_a \bar{v}_a^j \ln \bar{z}_a \,, \\
\bar{X} &= \sum_a m_a \bar{z}_a \,.
\end{aligned}
\tag{19}
$$

Note that the first term in the time transformation is the post-Newtonian analogue of the Eddington (1922) transformation. In carrying out the normal coordinate transformation,

$$
g_{\bar{\alpha}\bar{\beta}}(\bar{x}^{\bar{\gamma}}) = \frac{\partial x^\mu}{\partial \bar{x}^{\bar{\alpha}}} \frac{\partial x^\nu}{\partial \bar{x}^{\bar{\beta}}} g_{\mu\nu}(x^\lambda) \,,
\tag{20}
$$

to 1.5PN order, it is also necessary to reexpress the potentials in terms of the new coordinates. For example, the Newtonian potential changes according to

$$
U \to U - \epsilon(U^2 + \Phi_2 + \Phi_W) - 2\epsilon^{3/2}[L^0 \dot{U} + L^j U_{,j} - \Sigma(L^j)_{,j} + \Sigma(v^j L^0)_{,j}] \,,
\tag{21}
$$

where all quantities on the right side are barred, and where

$$
\Sigma(f) \equiv \sum_a \frac{m_a f(t, \mathbf{x}_a)}{z_a} \,.
\tag{22}
$$

The potentials Φ_2 and Φ_W are defined below.

A further coordinate transformation, given by

$$
\bar{t} = t' - \epsilon^2 (4U' L'^0 + 2X'_{,j} L'^0_{,j} - 2M'^j_{,j}) \,,
\tag{23}
$$

where $M^j = \sum_a m_a X_{,j}(\mathbf{x}_a) \ln z_a$, simplifies the 1.5PN terms in g_{0j} and g_{00}.

The post-Newtonian part of the metric will be discussed in Sec. 2.2, while the 1.5PN part will be discussed in Sec. 2.5.

2.2. PPN Parameters

Following the coordinate transformation of Eqs. (18), the metric to PN order takes the form

$$
\begin{aligned}
g_{00} &= -1 + 2U - 2U^2 - 3\Phi_1 - 2\Phi_2 + 6\mathcal{A} - 2\Phi_W \,, \\
g_{0j} &= -4V^j - \frac{7}{2} W^j \,, \\
g_{ij} &= \delta_{ij}(1 + 2U) \,,
\end{aligned}
\tag{24}
$$

where we drop the explicit use of ϵ, and where the potentials are given by

$$\begin{aligned}
\Phi_1 &= \sum_a \frac{m_a v_a^2}{z_a} , \quad \Phi_2 = \sum_a \frac{m_a U(\mathbf{x}_a)}{z_a} = \sum_{a,b\neq a} \frac{m_a m_b}{z_a z_{ab}} , \\
\mathcal{A} &= \sum_a \frac{m_a (\mathbf{v}_a \cdot \mathbf{z}_a)^2}{z_a^3} , \quad \Phi_W = \sum_{a,b\neq a} \frac{m_a m_b \mathbf{z}_a}{z_a^3} \cdot \left(\frac{\mathbf{z}_{ab}}{z_b} - \frac{\mathbf{z}_b}{z_{ab}} \right) , \\
V^j &= \sum_a \frac{m_a v_a^j}{z_a} , \quad W^j = \sum_a \frac{m_a \mathbf{v}_a \cdot \mathbf{z}_a z_a^j}{z_a^3} ,
\end{aligned} \tag{25}$$

where $\mathbf{z}_{ab} = \mathbf{z}_a - \mathbf{z}_b$.

We now compare this metric with the PPN metric for point masses given in Will (1993, 2006). Transforming from the perfect fluid version of U to the point mass version using $U_{\rm fluid} = U_{\rm point} - \frac{1}{2}\Phi_1 - 3\gamma\Phi_2$, and working in the universal rest frame, it is a simple matter to read off the PPN parameters,

$$\begin{aligned}
\gamma &= 1 , \quad \beta = 1 , \quad \xi = 1 , \\
\alpha_1 &= 0 , \quad \alpha_2 = 0 , \quad \alpha_3 = 0 , \\
\zeta_1 &= -4 , \quad \zeta_2 = -1 .
\end{aligned} \tag{26}$$

Because we are dealing with point masses rather than perfect fluids, the PPN parameters ζ_3 and ζ_4, associated with energy density and pressure are not determined.

The parameters γ and β are the same as in GR. The potential Φ_W is the infamous "Whitehead" potential, which did not appear in earlier versions of the PPN framework (Nordtvedt 1968b; Will 1971a; Will & Nordtvedt 1972; Will 1974). It was later seen to be a generic consequence of any "quasi-linear" theory of gravity (Will 1973). The original PPN framework was then extended to incorporate naturally this potential with its associated "Whitehead parameter", ξ. In Whitehead's theory, $\xi = 1$, while in GR, $\xi = 0$. The parameters α_i all vanish, as they do in GR, indicating that the theory satisfies a kind of Lorentz invariance for gravity, and has no "preferred-frame" effects. This is not surprising, given that it is constructed using a flat background Minkowski metric. Here we ignore any coupling between local gravity and a background cosmological solution for the metric, which can in fact lead to non-zero α's, even with a flat background metric (see Lee *et al* (1976) for an example). The "conservation-law" parameters ζ_i are non-zero, indicating that the theory lacks global conservation laws for momentum and angular momentum; in GR, the ζ's all vanish. In Sec. 3 we will see that many of these values are in violation of experimental bounds.

2.3. Static spherically symmetric metric

For a single, static point mass M, the metric is particularly simple. Placing the mass at the origin of coordinates, we see that $(y_a^-)^0 = y = r$, $(y_a^-)^j = x^j$, and $w_a^- = y = r$. The metric then is $g_{00} = -1 + 2M/r$, $g_{0j} = -2Mx^j/r^2$, $g_{ij} = \delta_{ij} + 2Mx^ix^j/r^3$. The coordinate transformation (7) converts the metric to the Schwarzschild metric of GR (Eddington 1922). This was the basis of the claim made in the early years of Whitehead's theory that it satisfied all the "classic" tests.

However, real bodies such as the Sun and Earth are not point masses, but are finite sized objects made up of many masses. Working in the PN limit and assuming a spherically symmetric collection of masses centered at the origin, it is easy to show that, for a field point outside the body,

$$U = M/r\,, \quad X = Mr + I/3r\,, \quad \Phi_W = -\Phi_2 - MI/3r^4\,, \tag{27}$$

where the latter follows from manipulating the identity $\nabla^2(\Phi_W + 2U^2 - 3\Phi_2) = -2X_{,ij}U_{,ij}$, and where $M = \sum_a m_a$ and $I = \sum_a m_a r_a^2$ are the total mass and spherical moment of inertia of the body. All other post-Newtonian potentials vanish. Thus, in the PPN framework, the metric for a finite spherically symmetric static body becomes (Harvey 1964; Synge 1952)

$$\begin{aligned} g_{00} &= -1 + 2M/r - 2\beta(M/r)^2 + 2\xi MI/3r^4\,, \\ g_{0j} &= 0\,, \\ g_{ij} &= \delta_{ij}(1 + 2\gamma M/r)\,. \end{aligned} \tag{28}$$

Recall that $\gamma = \beta = \xi = 1$ in Whitehead's theory. The pericenter advance per orbit of a test particle moving on a geodesic of this metric is given by

$$\begin{aligned} \Delta\omega &= \frac{6\pi m}{p}\left[(2 + 2\gamma - \beta) + \frac{2\xi}{3}\frac{I}{mp^2}(1 + \frac{1}{4}e^2)\right]\,, \\ &= \frac{6\pi m}{p}\left[1 + \frac{2}{3}\frac{I}{mp^2}(1 + \frac{1}{4}e^2)\right]\,, \end{aligned} \tag{29}$$

where $p = a(1-e^2)$, with a and e being the semi-major axis and eccentricity of the orbit, and where the second line is the Whitehead prediction. The size-dependent term in $\Delta\omega$ has a negligible effect on the perihelion advance of Mercury, and so Whitehead's theory agrees with the data; however that term *will* have measurable consequences for ranging of the Earth-orbiting LAGEOS satellites (Sec. 3.4).

2.4. Anisotropy in the locally measured G

Although we have set the fundamental coupling constant G equal to unity by making a specific choice of units, it turns out that, in many alternative theories, the "locally measured" G may vary. By locally measured G we mean the output of a Cavendish-type experiment, whereby one measures the force between a test body and a source body separated by a chosen distance. The result may depend on the velocity of the laboratory relative to a preferred frame, if any of the α PPN parameters is non-zero, and may also depend on the presence of matter outside the laboratory.

In the case of Whitehead's theory, there are no preferred-frame effects, but there are "preferred location" effects. Specifically (see Eq. (6.75) of Will (1993)), the locally measured G is given by

$$G_{\rm local} = 1 + \frac{7}{3}U_{\rm ext} + \left(1 - \frac{3I}{MR^2}\right)\hat{e}^i\hat{e}^j U_{\rm ext}^{<ij>} \,, \tag{30}$$

where I, M and R are the spherical moment of inertia, mass and radius respectively of the source body in the Cavendish experiment,

$$U_{\rm ext} = \sum_a \frac{m_a}{r_a} \,, \quad U_{\rm ext}^{<ij>} = \sum_a \frac{m_a}{r_a}\left(\hat{n}_a^i\hat{n}_a^j - \frac{1}{3}\delta^{ij}\right) \,, \tag{31}$$

with the sum extending over all masses external to the laboratory, and where $\hat{e}^i$ and $\hat{n}_a^i$ are unit vectors pointing from the source body to the test body and to the a-th external body, respectively. Angular brackets around the indices denote a symmetric, trace-free (STF) tensor. Equation (8) is the special case of Eq. (30) for a single external body, and for a point source mass ($I = 0$). The most important effect is the anisotropy in $G_{\rm local}$, which can lead to anomalous Earth tides in geophysics (for the Earth, $I \approx 0.5MR^2$). Notice that only the $l = 2$, or quadrupole anisotropy in the external matter distribution contributes.

2.5. Gravitational radiation reaction

We focus now on the 1.5PN terms in the metric. Combining the relevant terms from Eqs. (16) with the 1.5PN terms generated by the coordinate transformations (18) and (23), we obtain,

$$\begin{aligned}
h_{00}^{(5/2)} &= \sum_a m_a\left[\frac{8}{3}\dot{\mathbf{a}}_a\cdot\mathbf{z}_a - 2\mathbf{v}_a\cdot\mathbf{a}_a + 6\frac{v_a^2\mathbf{v}_a\cdot\mathbf{z}_a}{z_a^2} - 12\frac{\mathbf{v}_a\cdot\mathbf{z}_a\mathbf{a}_a\cdot\mathbf{z}_a}{z_a^2} - 4\frac{(\mathbf{v}_a\cdot\mathbf{z}_a)^3}{z_a^4}\right] \\
&\quad +4L^0\dot{U} - 4L^jU_{,j} + 4\Sigma(L^j)_{,j} - 4\Sigma(v^jL^0)_{,j} \,, \\
h_{0j}^{(2)} &= \sum_a m_a\left[a_a^j + 4\frac{\mathbf{a}_a\cdot\mathbf{z}_a z_a^j}{z_a^2} - \frac{v_a^2 z_a^j}{z_a^2} + 2\frac{(\mathbf{v}_a\cdot\mathbf{z}_a)^2 z_a^j}{z_a^4} - 4\frac{\mathbf{v}_a\cdot\mathbf{z}_a v_a^j}{z_a^2}\right] \\
&\quad -2\dot{L}^j + 4L^0U_{,j} \,, \\
h_{ij}^{(3/2)} &= 0 \,,
\end{aligned} \tag{32}$$

where the superscript (n) denotes the order of ϵ. With these expressions and the geodesic equation, it is straightforward to derive the 1.5PN contributions to the equation of motion of a body in the presence of other bodies,

$$\frac{dv^j}{dt} = \frac{1}{2}h_{00,j}^{(5/2)} - h_{0j,0}^{(2)} - h_{0[j,k]}^{(2)}v^k \,. \tag{33}$$

We restrict attention to a binary system, and evaluate the terms in Eq. (33) at body 1 (as usual, dropping contributions to potentials due to body 1 itself). We use the fact that, at body 1, $L^0 = m_2\ln r$, $L^0_{,j} = m_2x^j/r^2$, $L^j = m_2v_2^j\ln r$, $\dot{L}^j = m_2a_2^j\ln r - m_2v_2^j\mathbf{v_2}\cdot\mathbf{x}/r^2$, and so on, where now

$x^j = x_1^j - x_2^j$ and $r = |\mathbf{x}_1 - \mathbf{x}_2|$; we also recall that $\sum_a m_a a_a^j = 0$ from conservation of momentum at Newtonian order. The surprising result is that, despite many cancellations, there is a residual acceleration at 1.5PN order, given by

$$a_1^j = 8m_1 m_2 \frac{\dot{r} x^j}{r^4} \,. \tag{34}$$

The acceleration for body 2 is found by interchanging m_1 and m_2 and letting $x^j \to -x^j$. The relative acceleration $a^j = a_1^j - a_2^j$ is then given by

$$a^j = 16\mu m \frac{\dot{r} x^j}{r^4} \,, \tag{35}$$

where $m = m_1 + m_2$ and $\mu = m_1 m_2/m$ are the total and reduced mass of the system, respectively. This radiation reaction term does not affect the orbital angular momentum, but it does cause an *increase* in the orbital energy at the rate $dE/dt = 16\mu^2 m \dot{r}^2/r^3$.

We will see in Sec. 3.3 that this has disastrous consequences for Whitehead's theory.

2.6. Failure of momentum conservation

In gravitational theories that lack suitable conservation laws for total momentum of gravitating systems, a binary system could suffer an anomalous acceleration of its center of mass, given in the PPN framework by

$$A_{\rm CM} = \frac{1}{2}(\zeta_2 + \alpha_3)\frac{m}{a^2}\frac{\mu}{m}\frac{\delta m}{m}\frac{e}{(1-e^2)^{3/2}} \,, \tag{36}$$

where $\delta m = m_1 - m_2$ and the acceleration is directed toward the pericenter of the lighter body. In GR (and in any theory based on an invariant action) the effect vanishes, but in Whitehead's theory, it does not.

Levi-Civita (1937) once claimed that this center-of-mass effect occurred in GR, but Eddington & Clark (1938) spotted his error and confirmed that it did not. Clark (1954) later showed that the effect did occur in Whitehead's theory, in agreement with Eq. (36). At the time, of course, there was no hope of detecting the effect using known binary systems. However, the binary pulsar (Sec. 3.5) provides a particularly stringent bound on this effect.

3. Experimental tests of Whitehead's theory

3.1. Gravimeter tests of the anisotropy in $G_{\rm local}$

If $G_{\rm local}$ is anisotropic because of the presence of an external mass, then there will be anomalous tides of the solid Earth, superimposed on the normal luni-solar tides (see Nordtvedt & Will (1972); Will (1993) for detailed discussion). The latter are of typical amplitude $\Delta g/g \sim 10^{-8}$ (here g is the local acceleration as measured by a gravimeter). If the external body is the sun itself, then $U_{\rm ext} \sim 10^{-8}$, and the G anisotropy will produce a tidal signal of comparable amplitude and of the same frequencies as the solar tide. It is very unlikely that Whitehead's theory would survive a comparison between the measured solar Earth tide and standard tidal theory with such a large additional amplitude. However, the bound one could achieve has never been investigated in detail, because a cleaner test is provided by looking at the so-called sidereal tides.

If the external mass is that of the galaxy, then $U_{\rm ext} \sim 5 \times 10^{-7}$, and the direction is fixed in space. This produces tides at frequencies associated with the sidereal day rather than the solar day of the solar tide, and these can be compared with known sidebands of the coupled lunar and solar tides. Measurements by Warburton & Goodkind (1976) using superconducting gravimeters showed no evidence of anomalies, and placed the bound on the Whitehead parameter $|\xi| < 10^{-3}$, as compared with the Whitehead value of unity. This improved upon the earlier bounds of Will (1971b), which were based on the existing tidal literature.

This was considered a fatal blow to the theory, but it did assume an amplitude 5×10^{-7} for the anisotropic part of the galactic potential. That value came from relating the solar system's orbital velocity in the galaxy to the potential via $v^2 \sim U_{\rm ext}$. This was criticized (Mentock 1996) because it concentrated the mass of the galaxy at the center, whereas we now know that the bulk of the mass of the galaxy is in a roughly spherical halo of stars and dark matter, substantially larger in size than the visible Milky Way.

However it can be shown using a simple density model for the galaxy that the original estimate for the anomalous tidal amplitude holds up within a factor of two. First, we note that the "trace-free" tensor potential $U^{<ij>} = -X_{,<ij>}$, where X is the "superpotential" defined in Eq. (19). For a spherically symmetric distribution of matter, X is given by

$$X = rm(r) + \frac{1}{3r}\int_0^r 4\pi\rho' r'^4 dr' + \frac{1}{3}\int_r^\infty 4\pi\rho' r'(r^2 + 3r'^2) dr' \,, \tag{37}$$

where ρ is the mass density and $m(r)$ is the mass inside radius r. Then, for spherical symmetry,

$$\begin{aligned} X_{,<ij>} &= \hat{n}^{<ij>}(d^2X/dr^2 - r^{-1}dX/dr) \\ &= -\hat{n}^{<ij>}\left[\frac{m(r)}{r} - \frac{I(r)}{r^3}\right] \,, \end{aligned} \tag{38}$$

where $I(r)$ is the spherical moment of inertia inside radius r. For flat or monotonically decreasing density distributions. the second term is always smaller than the first.

To compare with the earlier estimate we consider a specific density distribution given by $4\pi\rho = \alpha/r_c^2$, for $r < r_c$, and $4\pi\rho = \alpha/r^2$, for $r > r_c$, where r_c is a core radius meant to represent the mass of the inner part of the galaxy, and α is a parameter. The $1/r^2$ density distribution is meant to model the dark matter halo, and to yield a flat rotation curve for the outer reaches of the Milky Way, in rough agreement with observations. By noting that a circular orbit in a spherical potential satisfies, $v^2/r = a_r = m(r)/r^2$, and considering the case $r > r_c$, we can fit $\alpha = v^2/(1 - 2q/3)$, and find that

$$U_{\rm ext}^{<ij>} = \frac{2}{3} v^2 \hat{n}^{<ij>} \frac{1 - q + q^3/5}{1 - 2q/3} , \tag{39}$$

where $q = r_c/r$. For the case $r < r_c$, a similar calculation gives

$$U_{\rm ext}^{<ij>} = \frac{2}{5} v^2 \hat{n}^{<ij>} , \tag{40}$$

independent of r. Thus for $v \sim 220$ km/s, we find an amplitude $2 - 3 \times 10^{-7}$, fully consistent with the earlier estimate. Note from Eq. (38) that only the matter inside our radius has an effect on the anisotropy. Even though the galaxy and its halo are not strictly spherically symmetric, this is unlikely to alter the estimate significantly. The only way to suppress this effect is by some specific, fine-tuned distribution of external matter.

The conclusion stands: Whitehead's theory violates geophysical tide measurements by about a factor of 500.

3.2. Lunar laser ranging and the Nordtvedt effect

In many alternative theories of gravity, there is a violation of the weak equivalence principle for massive, self-gravitating bodies. Specifically, the passive gravitational mass m_p may differ from the inertial mass m_i according to

$$m_p = m_i (1 - \eta |E_g|/m_i) , \tag{41}$$

where

$$\eta = 4\beta - \gamma - 3 - \frac{10}{3}\xi - \alpha_1 + \frac{2}{3}\alpha_2 - \frac{2}{3}\zeta_1 - \frac{1}{3}\zeta_2 , \tag{42}$$

and E_g is the gravitational binding energy of the body. This is known as the Nordtvedt effect (Nordtvedt 1968a), and can cause a difference in acceleration of the Earth and the Moon toward the Sun, and a resulting perturbation of the Earth-Moon orbit with a specific signature. Over 35 years of lunar laser ranging have found no evidence for such an effect, and have placed the bound $|\eta| < 9 \times 10^{-4}$ (Williams *et al.* 2004). From the set of PPN parameter values for Whitehead's theory in Eq. (26), $\eta_{\rm Whitehead} = -1/3$ in strong disagreement with experiment.

3.3. The binary pulsar

Thirty years of timing of the binary pulsar 1913+16 have shown that its orbital period is decreasing at a rate $\dot{P}_b = -(2.4184 \pm 0.0009) \times 10^{-12}$, in agreement with the GR prediction for gravitational radiation damping within a fraction of a percent (Weisberg & Taylor 2005). Orbital damping has also been measured in the binary pulsars 1534+12 and the double pulsar 0737-3039AB, again in agreement with GR. Unfortunately, Whitehead's theory has both the wrong sign – anti-damping instead of damping – and the wrong magnitude, $\dot{P}_b \approx +4 \times 10^{-8}$. The magnitude is so large because the reaction is a 1.5PN effect, rather that a v^2-times smaller 2.5PN effect, as in GR. One could change the sign of the effect, but not its magnitude, by assuming advanced, rather than retarded interactions.

3.4. LAGEOS data

Since 1992, precise laser tracking of two Earth-orbiting Laser Geodynamics Satellites (LAGEOS I and II) has made possible tests of general relativity in the vicinity of the Earth, in addition to its primary geophysical goals. Notably, the tracking data have been used to give a preliminary test of the "dragging of inertial frames", or Lense-Thirring effect, in which the rotating Earth causes a small precession of the planes of the orbits of the satellites. The NASA-Stanford Gravity Probe B experiment also aims to measure this effect with higher accuracy using orbiting superconducting gyroscopes. The effect depends on the PPN parameters γ and α_1, so both Whitehead's theory and GR agree on the prediction for this effect. However, the orbit of the LAGEOS II satellite has a small eccentricity, unlike LAGEOS I, and so its advance of perigee is also measured, along with the "nodal" precession of the orbit plane.

Now, the multipole moments of the Earth's Newtonian gravity field also contribute to the nodal precessions and the perigee advance, indeed they overwhelm the relativistic effects. However, Ciufolini *et al.* (1997) found a particular linear combination of the three measurables, the two nodal precessions, $\dot{\Omega}_I$, and $\dot{\Omega}_{II}$, and the perigee precession of II, $\dot{\omega}_{II}$, in which the effects of the leading $l = 2$ and $l = 4$ Newtonian multipoles would precisely cancel. The combinations depend on the known inclinations of the orbits relative to the equator. The uncertainties in the measured values of the remaining $l \geq 6$ multipoles then become part of the error budget of the experiment.

The only difference in any of the relevant predictions between Whitehead and GR is the additional size-dependent term in the pericenter advance, Eq. (29). Because the LAGEOS II satellite is at two Earth radii, this can be a sizable effect (unlike the case with Mercury). Thus, the specific linear combination of predicted effects used by Ciufolini *et al.* gives the theoretical prediction (in milliarcseconds per year)

$$\begin{aligned} A_{\rm theory} &= \dot{\Omega}_I + 0.295\dot{\Omega}_{II} - 0.35\dot{\omega}_{II} \\ &= 60.2 - 109\xi + (\text{errors}) \end{aligned} \tag{43}$$

where we have kept the PPN Whitehead parameter ξ but used the GR/Whitehead values for γ, β and α_1, and where "error" denotes those due to the higher multipole moments. Using the actual tracking data, the measured value of this combination is $A_{\rm exp} = 66.6$ milliarcseconds per year, plus measurement errors. The combination of all the errors leads to a total estimated error of about 25 percent. Thus for the theory to match observation within 25 percent, the parameter ξ must lie in the range

$$-0.2 < \xi < 0.1\,, \tag{44}$$

which thus excludes Whitehead's theory. It is likely that this bound could be improved by making use of dramatically improved Earth gravity models that have been derived from the GRACE and CHAMP geodesy space missions, which have reduced the errors in the Earth's multipole moments by significant amounts.

3.5. Binary pulsars and momentum conservation

The binary pulsar B1913+16 provides an excellent system to test the momentum non-conserving effect described in Sec. 2.6, because it is highly relativistic, and because of the ability to do precise timing. For a moving system all measured periods will be offset via the Doppler effect ($\Delta P/P \sim v/c$); accordingly, in an accelerating system periods will suffer a drift $dP/dt \sim (a/c)P$, and in a system with a changing acceleration, there will be a $d^2P/dt^2 \sim (\dot{a}/c)P$. In the binary pulsar, the center of mass acceleration predicted by Whitehead's theory changes because it is directed toward the periastron of the system, which rotates by 4 degrees per year. Indeed, in the 30 years since discovery, the center-of-mass motion (were it to exist) would have almost reversed itself. Yet precise timing of the pulsar 1913+16 has shown no evidence of any change in its spindown rate dP/dt, leading to an upper bound $|d^2P/dt^2| < 8.5 \times 10^{-32}$ s^{-1} (Manchester *et al.* 2005). Using the neutron star masses and orbital elements inferred from the timing data, together with Eq. (4) of Will (1992), we find the predicted value $d^2P/dt^2 \simeq 2.1 \times 10^{-25} \zeta_2 \cos\omega$ s^{-1}, where ω is the periastron angle (we adopt the Whitehead value $\alpha_3 = 0$). With $\cos\omega$ varying between -1 and $+0.59$ over that period, we find the bound $|\zeta_2| < 8 \times 10^{-7}$. Notice that the mass values used were inferred using GR; in Whitehead's theory, it is conceivable that these values could be different from the GR values (as occurs in other theories that violate the strong equivalence principle). However, to evade this bound, either the inferred masses would have to be 10^6 times smaller, or they would have to be the same to a part in 10^6. This seems highly unlikely.

4. Cosmological Considerations

In addition to passing stringent tests at terrestial, solar system, and galactic scales, in order to be viable, a theory of spacetime and gravity must agree with the basic facts of cosmology: the expansion of the universe and the existence of the Cosmic Microwave Background. Of course neither was known when Whitehead formulated his theory. However at present, we are entering an era in which cosmological observations are becoming increasingly detailed and precise (Spergel *et al.* 2006).

Already during the 1950's Synge (1954), using the spherically symmetric continuum version of Whitehead's theory developed by Rayner (1954), derived the form the Friedmann-Lemaitre metric takes according to Whitehead. If $\tau = \sqrt{-\eta_{\mu\nu}x^\mu x^\nu}$, the curved metric is

$$ds^2 = -\left(1 - \frac{3A}{\tau}\right) d\tau^2 + \tau^2 \left(1 + \frac{A}{\tau}\right) (d\chi^2 + \sinh\chi(d\theta^2 + \sin^2\theta d\phi^2))\,, \tag{45}$$

with the density

$$\rho = \frac{K}{\tau^2}\,, \tag{46}$$

and $A = 8\pi GK/9$, with K a constant. Note that Synge's version of Whitehead's Universe, which has hyperbolic, $k = -1$ spatial cross sections, becomes empty and flat at late times, becoming more and more Milne-like.

By contrast, current observations (Spergel *et al.* 2006) strongly indicate that our universe is currently of Friedmann-Lemaitre form with flat spatial sections and scale factor $a(\tau)$ with jerk (Blandford 2004)

$$j = \frac{a^2}{\dot{a}^3}\frac{d^3a}{d\tau^3} = 1\,, \tag{47}$$

and thus given by

$$a(\tau) = \sinh^{2/3}[(3\Lambda/4)^{1/2}\tau]\,, \tag{48}$$

where Λ is the cosmological constant. As proper time τ goes by, the universe is more and more accurately De-Sitter like, with

$$a(\tau) = e^{(\Lambda/3)^{1/2}\tau}\,. \tag{49}$$

It seems that to be viable, Whitehead's theory requires, at the very least, a modification that incorporates the same effects as the cosmological term in Einstein's theory. The principal motivation behind Whitehead's alternative to Einstein's theory was the desire to retain fixed, non-dynamical, background-independent, causal relations between spacetime events which do not depend upon one's location in spacetime. Presumably, purely on the same aesthetic or philosophical grounds, one might argue that, as a fixed set of spatio-temporal relations, those of De-Sitter spacetime or of anti-De-Sitter spacetime are to be preferred to those of Minkowski spacetime since the underlying isometry groups in the former two cases are simple, rather than being a mere semi-direct product in the latter. Be that as it may, early on, Temple (1923) pointed out that this aim could just as readily be achieved by adopting the causal relations of a fixed maximally symmetric spacetime of constant curvature, e.g. a De-Sitter spacetime, as it could by insisting that they were the same as Minkowski spacetime. With this in mind, Temple sketched a generalisation of Whitehead's theory to incorporate a De-Sitter background which received enthusiastic support from Whitehead himself. An interesting Machian argument in its favour was made by Band (1929a), who pointed out that for positive cosmological constant it described a finite universe relative to which one could define an absolute acceleration. Actually Band claimed (Band 1929b) that Whitehead's theory was in gross violation of experiment. Later, Rayner (1955b) pointed out what he claimed were some errors in Temple's formulae.

Rather than recall the details of Temple's construction, which appears to have been almost completely forgotten, perhaps because the reference to it in Synge's influential reformulation of Whitehead's theory in modern notation (Synge 1952) is incorrect, we shall content ourselves with the remark that the obvious statement of the theory[2] is that it amounts to linearising Einstein's theory with a cosmological constant around a De-Sitter background. This interpretation is consistent with Temple's finding that the perihelion advance agrees with that obtained by Eddington for the Schwarzschild-De-Sitter metric. If one accepts our interpretation, then the fact that the Kerr-De-Sitter solution is also of Kerr-Schild form (Carter 1968) shows that Eddington and Rayner's observations (Eddington 1922; Rayner 1955a) may be extended to the full set of rotating solutions in a background De-Sitter spacetime.

However, although incorporating a cosmological term may conceivably render Whitehead's theory in better accord with cosmological data, it will do nothing to alter the fact that it is in flagrant contradiction with observations at solar system and galactic scales, since the effects of any cosmological modification at these scales are negliglible.

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[2]which however appers to differ in detail from the approach advocated in Temple (1923); Rayner (1955b)

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