# Substantive general covariance meets naturality*

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**Abstract**

Almost since its inception, the question of whether there is any substantive sense in which general relativity is distinguished from other theories of space and time has exercised physicists and philosophers alike (Norton 1993). In recent decades, this question has been tied up with trying to identify a sense in which general relativity is 'background independent' (Read 2023). While many proposals for background independence are now on the table, all have their discontents, and it is not obvious in any case that they are satisfied *only* by general relativity. In this article, we review recent approaches to substantive general covariance and background independence, and spend significant time bringing this prior work into contact with the formalism of 'natural theories', which has recently been proposed by Weatherall and March (forthcoming) to be a promising way of cashing out general covariance.

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# 1 Introduction

Cashing out a 'substantive' sense of general covariance which makes good on a widely-shared intuition that general relativity is 'special' by dint of its manifesting this quality is a task which—infamously—has at once and for many decades both energised and confounded physicists and philosophers (see Norton (1993), Pooley (2010), and Teitel (2019)). Around the turn of the 21<sup>th</sup> century, the quest for a substantive notion of general covariance underwent somewhat of a rebranding: substantive general covariance is now just as likely to go under the moniker of 'background independence' (for a recent book-length survey of which see Read (2023)). But this rebranding belies essential continuity of the fundamental objective, which remains to identify a sense in which general relativity is distinguished from many other theories of space and time, roughly (and intuitively) due to the fact that it is liberated from coordinate systems, or evinces suitable coupling between geometry and matter, or involves only quantities with 'well defined' transformation rules, or something along those lines.

Confronted with the panoply of proposals for cashing out this notion (which we will refer to interchangeably as 'substantive general covariance' and 'background independence' in what follows)—see *inter alia* Anderson (1967), Belot (2011), Freidel and Teh (2022), Giulini (2007), Pooley (2017), Rickles (2008), and Rovelli (1997)—Read (2023) moots for pluralism: perhaps there is no 'one true' notion of background independence, and perhaps whether general relativity manifests this quality (and is distinguished from other theories in this regard) is in the end a knottier and less clear business than one might have hoped.[1] Our purpose in this article is not merely to repeat the survey and study of the approaches to background independence and general covariance provided by Read (2023), but rather in addition to fold into these discussions the notion of 'naturality', which finds detailed presentations in mathematics and physics texts (most notably Fatibene and Francaviglia (2003) and Kolář et al. (1993)) but which has recently been propounded in the philosophy literature by March and Weatherall (forthcoming) and Weatherall and March (forthcoming) as a promising way of making sense of the notion of general covariance.[2]

Very roughly, the idea of naturality is that changes to the spacetime manifold (e.g. diffeomorphisms) should have concomitant changes to the values of fields defined thereon; if this were to fail, there would be a clear sense in which the theory under consideration (using this geometrical framework: the notion of a 'natural theory' is discussed in depth by Weatherall and March (forthcoming)) fails some more general version of 'diffeomorphism invariance', and so fails (it seems) to be generally covariant.[3,4] (The notion of dependence here is cashed out

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[1] Cf. the pluralism about determinism espoused by Manchak et al. (forthcoming), which we endorse.

[2] Weatherall and March (forthcoming) also make the case that naturality can shed light on other foundational principles such as 'minimal coupling'; we will not discuss these further in this article.

[3] The relationship between diffeomorphism invariance and general covariance is in fact somewhat delicate—see Pooley (2017) and Read (2023); this will be discussed further below.

[4] The concept of naturality emerged from the work of Nijenhuis (1952, 1972), which in turn

formally in the category-theoretic sense of functorality, as we will discuss below.) Weatherall and March (forthcoming, p. 3) indeed indicate that naturality is a plausible way of cashing out general covariance, writing that "[t]his construction, which we call 'naturalization', can be though of as a kind of Kretschmannization procedure (Kretschmann 1917), where one identifies background structure and makes explicit how that structure itself transforms under the action of smooth embeddings."[5]

What remains to be explored in depth, however, is the extent to which naturality—interesting and powerful concept as it is—is able to deliver fresh insights into debates regarding substantive general covariance/background independence. The purpose of the present article is to achieve exactly this; in particular, we will show that while naturality *à la* Weatherall and March (forthcoming) arguably falls short of what many physicists and philosophers seem to hanker after when it comes to delivering a *substantive* notion of general covariance, there are various ways in which to leverage the tools of naturality in order to provide just such accounts.[6] Here, we develop some such proposals, making contact with other proposals regarding background independence which already exist in the literature (especially that of Belot (2011)). In addition, we (a) make some remarks regarding naturality and the infamous 'hole argument' of general relativity (typically regarded as being one problematic upshot of general relativity's general covariance), (b) discuss some of the metaphysical aspects of natural theories in the sense of March and Weatherall (forthcoming) and Weatherall and March (forthcoming), and (c) anticipate and respond to some possible objections to the conceptual significance of naturality. The ultimate goal—as should already have been made clear in the foregoing—is to integrate naturality into the literature on general covariance and background independence, and to explore some of the ends to which it can be deployed.

As such, our plan for the article is as follows. In §2, we survey very briefly the lay of the land in the literature on general covariance and background independence. In §3, we introduce the tools of naturality more formally. In §4, we consider the extent to which naturality can shed new light on these debates regarding background independence, and make some novel proposals in this regard. In §5, we consider the interactions between naturality and the hole argument, picking up on some comments made by Weatherall and March (forthcoming, §7). In §7, we anticipate and respond to some worries regarding the foundational significance of naturality. In §8 we wrap up.

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developed out of the 'geometric objects' programme of Schouten and collaborators (see March (2025b) for some of the early history). We return to this in §6.

[5] One should keep in mind here Kretschmann's famous objection to Einstein on general covariance: namely, that *any* theory can be rendered generally covariant, in the sense of being written in a form which holds with reference to any member of a class of coordinate systems related by smooth but otherwise arbitrary transformations.

[6] Of course, if one thinks that naturality is the 'one true' way of cashing out general covariance, then one might not be maximally motivated to take up this task. Fair enough—but it seems to us that there is still more to be done on this front, and we expect that most players in this debate would agree with us. Our thanks to Jim Weatherall for discussion of these motivational issues.

# 2 Background independence: state of play

We begin by very briefly sketching the state of play in the philosophical literature on general covariance. Throughout this section, we follow the lead of Pooley (2017) and Read (2023); for the deeper history, the reader should consult Norton (1993).

In §2.1, following Pooley (2017), we distinguish between the notions of 'absolute objects' and 'fixed fields'. In §2.2, we introduce (again following Pooley (2017)) two versions of a special relativistic Klein–Gordon theory, which will prove helpful in providing a 'litmus test' of background independence in subsequent discussions. In §2.3, we clarify the difference between general covariance and diffeomorphism invariance, building on the previous two subsections. In §2.4, we review very briefly existing definitions of background independence in the literature, mostly following the presentation by Read (2023).

## 2.1 Fixed fields and absolute objects

As is standard in this literature, let us say that the 'kinematically possible models' (KPMs) of a theory are tuples $\langle M, O_1, \dots, O_n \rangle$, where $M$ is a differentiable manifold and the $O_i$ are geometric object fields on $M$; let us say that the 'dynamically possible models' (DPMs) of a theory are those KPMs in which the geometric object fields satisfy specified dynamical equations. (We illustrate the difference between KPMs and DPMs via examples in the next subsection.)

Following Pooley (2017) and Read (2023), let us say that a geometric object field $O_i$ on $M$ is a 'fixed field' just in case it is fixed identically in all KPMs of the theory; following Anderson (1967) (cf. Pitts (2006)), let us say that a field is an 'absolute object' just in case it is fixed (up to isomorphism—here understood to mean diffeomorphism) in all DPMs of the theory. So, if some field $O_i$ is a fixed field of some theory $T$, then if $\langle M, O_1, \dots, O_i, \dots, O_n \rangle$ is a KPM of the theory, then (say) $\langle M, d^*O_1, \dots, d^*O_i, \dots, d^*O_n \rangle$ *will not even be a KPM* unless $d^*O_i = O_i$.[7] Whereas if $O_i$ is an absolute object, then if $\langle M, O_1, \dots, O_i, \dots, O_n \rangle$ is a DPM of the theory, then $\langle M, O'_1, \dots, O'_i, \dots, O'_n \rangle$ is also a DPM only if $O'_i = d^*O_i$.[8]

Note that there are in fact two key differences between the field field concept and the absolute object concept. The first is that while fixed fields impose a condition at the level of KPMs, absolute objects impose a condition at the level of DPMs. The second is that the condition on field fields is strictly stronger than that for absolute objects, since the former insist that the field be *identically* the same in each model, whereas the latter requires only that the object be the same *up to isomorphism* in each model. In principle, one could countenance two

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[7] Throughout, '$d^*$' denotes the pushforward map.

[8] The very notion of a fixed field involves drawing distinctions which 'cut finer' than isomorphism; as such, it may be distasteful to authors such as Halvorson and Manchak (2025) and Weatherall (2018), who prefer a more 'categorical-structuralist' point of view (cf. Cheng and Read (forthcoming) and Read (2025b)). We return to these matters in §5, when we discuss the hole argument.

further positions which cut across these distinctions (i.e., an object fixed up to isomorphism in all KPMs, or an object fixed identically in all DPMs), although it is not obvious that there would be a great deal of additional conceptual insight to be gained from doing so.

## 2.2 Two versions of special relativity

To bring out the difference between fixed fields and absolute objects, it is helpful to distinguish between two different versions of a special relativistic Klein–Gordon theory, as do Pooley (2017) and Read (2023).[9]

The first such theory we call **SR1**, and has KPMs $\langle M, \eta_{ab}, \varphi \rangle$, where $\eta_{ab}$ is a fixed Minkowski metric field on the differentiable manifold $M$,[10] and $\varphi$ is a real scalar field on $M$. DPMs of this theory are given by the equation

$$\eta_{ab} \nabla^a \nabla^b \varphi = \mathbf{0}, \tag{1}$$

where $\nabla$ is the Levi-Civita derivative operator for $\eta_{ab}$.

This version of special relativity is to be contrasted with what we call **SR2**, which has KPMs $\langle M, g_{ab}, \varphi \rangle$, where now $g_{ab}$ is a generic Lorentzian metric field on the differentiable manifold $M$; DPMs for this theory are given by

$$g_{ab} \nabla^a \nabla^b \varphi = \mathbf{0}, \tag{2}$$

$$R^a{}_{bcd} = \mathbf{0}, \tag{3}$$

where of course the second condition imposes that the (Levi-Civita connection of) the metric be flat, and hence that the metric be the Minkowski metric, up to isomorphism.

In **SR1**, the metric $\eta_{ab}$ is a fixed field; in **SR2**, the metric field $g_{ab}$ is an absolute object but not a fixed field. As Pooley (2017) and Read (2023) state, one might have the intuition that **SR2** is not 'background independent' in the same manner as general relativity—perhaps due to the lack of non-trivial dynamical coupling between geometric and material degrees of freedom in this theory (as contrasted with general relativity).[11] To the extent that one has that intuition, one might then be inclined to find a sense of background independence which identifies **SR2** as background *dependent*—i.e., as not *substantively* generally covariant. We return to this issue several times in what follows.

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[9] Note that the nomenclature for versions of special relativity differs between Pooley (2017) and the earlier Pooley (2010); in this article, we use only the nomenclature of Pooley (2017), which is the same terminology as that used by Read (2023).

[10] Of course, there are many Minkowski metric fields on $M$, related by diffeomorphism. Since we are regarding $\eta_{ab}$ as a fixed field here, we really do need to make a choice between members of this equivalence class.

[11] One might adduce good reasons in favour of questioning the fundamentality of the spacetime–matter distinction: see Martens and Lehmkuhl (2020). We will not go further intro these reasons here.

## 2.3 General covariance and diffeomorphism invariance

One can leverage the work of the previous two subsections in order to provide a clear distinction between the notions general covariance and diffeomorphism invariance. Here is how Pooley (2017, p. 115) defines general covariance:

**Definition 1 (General covariance)** *A formulation of a theory is* generally covariant *iff the equations expressing its laws are written in a form that holds with respect to all members of a set of coordinate systems that are related by smooth but otherwise arbitrary transformations.*

Note that **SR1** and **SR2** are both generally covariant simply by virtue of their being written in a coordinate-free manner; this is one (anachronistic) manifestation of Kretschmann's objection to Einstein that *any* theory of space and time can be trivially rendered generally covariant. Since both **SR1** and **SR2** are written in this coordinate-free way, they of course both satisfy the above definition of general covariance.

Let us move on, then, to diffeomorphism invariance. Letting $F_i$ denote fixed fields of some theory and $D_i$ denote dynamical (i.e., non-fixed) fields of that theory, here is how Pooley (2017, p. 117) defines diffeomorphism invariance:

**Definition 2 (Diffeomorphism invariance)** *A theory $T$ is* diffeomorphism invariant *iff, if*

$$\langle M, F_1, \ldots, F_n, D_1, \ldots, D_m \rangle$$

*is a DPM of $T$, then so is*

$$\langle M, F_1, \ldots, F_n, d^* D_1, \ldots, d^* D_m \rangle,$$

*for all $d \in \mathrm{Diff}\,(M)$.*

The idea here is that applying arbitrary diffeomorphisms to the dynamical fields of the theory under consideration keeps one within the solution space of that theory. Note that while **SR1** fails this criterion (its equations are invariant only under Poincaré transformations plus—being massless—scale transformations), **SR2** satisfies it. So, to the extent that one considers the distinction between fixed fields and absolute objects to be meaningful (or at least worthwhile),[12] one can use it to track a difference between the notions of diffeomorphism invariance and general covariance.

In any case though, since **SR2** is (a) diffeomorphism invariant, but (b) intuitively background dependent, one might think that there must be some superior way of cashing out what background independence amounts to, which delivers the (again, intuitively) correct verdict that **SR2** is background *dependent*. It is to these alternative proposals that we now turn.

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[12] Cf. footnote 8.

## 2.4 Proposed definitions of background independence

Read (2023) surveys many different possible definitions of background independence; here we focus on three approaches which are particularly prominent in the literature:

1. A criterion that a theory not contain 'absolute objects'.
2. A criterion in terms of variational principles.
3. A proposal due to Belot (2011), in terms of matching 'geometrical degrees of freedom' and 'physical degrees of freedom'.

There are other recent proposals, e.g. that due to Freidel and Teh (2022) in terms of a theory having 'non-trivial corner charges', but for the sake of constraining the narrative we will not go into these further here—although see (in this case) Read (2023) and Struyve (2025) for critical engagement.[13]

### 2.4.1 No absolute objects

The first criterion for a definition of background independence going beyond the definitions of general covariance and diffeomorphism invariance presented thus far dates back to Anderson (1967), and has it that a theory is background independent just in case it possesses no absolute objects:[14]

**Definition 3 (Background independence, absolute objects)** *A theory is* background independent *iff it has no absolute objects in its formulation.*

(It is important to be clear here that we have in mind here the 'local' version of the 'no absolute objects' criterion proposed by Friedman (1983), rather than the earlier 'global' version proposed by Anderson (1967).[15]) The problem with this definition, *inter alia*, is that almost *all* field theories can be identified as having *some* absolute objects—see Pitts (2006), Pooley (2017), and Read (2023). So, while the proposal might *seem* to do a good job when it comes to (say) distinguishing theories such as **SR1** and **SR2** from general relativistic theories, it ultimately fails to afford a clean distinction between those theories which are background independent and those which are not.

### 2.4.2 Variational principles

The second proposal for making good on a notion of background independence proceeds via variational principles. The account along these lines proposed by Pooley (2017) can be stated as follows (Read 2023, p. 24):

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[13] For a recent survey of approaches to background independence following roughly the structure of Read (2023, ch. 3), see Vassallo (2026).

[14] All definitions which follow in this subsection are taken from Read (2023).

[15] Recognising this distinction resolves a challenge made by De Haro (2025) to Read (2023).

**Definition 4 (Background independence, variational)** *A theory is background independent iff its solution space is determined by a generally covariant action, (i) all of whose dependent variables are subject to Hamilton's principle, and (ii) all of whose dependent variables represent physical fields.*

Clause (ii) here plays a critical role since, in principle, *any* equation of motion can be introduced to a theory's action using Lagrange multipliers. There are various worries regarding this proposal, however—among others, (a) there are theories which are intuitively background dependent despite satisfying the definition, (b) what makes a variable 'physical' has not been made precise, and (c) there are theories which simply lack variational principles, so that the above criterion cannot apply to them. For further discussion, see Freidel and Teh (2022), Pooley (2017), Read (2023), and Struyve (2025).

### 2.4.3 Belot's proposal

The final proposal for cashing out background independence which we review in this article is due to Belot (2011). Notably, Belot maintains that whether a particular piece of structure in a physical theory qualifies as geometrical or physical is not exclusively a formal matter; rather, our interpretive convictions factor strongly into such judgements. This will be reflected in the definition of background independence which he ultimately lays out.

In order to articulate Belot's account of background independence, we first need to present his distinction between 'physical degrees of freedom' and 'geometrical degrees of freedom'. Consider a theory $T$ whose KPMs are tuples of the form $\langle M, O^G, O_1, \ldots, O_n \rangle$, where $M$ is a smooth manifold, and $O^G$ is a piece of structure identified antecedently as being 'geometrical'. Such $O^G$ may in principle vary across KPMs and DPMs of $T$, so let us call $\mathcal{G}$ the set of all $O^G$ across all dynamically possible models of $T$. Additionally, one may wish to regard certain elements of $\mathcal{G}$ as *equivalent* geometrical structures and equip $\mathcal{G}$ with an equivalence relation $\sim_G$ relating equivalent geometrical structures. Taking the quotient $\mathcal{G}/\sim_G$ then defines the reduced set of geometrical objects in $T$ called $\tilde{\mathcal{G}}$. Read then notes that "[t]he degrees of freedom needed to parametrize this latter set $\tilde{\mathcal{G}}$ are what Belot calls the *geometrical degrees of freedom*" (Read 2023, p. 29).

Turning now to physical degrees of freedom, we note that it is often the case that a given physical theory commits to excess mathematical structure in the sense that two mathematically distinct models may be regarded as representing the same physical states of affairs. Such models are called 'gauge-equivalent' and physical theories which contain such models are called 'gauge theories'.[16] We denote the relation of gauge equivalence by $\sim_P$, where $P$ in the subscript stands for *physical* equivalence. Belot's procedure for identifying physical degrees of freedom runs as follows. Take the set $\mathcal{D}$ consisting of all the DPMs of $T$ and take the quotient $\tilde{\mathcal{D}} := \mathcal{D}/\sim_P$. Then "the degrees of freedom needed

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[16] This is just one sense in which the term 'gauge theory' might be used. See Weatherall (2016b) for disambiguation.

to parametrize $\tilde{\mathcal{D}}$—the gauge-quotiented class of dynamically possible models [...]—are the theory's *physical degrees of freedom.*" (Read 2023, p. 29).

With this distinction in hand, Belot's four definitions which characterise the background independence of a given theory are as follows:

**Definition 5 (Full background dependence, Belot)** *A field theory is fully background dependent if it has no geometrical degrees of freedom: every solution is assigned the same spacetime geometry as every other solution.*

**Definition 6 (Full background independence, Belot)** *A field theory is fully background independent if all of its physical degrees of freedom correspond to geometrical degrees of freedom: two solutions correspond to the same physical geometry iff they are gauge equivalent.*

**Definition 7 (Near background dependence, Belot)** *A field theory is nearly background dependent if it has only finitely many geometrical degrees of freedom: quotienting the space of geometries that arise in solutions of the theory by the relation of geometrical equivalence yields a finite-dimensional space.*

**Definition 8 (Near background independence, Belot)** *A field theory is nearly background independent if it has a finite number of non-geometrical degrees of freedom: there is some $N$ such that for any geometry arising in a solution of the theory, the space of gauge equivalence classes of solutions with that geometry is no more than $N$-dimensional.*

Again, this presentation will suffice for our purposes; see Read (2023, ch. 3) for a detailed assessment and exploration of Belot's proposal. In brief: the flexibility of the proposal is a strength, and while it may seem to flounder on theories with multiple geometrical structures only some of which are fixed (or absolute), these issues can be addressed via only small modifications to the proposal. (We needn't go further into these proposed modifications in this article.)

# 3 Naturality defined

So be it for the state of play *vis-à-vis* substantive general covariance/background independence. Before addressing how naturality fits into these considerations, we must introduce the technical background to this concept; the purpose of this section is to do so.

In §3.1, we remind the reader of the relevant technical aspects of fibre bundles; in §3.2, we generalise to the notion of a 'natural bundle'. In §3.3, we introduce 'natural equations' and 'natural theories'. For further details regarding what is presented in this section, the reader is referred to March and Weatherall (forthcoming) and Weatherall and March (forthcoming).

## 3.1 Fibre bundles

Our first technical notion to be explained—that of a *fibre bundle*—is already quite familiar to philosophers of physics.[17] The intuition behind a fibre bundle is this: for some field $\Phi$ on $M$, one wants to encode within the mathematical model with which one is working all *possible* values of $\Phi$ at that point; one does this by appending to each $p \in M$ a 'fibre' of possible field values; across all of $M$, these fibres form a 'bundle'—hence the terminology.

More technically, a fibre bundle $B \overset{\pi}{\to} M$ with typical fibre $S$ consists of differentiable manifolds $B$ (the *total space*) and $M$ (the *base space*) and a smooth surjective map $\pi : B \to M$ with the property that for any $p \in M$, there exists an open neighbourhood $O$ of $p$ and a *local trivialisation* of $B$ over $O$, i.e. a diffeomophism (i.e. smooth map with smooth inverse) $\varphi : O \times S \to \pi^{-1}(O)$. One can envisage, 'living above' $M$, the entire bundle $B$, which consists of fibres $S$ associated with each $p \in M$; the projection map takes us from $B$ to $M$; locally, the bundle looks like a product $O \times S$.

A *section* of a fibre bundle associated with each $p \in M$ a unique point in the fibre $S$ with which $p$ is associated; metaphysically speaking, then, sections represent different possible distributions of material fields on the manifold, but all of these possibilities are now encoded in the overall structure of the fibre bundle.

Finally, we will need the notion of a *bundle morphism* $(\Psi, \psi)$, which is a pair of maps $\Psi : B \to B'$ and $\psi : M \to M'$ such that $\pi' \circ \Psi = \psi \circ \pi$.[18]

## 3.2 Natural bundles

Some further generalisation is now in order. What one would like to consider are different possible values of fields (as encoded in a fibre bundle, as discussed above) *on different possible manifolds*—i.e., different possible 'ways for spacetime to be'. One can make rigorous this desideratum by availing oneself of the resources of natural bundles, as presented in the philosophy literature by Weatherall and March (forthcoming).

To proceed, we require some category theory. A *category* is a collection of mathematical objects—the category's *objects*—along with a set of arrows between those objects which denote the objects which are regarded as being mathematically equivalent. So, for example, for manifolds $M$ equipped with Lorentzian metrics $g_{ab}$—so-called *Lorentzian manifolds* $(M, g_{ab})$—the appropriate category has objects which are pairs $(M, g_{ab})$ and morphisms which are *isometries*, i.e., smooth maps between metrics. A map between categories is called a *functor*.[19]

Now let $\mathcal{M}_n$ denote the category of smooth, $n$-dimensional manifolds, with

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[17] See e.g. Arntzenius (2012), Gomes (2025), Jacobs (2023), and Maudlin (2007), amongst many others.

[18] As always, '$\circ$' notation means: apply the rightmost map first.

[19] For further details on these notions at a level aligned with this article, see e.g. Weatherall (2016a).

smooth embeddings as morphisms.[20] And let $\mathcal{FB}$ denote the category the objects of which are smooth fibre bundles and the morphisms of which are smooth bundle morphisms (in the sense introduced above). A *natural bundle* (over $n$-manifolds) is then a functor $F : \mathcal{M}_n \to \mathcal{FB}$ such that (1) for every object $M$ of $\mathcal{M}_n$, $FM$ is a bundle whose base space is $M$, and (2) for every morphism $\varphi : M \to N$ of $\mathcal{M}_n$, the morphism $F\varphi$ of $\mathcal{FB}$ is of the form $(\varphi_*, \varphi)$, where the maps $\varphi_*$ induced from fibres of $FM$ to fibres of $FN$ are diffeomorphisms.[21]

The notion of a natural bundle gives us the two axes of modal variation which we declared ourselves to be after at the beginning of this subsection: for every object $M$ in $\mathcal{M}_n$—i.e., every possible arena of spacetime points under consideration—is associated via the functor $F$ a bundle $FM$ of possible values of fields on $M$.

## 3.3 Natural equations

With the background on fibre bundles and natural bundles in hand, we can consider defining dynamics for fields in this formalism. In order to do so, we first need to introduce the notion of a *jet bundle*. Here is the motivation for jet bundles. Suppose one has some bundle $B \xrightarrow{\pi} M$. If one has a section $s$ of that bundle, then one might want to speak not only of the values of $s$ at each $p \in M$, but also of its (first, second, etc.) derivatives there. A *k-jet* at $p$ encodes this structure: it encodes the possible values of a field and the possible values of its derivatives up to order $k$ at that point $p$. If one then repeats this for each $p \in M$, then one obtains the *k-jet bundle* over $M$, $J^k(B)$.

One significant merit of jet bundles is that they facilitate geometrical discussion of field values and their derivatives globally and without recourse to coordinates. But a further crucial merit for our purposes is that they facilitate a geometrical understanding of equations which describe the evolution of fields on $M$: a $k$th order system of partial differential equations on sections of $B \xrightarrow{\pi} M$ is nothing other than a closed embedded submanifold $E$ of $J^k(B)$. This should be relatively straightforward to see, since $E$ assigns to each $p \in M$ the space of possible field values and their derivatives at $p$ which are the solutions to that partial differential equation.

Finally, we can bring this into contact with the natural bundles framework: as Weatherall and March (forthcoming, pp. 7ff.) elaborate, given any natural bundle $F : \mathcal{M}_n \to \mathcal{FM}$ and a manifold $M$ which is an object of $\mathcal{M}_n$, we can define a system of partial differential equations on $FM \to M$ as a submanifold of a jet bundle over $FM$ of suitable order.[22] A *natural equation* on a natural bundle is an assignment, to every object in $\mathcal{M}_n$, of a submanifold of some finite order jet bundle over the natural bundle over that manifold in such a way that it

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[20] For the definition of a smooth embedding see e.g. nLab authors (2025). The details will not be relevant for this article.

[21] A diffeomorphism between manifolds is a smooth map between those manifolds with smooth inverse.

[22] Also satisfying appropriate regularity conditions: see Weatherall and March (forthcoming, p. 6).

is preserved by smooth mappings and their lifts.[23] A *natural theory* is a natural bundle and a natural equation on that bundle.

**SR1** fails to have this lift property, and hence is not a natural theory. By contrast—if one does not restrict *ab initio* to a particular manifold $M$—**SR2** *is* a natural theory (natural over the category of smooth manifolds, that is). As March (2025a, pp. 28–29) explains in more detail:

> As a second example, consider the theories which Pooley (2017) and Read (2023) call SR1 and SR2. SR1 is a second-order theory formulated on the natural bundle $K$ of real scalar fields, for a (fixed) Minkowski spacetime background $(M, \eta_{ab})$, with $E \subset J^2 KM$ given by the source-free Klein–Gordon equation $\nabla_n \nabla^n \varphi = 0$ determined by $\eta_{ab}$ and its associated Levi-Civita connection $\nabla$. This theory is not natural, for exactly the same reason as the Maxwell example discussed above. SR2, meanwhile, is a second-order natural theory $(\mathfrak{E}, K \times G)$ on the bundle $K \times G : \overline{\mathcal{M}}_n \to \mathcal{F}B$, with $\mathfrak{E}$ the natural equation defined by taking the source-free Klein–Gordon equation, at each point, relative to the metric and its Levi-Civita derivative operator at that point, whenever that Levi-Civita derivative operator satisfies $R^a{}_{bcd} = \mathbf{0}$ there. It is straightforward to see that $(\mathfrak{E}, K \times G)$ is a (second-order) naturalization of $E$ over $G$.

Clearly, though: if **SR2** is natural, then the notion of naturality butts against various authors' intuitions that this theory is *not* substantively generally covariant (i.e. background independent). We return to this issue in the next section.

## 3.4 Gauge naturality

Before closing this section, there is one further concept which we should introduce: the generalisation of naturality known as *gauge naturality*. Before we get to the definition of gauge naturality, we should elaborate on the motivations for this concept. As March and Weatherall (forthcoming, pp. 8, 13) observe, classical Yang–Mills theory is *not* natural in the sense introduced above:[24]

> The bundles encountered in Yang–Mills theory are not generally constructed from (just) the structure of the base space. This, in turn, means that there is no generally applicable and uniform—i.e., no natural—way to lift diffeomorphisms to act on the fibers of these bundles. (March and Weatherall forthcoming, p. 13)

This issue motivates the move to the concept of *gauge* naturality. Here is how March and Weatherall (forthcoming, pp. 14–15) explain the transition:

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[23] For further details, see Weatherall and March (forthcoming, p. 8).

[24] Cf. Dewar (2020).

> The key move is to change the category that acts as the domain of the natural bundle functor, so that the objects in that category are not the base space of the bundle under consideration but rather principal bundles, for some fixed structure group $G$, over that base space.

Following March and Weatherall (forthcoming, p. 15), a precise definition of gauge naturality is as follows. We define a category $\mathcal{PB}_n(G)$ whose objects are principal $G$-bundles over $n$ dimensional manifolds and whose arrows are principal bundle morphisms whose action on the base space is a smooth embedding. A *gauge natural bundle* is a functor $F : \mathcal{PB}_n(G) \to \mathcal{FB}$ satisfying the following conditions:

1. the action of $F$ on objects preserves their base space, i.e., it takes principal bundles over a manifold $M$ to fibre bundles over $M$;
2. the action of $F$ on arrows preserves their action on the base space; and
3. for every object $\pi : P \to M$ of $\mathcal{PB}_n(G)$ and open set $U \subseteq M$, the inclusion arrow $(i, 1_M)$, which takes the sub-bundle $\pi^{-1}[U] \to U$ into $P \to M$, is mapped to the inclusion arrow taking $q^{-1}[U] \to U$ into $F(\pi : P \to M)$, where $q$ is the projection map associated with $F(\pi : P \to M)$.

Conceptually, what we are doing here is checking that the objects of the theory under consideration (*viz.*, the connection, in the case of classical Yang–Mills theory) transform covariantly with principle bundle automorphisms—even though they might not do so (as is indeed the case for Yang–Mills connections) with respect to just diffeomorphisms on the base manifold. March and Weatherall (forthcoming) propose gauge naturality as a generalisation of naturality, and hence of general covariance, suited to gauge theories such as Yang–Mills theory. In our view, there is more to be said regarding this claim. Although we focus in the next few sections on the notion of naturality only (rather than gauge naturality), we return to these issues in §6.

# 4 Naturality and general covariance

Now that we have both surveyed the state of play in existing discussions of background independence (§2) and introduced the technical notions of (gauge) naturality and natural theories (§3), we are in a position to assess and evaluate how these two threads interact with each other.

In §4.1, we make some preliminary remarks regarding claims by March and Weatherall (forthcoming) on general covariance. In §4.2 we compare general covariance-*qua*-naturality with the existing proposals for cashing out background independence, especially the proposal due to Belot (2011). In §4.3 we appropriate the insights from those comparisons in order to propose a notion of *substantive* general covariance using the resources of natural bundles and natural theories.

## 4.1 Naturality and general covariance

As mentioned above, March and Weatherall (forthcoming) propose to understand general covariance in terms of (gauge) naturality. (As mentioned in §3.4, we focus for the time being on naturality, and defer discussion of gauge naturality to §6.) But when—with naturality in mind—March and Weatherall (forthcoming) write that "we will consider just minimal necessary conditions for general covariance", this claim has to be treated with a certain degree of care.

The reasons for this are twofold. On the one hand, the more 'standard' definition of general covariance which one can find in Definition 1 is strictly weaker than the notion of naturality, as one can see immediately in the fact that it is satisfied by *both* **SR1** and **SR2**. As such, when March and Weatherall (forthcoming) speak of "minimal necessary conditions for general covariance", they cannot (despite their reference to 'Kretschmannian general covariance') have in mind such a definition—rather, they are better read as offering minimal necessary conditions for a *substantive* notion of general covariance.[25]

And yet, it is clear that naturality cannot be *sufficient* for a substantive notion of general covariance—the reason begin that we have already seen that naturality fails the 'litmus test' of **SR2**, in the sense that it arbitrates that that theory *is* generally covariant (by dint of its being natural). This raises the following question: can one avail oneself of the resources of natural bundles and natural theories in order to provide a definition of background independence which cuts more finely than naturality alone?

Before we get to answering this question, let us make one further observation. In a clear sense, cashing out general covariance in terms of naturality aligns closely with the notion of diffeomorphism invariance presented by Pooley (2017) in the form of Definition 2—but now allowing the manifold to vary (recall that a natural bundle is a map from $\mathcal{M}_n$—the category of smooth, $n$-dimensional manifolds, with smooth embedding as morphisms—to $\mathcal{FB}$). Cf. March and Weatherall (forthcoming, p. 12), who write: "The objects under consideration in a generally covariant theory have to exhibit "diffeomorphism" covariance in the sense made precise by the fact that a natural bundle is functorial over smooth manifolds." In this sense, naturality is a very natural(!) improvement on the notion of diffeomorphism invariance that is Definition 2. But it suffers from the same discontents, insofar as both definitions are unable to identify theories such as **SR2** as *not* substantively generally covariant (i.e., as background *dependent*). To repeat, though: not being committed to a particular manifold is a very natural thing to demand in the context of discussions of background independence. So plausibly: for substantive general covariance, one needs naturality, plus something extra.[26]

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[25] *Vis-à-vis* the 'standard' definition of general covariance that is Definition 1, one option would be to read March and Weatherall (forthcoming) as not engaging in conceptual *analysis*, but rather in conceptual *engineering* (cf. Cappelen (2018)). We will not pursue this thought further here.

[26] Read (2023, ch. 4) considers various different versions of Newton–Cartan theory—i.e., the curved spacetime version of Newtonian gravity (on which see Malament (2012, ch. 4))—and

## 4.2 Comparison with existing analyses

*Prima facie*, the criterion of naturality differs from that of the three approaches to background independence presented in §2.4. For example, a theory can contain absolute objects and yet be natural: this is the case for **SR2**, as well as for various versions of general relativity (see Pitts (2006)). (This latter point, indeed, is one of a litany of problems for the 'absolute objects' proposal that is Definition 3: see Pitts (2006), Pooley (2017), and Read (2023) for further discussion.) Moreover, theories might be natural—in the sense of having natural equations—and yet either lack Lagrangian formulations or possess Lagrangian formulations with 'unphysical fields' which violate Definition 4: the version of **SR2** which uses an 'unphysical' Lagrange multiplier field to enforce ${R^a}_{bcd} = 0$ would be one example of the latter.[27]

While Belot's proposed definitions of background independence might also appear conceptually removed from the concept of naturality, there is perhaps more to explore here. To this end, it is worth noting that all three of Belot's proposal, Definition 4 (in terms of admitting a Lagrangian formulation with no unphysical fields), and the absolute objects proposal have a common conceptual core: all three seek to capture (in different ways) the idea that in a background independent theory, all the geometrical structures of a theory (and all the degrees of freedom of those geometrical structures) should be 'dynamical' in some sense. (To repeat, each of these proposals cashes out the relevant notion of 'dynamical' in different ways—for Belot, it has to do with covarying with physical degrees of freedom, for the absolute objects proposal, it has to do with varying (locally) model-to-model, for the Lagrangian proposal, it has to do with being subject to Lagrangian variation.) In what follows, and drawing from ideas from Geroch (1996), we wish to explore what can be said about background independence in terms of a slightly different notion of what it is for the geometrical structures of a theory to be 'dynamical', which fits closely with the focus of §3 on the dynamical equations of a physical theory—namely, in terms of those geometrical structures admitting couplings to matter fields.

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notes that some of them have fixed fields and others do not. The tools of naturality allow one to draw distinctions between these versions of the theory a little more cleanly: (i) versions of Newton–Cartan theory with fixed fields are not natural; (ii) versions of Newton–Cartan theory without fixed fields are natural; (iii) even then, such versions might be more akin to **SR2** in the sense that (say) the spatial and temporal metrics lack their own dynamics; (iv) one could nevertheless build natural versions of Newton–Cartan theory in which these fields do have their own dynamics (this might include 'Type II Newton–Cartan theory', reviewed in Hartong et al. (2023) and which is discussed by Read (2023, ch. 4)); (v) all of this provides another illustration of the point made by March (2025a, p. 29) that naturalisation is not unique. (We discuss Newton–Cartan theory further in §4.3 below.)

[27] One obvious task to pursue would be to extend the formalism of natural theories to those written in the Lagrangian framework—see Anderson and Pohjanpelto (2012) and Krupka (2015). We will leave this technical project for another day, and note only that a 'natural' Lagrangian might still violate some of the prohibitions (on 'unphysical fields' in particular) present in Definition 4.

### 4.3 A natural approach to background independence

To proceed with our new proposal for background independence in terms of the machinery of natural theories, we need to restrict attention to theories the equations of which are *quasilinear*.[28] Recall that given any bundle $B \xrightarrow{\pi} M$, the bundle $J^kB \to J^{k-1}B$ (for any $k$) is an affine bundle, with modelling vector bundle $S^kT^*M \otimes V^*B/U$. A quasilinear ($k$th-order) equation $E \subset J^kB \to J^{k-1}B \to ... \to B \xrightarrow{\pi} M$ is one for which the submanifold $E$ is actually an affine subbundle of $J^kB \to J^{k-1}B$.

For our purposes, the point of this restriction is that quasilinear equations come equipped with a notion of a *space of sources*. To see this, observe that since $J^kB \to J^{k-1}B$ is an affine bundle, any affine subbundle $E$ thereof gives rise to an affine bundle morphism $q$—effectively, a quotient mapping—from $\pi_B^{(k,k-1)} : J^kB \to J^{k-1}B$ to a vector bundle over $J^{k-1}B$ (naturally considered as an affine bundle), whose fibre, at each point in $J^{k-1}B$, is the quotient space of the fibre of $J^kB$ at that point by the fibre of $E$ at that point, i.e. the quotient bundle $(VB \otimes S^kT^*M)/U$, where $U$ is the modelling vector bundle for $\pi_{B|E}^{(k,k-1)} : E \to J^{k-1}B$. This means that, given a section $\psi : M \to B$, $E$ can be expressed as an 'equation' as follows: $q(j^k\psi) = \mathbf{0}$. But now suppose that we move to a different ($k$th-order, quasilinear) equation, with the same modelling vector bundle $U$. Then we would have $q(j^k\psi) = j$, where $j$ is any section of $(VB \otimes S^kT^*M)/U \to J^{k-1}B$. Thus we can think of sections of $(VB \otimes S^kT^*M)/U \to J^{k-1}B$ as giving rise to a family of 'sourced' equations, relative to our original 'unsourced' equation $E$.

The next point to note is that, in general, what are considered the permissible 'sources' for an equation do not span the full space $S^kT^*M \otimes V^*B/U$. This is perhaps clearest when one considers examples. The vacuum Einstein's equation can be formulated as a first-order (natural) equation on the bundle $Q \times G$ whose fibres are 50-dimensional manifolds of pairs $(\nabla, g_{ab})$ of Lorentzian metrics and affine connections. The equation (at each $M$) consists of all points of the 200-dimensional $J^1(Q \times G)$ satisfying the conditions

$$R_{ab} = \mathbf{0}, \tag{4}$$

$$\nabla_a g_{bc} = \mathbf{0}. \tag{5}$$

This space is an affine subbundle of $J^1(Q \times G) \to Q \times G$, with fibre dimension 90; thus the quotient space $S^1T^*M \otimes V^*B/U$ has fibre dimension 110. But the space of sources standardly considered for Einstein's equation is the 10-dimensional subspace of this, consisting of tensors of the form $T_{ab} - 1/2 g_{ab} T^n{}_n$, where $T_{ab}$ is symmetric.

We now return to background independence. Let $E$ be a quasilinear equation on a bundle $F \to B \xrightarrow{\pi} M$, and suppose that we have identified the fields in $B$ as 'geometrical' (in some appropriate sense or other). Let the 'geometrical

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[28] This is not a substantive restriction, since almost all field equations considered in physics are quasilinear or can be recast as such, including Klein–Gordon theory, Maxwell theory, Einstein's equation, special relativity *à la* **SR2**, etc.—see Geroch (1996).

degrees of freedom' of the theory be $b$, the dimension of space of independent geometrical degrees of freedom of the theory (at each point).[29] Now suppose that space of permissible sources for $E$ form a subbundle of $S^kT^*M \otimes V^*B/U$ with fibre dimension $d$. Then we could call $d$ the 'dynamical degrees of freedom' of the theory—representing the space of allowed couplings of geometry to matter fields (at each point) which could enter into the equation $E$. Finally, we could say that every geometrical degree of freedom of the theory corresponds to a dynamical degree of freedom iff $g \leq d$. A theory *none* of whose geometrical degrees of freedom were dynamical degrees of freedom would be one for which $g \neq 0$ but $d = 0$. A theory *some* of whose geometrical degrees of freedom were dynamical degrees of freedom would be one for which $0 < d < g$.

In effect, we propose that combining this notion of 'degrees of freedom matching' inspired by Belot (2011) with the notion of naturality offers a plausible explication of what it is for a theory to be background independent, which can capture various authors' intuitions that (e.g.) **SR2** is background dependent whereas general relativity is background independent, whilst avoiding many of the problems facing Definitions 3, 4, and Belot's proposal. Explicitly, we could say that a natural theory is fully background independent iff all of its geometrical degrees of freedom correspond to dynamical degrees of freedom. A theory which was not natural, or none of whose geometrical degrees of freedom were dynamical degrees of freedom, would be fully background dependent. And a natural theory some of whose geometrical degrees of freedom were dynamical degrees of freedom would be neither fully background independent nor fully background dependent.

Let's see how this works with some examples. The space of permissible sources for the vacuum Einstein's equation is the 10-dimensional space of tensors of the form $T_{ab} - 1/2 g_{ab} T^n{}_n$, where $T_{ab}$ is symmetric, making $d = 10$. Since $g = 10$, this makes the theory fully background dependent. By contrast, for the equation $R^a{}_{bcd} = \mathbf{0}$, the space of permissible sources is (apparently) 0-dimensional—reflecting the fact that there are no known matter theories which couple directly to Riemann tensor. But the geometrical degrees of freedom are the same as for the vacuum Einstein equations. This means that $g = 10$ whilst $d = 0$, making **SR2** fully background dependent.

An interesting intermediate case is given by Newton–Cartan theory.[30] This is a theory with models $\langle M, t_a, h^{ab}, \nabla, T^{ab} \rangle$, where $t_a$ and $h^{ab}$ are orthogonal temporal and spatial metrics ($t_n h^{na} = \mathbf{0}$), $\nabla$ is a compatible affine connection ($\nabla_a t_b = \mathbf{0}$ and $\nabla_a h^{bc} = \mathbf{0}$), and $T^{ab}$ is the mass–momentum tensor for whatever matter fields are present. The vacuum field equations are given by

$$R_{ab} = \mathbf{0}, \tag{6}$$

$$R^a{}_b{}^c{}_d - R^c{}_d{}^a{}_b = \mathbf{0}, \tag{7}$$

$$R^{ab}{}_{cd} = \mathbf{0}. \tag{8}$$

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[29] Which geometrical degrees of freedom of a theory are *independent* will be an interpretative matter—for discussion, see e.g., March (forthcoming).

[30] Cf. footnote 26.

Here, we have $g = 1 + 6 + 9 = 16$ (since only 9 degrees of freedom of the affine connection are left unconstrained by the metric compatibility conditions). On the other hand, the space of permissible sources for the vacuum Newton–Cartan field equations is one-dimensional, i.e., we may couple the vacuum geometrised Poisson equation $R_{ab} = \mathbf{0}$ to a term of the form $4\pi\rho t_a t_b$ for some scalar field $\rho$, and so we have $d = 1$. This makes the theory neither fully background dependent nor fully background independent. We take this result to be intuitively the correct verdict: Newton–Cartan theory thus construed is natural, and *some* of the degrees of freedom of the Newton–Cartan affine connection are dynamical, in the sense of being subject to coupling to matter fields, but not *all* of them are. By contrast, Newtonian gravitation set on Galilean spacetime—in which the (vacuum) field equations are replaced by $R^a{}_{bcd} = \mathbf{0}$—is fully background dependent, since again we have $d = 0$.[31]

There are several features of this proposal worth highlighting. To begin, the notion of background dependence here is insensitive to the *actual* couplings of matter fields to spacetime geometry present in the theory—rather, what matters is the space of *possible* couplings of matter fields to spacetime geometry that are being countenanced. In other words, whether or not a theory is background dependent, on this proposal, depends on whether the dynamics governing the 'geometrical' fields permit that 'same equation' to couple to matter sources, and in what ways. This has two consequences. The first is that theories such as the one Pooley (2017) calls **GR2**—which consists of the vacuum Einstein equation along with the Klein–Gordon equation—are background independent in just the same way as is standard general relativity. The second is that this definition of background (in)dependence requires that we have some independent handle on the space of permissible sources for an equation. One might worry that this makes the definition of background (in)dependence insufficiently 'intrinsic', since it requires some external input about what the space of permissible couplings to matter of the theory is. But we do not think that this is such a problem. That is, most field equations considered in physics—such as Einstein's equation, or the geometrised Poisson equation, or Maxwell's equation, or the field equations of **SR2**—are standardly presented with a space of permissible sources *built in*, i.e., in textbook general relativity, one finds $R_{ab} = T_{ab} - 1/2 g_{ab} T^n{}_n$ rather than $R_{ab} = \mathbf{0}$. Or in other words, in practice, we generally do seem to have a handle on the space of permissible sources for an equation.

Next, we note that this proposal provides a way around some of the issues facing Belot's original characterization of the 'geometrical' and 'physical' degrees of freedom of a theory. For example, as Read (2023) notes, one might worry that even in general relativity, distinct (i.e., non-isomorphic) configurations of matter (e.g., for electromagnetic fields) can give rise to the *same* stress–energy tensor, and so there is not (always) a one-to-one correspondence between the physical and geometrical degrees of freedom of the theory, making it not fully background independent. But on our characterization of the geometrical and

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[31] As such, this modification of Belot's proposal avoids some of the critiques of the original proposal raised in the context of Newton–Cartan theory by Read (2023, ch. 3).

dynamical degrees of freedom of the theory, this worry is averted—what matters is the space of sources which can couple to the geometry of the theory, regardless of how one is supposed to associate particular matter field configurations with a particular source term. On the other hand, combining the notion of 'degrees of freedom matching' with the notion of naturality provides a way out of some cases where Belot's definition of (full) background independence seems too weak. For example, the case that Read (2023) considers of a theory of a Lorentzian metric and scalar field, subject to the conditions that in every model, both $g_{ab}$ and $\varphi$ adopt inequivalent configurations, will not be natural (on any way of specifying dynamics for this theory), and so is not background independent.

That said, there are also some cases where the advantages of our approach over Belot's seem less clear-cut. For example, the asymptotically flat sector of general relativity—which Belot's proposal adjudges to be nearly background independent, at least if models related by a diffeomorphism which is not asymptotic to the identity at spatial infinity are regarded as gauge inequivalent—will come out as background *dependent* on the account proposed here, since there is no natural way to define spatial infinity (and *a fortiori* flatness at spatial infinity) across all manifolds. This reveals rather different diagnoses between Belot's proposal and our own as to what the failure of full background independence in asymptotically flat general relativity amounts to. For Belot, the 'background' in asymptotically flat general relativity amounts to a choice of a particular flat metric at spatial infinity.[32] For us, it amounts to a choice of a particular spacetime manifold (with its associated definition of spatial infinity). On the other hand, the failure of asymptotically flat general relativity to be a natural theory does not depend on whether one shares Belot's view that models related by a diffeomorphism which is not asymptotic to the identity at spatial infinity are gauge inequivalent—and so in that sense, the verdict issued by our proposal here is more robust.

## 5 The hole argument

No discussion of general covariance would be complete without a concomitant discussion of its most formidable offspring: the hole argument. For background on the hole argument, see Norton et al. (2023) and references therein; suffice it to say for our purposes here that the general covariance (understood now as diffeomorphism invariance) of general relativity seems to permit arbitrary smooth deformations of spacetime structure (and material contents thereon) to the future of some spacelike hypersurface (assuming indeed that the spacetime is foliable into hypersurfaces), thereby leading to worries about indeterminism (and underdetermination: see Earman and Norton (1987) and Pooley and Read (2025)).

To see this in a little more detail: the problem here is essentially that, given the diffeomorphism invariance of general relativity, both $\langle M, g_{ab}, \Phi \rangle$ and $\langle M, d_* g_{ab}, d_* \Phi \rangle$—where $M$ is a differentiable manifold, $g_{ab}$ is a Lorentzian metric

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[32] Cf. Fletcher (2020).

on $M$, $\Phi$ is a placeholder for matter fields, and $d_*$ denotes the pushforward map under some diffeomorphism $d : M \mapsto M$—are DPMs of the theory. So if $d$ is non-trivial to the future of some spacelike hypersurface $\Sigma$, we have the above-mentioned problem of indeterminism: the theory seems to fail to determine uniquely what will happen to the future of (the moment represented by) $\Sigma$.

Evidently, part of the issue here is that, in the above formulation, the metric field $g_{ab}$ is not a fixed field, and as such both of the above models are DPMs. But if $g_{ab}$ *were* a fixed field, it would not be the case that both of the above models were legitimate solutions of the theory, unless it were the case that $d_* g_{ab} = g_{ab}$. This, in general, would suffice to *block* the hole argument. There is a general lesson here: theories which have fixed fields (and so which are not natural in the sense of March and Weatherall (forthcoming)—recall our above discussion of **SR1**, and we come return to this issue in greater detail below) in general do not face the challenge of the hole argument.[33]

This point chimes well with the observation made by Weatherall and March (forthcoming), that natural theories are confronted with hole argument-type problems, whereas non-natural theories are not. To make this point, Weatherall and March (forthcoming, p. 20) begin by defining a notion of 'sufficient richness':

> Let us say that a natural theory $E$ on a natural bundle $X\colon \mathcal{M}_n \to \mathcal{FB}$ is 'sufficiently rich' if it admits solutions $\varphi\colon M \to XM$ for which there exist diffeomorphisms $\chi\colon M \to M$ such that $\chi_* \circ \varphi \neq \varphi \circ \chi$. To say that a theory is sufficiently rich means that it admits solutions that are non-constant, in the sense that they vary from point to point by some standard of comparing points.

Weatherall and March (forthcoming, p. 20) then go on to prove the following theorem: no sufficiently rich natural theory admits a well-posed initial value formulation. This of course squares perfectly with our above observation that theories with fixed fields do not typically face hole argument-type problems, whereas those without fixed fields typically do. (The reason of course being that the former are typically not natural, whereas the latter typically are: recall again our running example of **SR1** and **SR2**.)[34]

On this result, Weatherall and March (forthcoming, p. 3) write that:

> We take this result to provide insight into the structure of natural equations, but also to advance the literature on the status of the hole argument. In particular, it isolates the sense in which the hole argument rests on an 'error' (but see Weatherall (2018); Bradley and Weatherall (2022)), from the point of view of ordinary mathematical

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[33] There are some delicate issues here to do with Newton's Corollary VI in the context of non-relativistic gravity: see Cheng and Read (2022) and Saunders (2003) for discussion. We won't go into this further here.

[34] When Weatherall and March (forthcoming, p. 20) write that "The proof of this theorem should seem familiar: it is just the hole argument.", this is arguably too fast given the various *interpretative* inputs which are also necessary to get the argument off the ground: see Menon and Read (2023) and Pooley (2022) for further discussion.

> practice, which is that the sense of uniqueness of solution at issue in the argument not natural.

Later, they make a related point (p. 23):

> Formalist responses to the hole argument, [footnote suppressed] meanwhile, can be viewed as insisting that well-formed mathematical claims must be suitably natural—and therefore that deploying something like well-posedness in the context of a natural equation involves a subtle mathematical error, a mismatch between the sort of equation under consideration and the compatible uniqueness properties of solutions.

Again, these claims are worth unpacking. The first point to make is that the scare quotes on 'error' in the first of the above two quotations are well-placed: there is no error *sensu stricto* in the hole argument: one will not 'go to jail' for comparing mathematical models (built using standard set-theoretic foundations: see Cheng and Read (forthcoming) and Read (2025b)) using the identity map $1_M$, despite this not being an isometry of the Lorentzian manifolds under consideration (cf. Menon and Read (2023) and Pooley and Read (2025)). The second point to make is that this 'error'—i.e., indulging in constructions which are not natural—is *prima facie* not the same as the arguments presented previously by 'formalists' about the hole argument—e.g., it is not the same as (i) the 'equivocation argument' from Weatherall (2018), that the hole argument relies on a problematic equivocation of maps between manifolds, or (ii) the 'argument from mathematical structuralism' from Weatherall (2018), that one is only permitted to compare models of a theory via maps which witness their being isomorphic, or (iii) the 'argument from semantic ascent' from Bradley and Weatherall (2022), which is that set-theoretic constructions go beyond the 'language' of general relativity. The former two of these arguments have been tackled by Pooley and Read (2025); the third has been tackled by Cheng and Read (forthcoming)—but in any case, our point here is not to entertain whether these arguments do or do not succeed, but merely to point out that the 'argument from naturality' is (*prima facie*) a different argument again, best understood as bearing a 'family resemblance' to other formalist arguments which have been offered in the recent literature.[35]

In the end, the observations from Weatherall and March (forthcoming) regarding naturality and determinism are sufficiently interesting to stand on their own two feet, and there are certainly a number of conceptual and pragmatic reasons which speak in favour of the use of natural theories, many of which we have already discussed above. What we pause over here is only that the remarks which Weatherall and March (forthcoming) make in the context of naturality and the hole argument are a direct continuation of other 'formalist' arguments against the hole argument. And in any case, in the absence of further details,

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[35] Other 'formalist' approaches, e.g. those of Dougherty (2020), Ladyman and Presnell (2020), Mundy (1992), and Shulman (2017), are different yet again.

these points are unlikely to be sufficient to convince those authors who continue to believe that the hole argument remains a genuine philosophical problem.

## 6 The metaphysics of gauge natural theories

Let us return now to the concept of gauge naturality, which was introduced in §3.4. Why should we regard gauge naturality as being an interesting and philosophically fecund successor to naturality, and in turn (if we follow the lead of March and Weatherall (forthcoming)) to general covariance? Here is a motivation from March and Weatherall (forthcoming, p. 18) to care about gauge naturality:

> Most importantly, it is principal bundle morphisms that play the role of smooth maps on the base space in considerations of general covariance for associated vector bundles. This means that the coordinating role of the principal bundle is not just to determine what it means for the same connection to act on different vector bundles, but also what it means to act on sections of different bundles with a single coordinate transformation or smooth (bundle) map.

As one can see, the first sentence here points out that principal bundle automorphisms inherit the role of diffeomorphisms on the base space, the latter of course (as we have seen above) having something to do with the notion of general covariance (although, as we have also seen, even those connections—i.e., the connections between diffeomorphism invariance and (substantive) general covariance—are less direct than one might have thought initially). But arguably there is more to be said here regarding the metaphysics—for whether the gauge naturality of Yang–Mills theory (say) suffices to ameliorate all philosophical (and in particular metaphysical) worries about its non-naturality will depend upon one's metaphysical commitments.

To see this, first note that a failure of naturality amounts not merely to a failure of (a plausibly necessary condition for a substantive notion of) general covariance, but also to a failure to be working with *geometric objects*.[36] The concept of a geometric object goes back to the work of Schouten (1926) and collaborators in the 1920s; essentially, the idea is that a geometric object must have 'well-defined transformation rules', in the sense that transforming the components of that object from some coordinate system $A$ to some coordinate system $C$ should yield the same results as transforming from $A$ to $C$ via the intermediary of another coordinate system $B$ (for a clear presentation of this idea, see Duerr (2021)). The notion of a geometric object was in fact ultimately updated into the formalism of fibre bundles and natural bundles by Nijenhuis (1952, 1972)—a former student of Schouten—(and later e.g. Fatibene and Francaviglia (2003) and Kolář et al. (1993)), and indeed the connections here should be obvious: if diffeomorphisms on the base space do not 'lift' to

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[36] Cf. footnote 4.

diffeomorphisms in the bundle, then the object whose field values are sections of that bundle, or connections on that bundle, etc., will fail to be a geometric object on the base space.

With this in mind, let's return to gauge naturality and matters of metaphysics. Suppose that one has a roster of basic metaphysical commitments roughly along the lines of Lewis' 'Humean mosaic', according to which "all there is to the world is a vast mosaic of local matters of particular fact, just one little thing and then another" (Lewis 1986, p. ix). In that case, one will (let's grant for the time being) be committed to a four dimensional manifold, with various fields defined thereon. But in that case, one will worry that one cannot be committed to (say) Yang–Mills connections, as they fail to be natural over the manifold. This failure of naturality means that they fail to induce geometric objects on the manifold without the addition of extra structure (in this case, e.g. a choice of local section along which to pull the connection back to the manifold, yielding a gauge potential). The problems with an object's components lacking well-defined transformation rules can be understood in terms of Klein's Erlangen approach to geometry, which characterises geometrical structures as invariants under certain coordinate transformations (see Read (2022) and Wallace (2019)): if this fails, then it is simply not clear what the invariant structure associated with those transformations is supposed to be.

If one has this Humean picture, and yet wishes to retain ontological commitment to these kinds of objects, then it seems that one is confronted with a threefold choice when considering theories such as Yang–Mills theory which are not natural but which are nevertheless gauge natural:

1. Retain the Humean vision, but enrich the dimension of the manifold about which is a substantialist to include the entire bundle structure. Then, objects such as Yang–Mills connections will—by virtue of the the theory being gauge natural—be geometric objects over one's updated basic metaphysical arena. (This chimes with the bundle substantialism of Arntzenius (2012) and Jacobs (2023). Note that Jacobs (2023) moots for even further geometric ascent, choosing (for reasons to do with having a local and separable ontology) to reify another bundle, known as the 'bundle of connections'.)

2. Reject Lewis' Humean vision, and accept that there are primitive facts—about Yang–Mills connections, etc.—which do not supervene on the mosaic. (Perhaps something of Humeanism could still be retained in the form of the version endorsed by e.g. Demarest (2017).)

3. Understand Yang–Mills connections as encoding *relational* facts between points on the spacetime manifold. (For brief discussion of this option, see Hirsch (2017).)

These options can be brought out by considering a closely related issue, discussed by Jacobs and Read (2025): that regarding the ontological status of gravitational energy in general relativity. There, the worry is that the obvious candidates to represent local gravitational stress-energy in general relativity—so-called 'pseudotensors'—are not geometric objects on the manifold, lacking well-defined transformation properties, etc. One way to tackle this problem is to note that different pseudotensors are pullbacks (along different local sections) of an object defined on the bundle of linear frames over the manifold known as the 'Sparling form'. As such, if one is willing to be a bundle substantialist about the frame bundle, one can render local gravitational stress-energy a *bona fide* geometric object by associating it with the Sparling form.[37] Another option would simply be to accept that there are facts about local gravitational stress energy which do not supervene on the mosaic: but this might well strike one as problematically 'spooky'. And a third option would be to invoke the (in our view) under-explored resources of Palmer (1978a,b, 1979, 1980), who renders local gravitational stress-energy in general relativity less problematic (i.e. a geometric object) by associating it with a *bitensor*—again a relational object (since bitensors are functions of two spacetime coordinates) on the manifold.

In any case, coming back to Yang–Mills theory and gauge naturality: our point is simply that while it is all well and good to point to some successor of the naturality concept, one should not lose sight of the reasons why that was important to begin with (because, *inter alia*, it affords an ontology of geometric objects on spacetime which would otherwise be unperspicuous), and one should be attuned to whether the virtues of the naturality concept transfer over to the next context (namely, that of gauge naturality). As we have seen, when one tries to assess the ontology of gauge natural bundles, one is confronted with a range of options, none of which might seem to be maximally palatable. For, as we have seen, it seems that one must either enrich one's basic ontological arena (*per* (1)), or buy into a non-Humean ontology (*per* (2)), or buy into a new kind of relationalist ontology (*per* (3)). To some extent, these options have already been countenanced and evaluated by philosophers in e.g. Arntzenius (2012), Jacobs (2023), Jacobs and Read (2025), and Maudlin (2007), but we see now the connection with gauge naturality—and how gauge naturality as a mathematical concept relates to these metaphysical issues.

## 7 Category theory as a tool

The notion of a natural bundle involves a functor from the category of smooth manifolds to the category of fibre bundles over that manifold. However, as we have already pointed out in §5, in order to avail oneself of these resources one need not change one's approach to the objects in those categories: in this article, for example, we have been assuming standard set-theoretic foundations, and so that these objects are still structured sets (this is the same approach that one

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[37] Jacobs and Read (2025) make the case that this particular kind of fibre bundle substantivalism might be less problematic that other kinds, including fibre bundle substantialism about the principal bundles used in Yang–Mills theory. The reason for this is that the bundle of linear frames over a manifold is implicitly definable from that manifold alone, whereas this is not the case for e.g. Yang–Mills theory.

can find in e.g. March (2025c) and Read (2025a)).

But if one proceeds in this way, then one might worry that the tools of natural bundles are insufficiently *intrinsically* categorical, and one might instead hope for a reformulation of such objects in terms of some appropriately 'categorical' foundations (see Gajic (2024) for an exploration of the options here). We have no bone to pick with authors who choose to follow this route (perhaps including Dougherty (2020)[38])—we would only counter that, *regardless* of one's views on the foundations of mathematics, the tools of category theory have found wide application to many areas of the foundations of physics: not just naturality, but also e.g. theoretical equivalence (see March (forthcoming) and Weatherall (2016a,b)), the philosophy of symmetries (see March (2025c), March and Read (2025), and Read (2025a)), quantisation (see Feintzeig (2023, 2024, 2025)), and so forth. In our view, as it were, a bird in the hand (i.e., category-theoretic tools to deliver foundational insights on physics) is worth two in the bush (i.e., the above, plus *bona fide* categorical foundations and reformulation of physical theories and foundational problems in such a language).

## 8 Close

Let's wrap up. In this article, we've reviewed the state of play in the literature *vis-à-vis* proposals for cashing out a notion of substantive general covariance or (equivalently) background independence (§2); we've also explored how naturality (defined in §3) fits into this framework, and have proposed a new approach to background independence which makes use of the tools of naturality (§4). We have discussed the significance of (gauge) naturality for the hole argument (§5) and for metaphysics more generally (§6), and the manner in which naturality avails itself of category-theoretic resources (§7). All-in-all, we hope that this work serves not only to tie up a number of loose ends in the literature on this topic, but also to pave the wave for an exploration of related issues in the future. To take just one example, consider spinors: being neither natural nor gauge natural,[39] how is one to understand the physics and metaphysics of spinorial theories, and the status of those theories as substantively generally covariant or otherwise?

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[38] In personal correspondence, Dougherty has written to us that: "Whether a functor is a natural bundle depends on its set-theoretic realization, and this is the kind of fact that category theory is designed to be insensitive to."

[39] See e.g. Fatibene and Francaviglia (2003).

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