# Condensed Matter Physics and the Nature of Spacetime

This essay considers the prospects of modeling spacetime as a phenomenon that emerges in the low-energy limit of a quantum liquid. It evaluates three examples of spacetime analogues in condensed matter systems that have appeared in the recent physics literature, and suggests how they might lend credence to an epistemological structural realist interpretation of spacetime that emphasizes topology over symmetry in the accompanying notion of structure.

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## 1. Introduction

In the philosophy of spacetime literature not much attention has been given to concepts of spacetime arising from condensed matter physics. This essay attempts to address this. I look at analogies between spacetime and a quantum liquid that have arisen from effective field theoretical approaches to highly correlated many-body quantum systems. Such approaches have suggested to some authors that spacetime can be modeled as a phenomenon that emerges in the low-energy limit of a quantum liquid with its contents (matter and force fields) described by effective field theories (EFTs) of the low-energy excitations of this liquid. While directly relevant to ongoing debates over the ontological status of spacetime, this programme also has other consequences that should interest philosophers of physics. It suggests, for instance, a particular approach towards quantum gravity, as well as an anti-reductionist attitude towards the nature of symmetries in quantum field theory. Moreover, while the topic of EFTs in the philosophy of quantum field theory literature has been given some attention (*e.g.*, Castellini 2002, Hartmann 2001, Huggett and Weingard 1995), surprisingly little has been said about how EFTs arise in condensed matter systems.

The plan of the essay is as follows. Section 2 sets the stage by describing the nature of EFTs in condensed matter systems. Section 3 looks at three examples of spacetime analogues in condensed matter systems that have appeared in the physics literature: analogues of general relativistic spacetimes in superfluid Helium 4 associated with the "acoustic" spacetime programme (*e.g.*, Barceló *et al.* 2005), analogues of the Standard Model of particle physics in superfluid Helium 3 (Volovik 2003), and twistor analogues of spacetime in 4-dimensional quantum Hall liquids (Sparling 2002). Section 4 examines the notion of low-energy emergence that some authors have associated with these examples, indicating how it is distinct from a "received view" of emergence associated with phase transitions in condensed matter systems, and situating it in the general debate over the concept of emergence in the philosophy of science literature. The key claim is that low-energy emergence is minimally *epistemological* in nature. Finally, Section 5 examines the notion of universality associated with the above examples and indicates how it can inform a concept of dynamical structure that might provide the basis for an *epistemological structural realist* interpretation of spacetime. A key characteristic of this interpretation is that it emphasizes topology over symmetry in the accompanying notion of structure.

## 2. Effective Field Theories in Condensed Matter Systems

The condensed matter systems to be discussed below are highly-correlated quantum many-body systems; that is, many-body systems that display macroscopic quantum effects. Typical examples include superfluids, superconductors, Bose condensates, and quantum Hall liquids. In general, an effective field theory of such a system describes the dynamics of the states with energy close to zero. These low-energy states can take the following forms (Volovik 2003, pg. 4):

(i) Bosonic collective modes of the ground state of the system.
(ii) Fermionic excitations of the system above its ground state, referred to as "quasiparticles".
(iii) Topological defects of the ground state, the simplest taking the form of vortices.

Intuitively, one considers the system in its ground state and tickles it with a small amount of energy. The ripples that result then take one of the above three forms. An effective field theory of such low-energy states is obtained by constructing a *low-energy approximation* of the original theory. In the Lagrangian formalism, one can expand the Lagrangian of the system in small fluctuations in the field variables about their ground state values, and then integrate out the high-energy fluctuations. An example of this will be the EFT for superfluid Helium 4 below. Alternatively, in the Hamiltonian formalism, one can linearize the energy of the system about the points where it vanishes, and then construct the corresponding low-energy Hamiltonian. An example of this will be the EFT for superfluid Helium 3.

As will be readily apparent below, the Lagrangian (or Hamiltonian) of an EFT (the "effective Lagrangian") will typically differ formally from the Lagrangian of the original theory to the extent that the former cannot be embedded in the latter in the sense of a *sub-theory* (in arguably either syntactic or semantic senses of the latter). Informally, the EFT will entail *different* dynamical laws. Thus under typical notions of reduction, an EFT cannot be said to *reduce* to the original theory. This will be important in the discussion of the associated notion of low-energy emergence in Section 4.

Note finally that not all systems will admit low-energy approximations. Whether an EFT for a given system can be constructed is a contingent matter and depends on the system's dynamics. For fermionic systems, it turns out that the existence and type of an EFT in particular depends on the topology of the system's momentum space. This is related to techniques used in renormalization group theory and will be explained in more detail in Section 5 below where it will inform a notion of dynamical structure.

## 3. Spacetime Analogues in Superfluid Helium and Quantum Hall Liquids

This section critically reviews three examples of spacetime analogues in condensed matter systems. The first concerns acoustic spacetimes in superfluid Helium 4, the second concerns the Standard Model and superfluid Helium 3, and the last concerns twistors and quantum Hall liquids.

### 3.1. "Acoustic" Spacetimes and Superfluid Helium 4

The ground state of superfluids and conventional superconductors is believed to be a Bose condensate. In the case of superfluid Helium 4, this condensate consists of $^4He$ atoms (Helium isotopes with four nucleons) and can be characterized by an order parameter that takes the form of a “macroscopic” wavefunction $\varphi_0 = (\rho_0)^{1/2}e^{i\theta}$ with condensate particle density $\rho_0$ and coherent phase $\theta$ (the latter can be viewed as a measure of how correlated the constituents of the condensate are). An appropriate Lagrangian describes *non-relativistic* neutral bosons (*viz.*, $^4He$ atoms) interacting *via* a spontaneous symmetry breaking potential with coupling constant $\alpha^2$ (see, *e.g.*, Zee 2003, pp. 175, 257),

$$\mathcal{L}_{4He} = i\varphi^\dagger\partial_t\varphi - \frac{1}{2m}\partial_i\varphi^\dagger\partial_i\varphi + \mu\varphi^\dagger\varphi - \alpha^2(\varphi^\dagger\varphi)^2 \ , \qquad i = 1, 2, 3. \tag{1}$$

Here $m$ is the mass of a $^4He$ atom, and the term involving the chemical potential $\mu$ enforces particle number conservation. This is a thoroughly non-relativistic Lagrangian invariant under Galilean transformations. Importantly, it encodes both the normal state of the liquid and the superfluid state, as well as the phase transition between these two states accompanied by the spontaneously broken symmetry.

A low-energy approximation of (1) can be obtained in a two-step process[1]:

(a) One first writes the field variable $\varphi$ in terms of density and phase variables, $\varphi = (\rho)^{1/2} e^{i\theta}$, and expands the later linearly about their ground state values, $\rho = \rho_0 + \delta\rho$, $\theta = \theta_0 + \delta\theta$ (where $\delta\rho$ and $\delta\theta$ represent fluctuations in density and phase above the ground state).

(b) After substituting back into (1), one identifies and integrates out the high-energy fluctuations.

Since the ground state (as given by the macroscopic wavefunction of the condensate) is a function only of the phase, low-energy excitations take the form of phase fluctuations $\delta\theta$. To remove the high-energy density fluctuations $\delta\rho$, one "integrates" them out: One way to do this is by deriving the Euler-Lagrange equations of motion for the density variable, solving for $\delta\rho$, and then substituting back into the Lagrangian. The result schematically is a sum of two terms: $\mathcal{L}_{4He} = \mathcal{L}_0[\rho_0, \theta_0] + \mathcal{L}'_{4He}[\delta\theta]$, where the first term describes the ground state of the system and is formally identical to (1), and the second term, dependent only on the phase fluctuations, describes low-energy fluctuations above the ground state. This second term represents the effective field theory of the system and is generally referred to as the effective Lagrangian. To second order in $\delta\theta$, it takes the following explicit form:

$$\mathcal{L}'_{4He} = \frac{1}{4\alpha^2}(\partial_t\theta + v_i\partial_i\theta)^2 - \frac{\rho_0}{2m}(\partial_i\theta)^2 \ , \tag{2}$$

with $\delta\theta$ replaced by $\theta$ for the sake of notation. Here the second order term depends explicitly on the superfluid velocity $v_i \equiv (1/m)\partial_i\theta$. One now notes that (2) is formally identical to the Lagrangian that describes a massless scalar field in a (3+1)-dim curved spacetime:

$$\mathcal{L}'_{4He} = \frac{1}{2}\sqrt{-g}g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta \ , \qquad \mu, \nu = 0, 1, 2, 3 \tag{3}$$

where the curved effective metric depends explicitly on the superfluid velocity $v_i$:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = (\rho/cm)[-c^2dt^2 + \delta_{ij}(dx^i - v^i dt)(dx^j - v^j dt)] \ , \tag{4}$$

[1] The following draws on Wen (2004, pp. 82-83) and Zee (2003, pp. 257-258).

where $(-g)^{1/2} \equiv \rho^2/m^2c$, and $c^2 \equiv 2\alpha^2\rho/m$ (see, *e.g.*, Barceló *et al.* 2001, pp. 1146-1147). One initial point to note is that, if the original Lagrangian had been expanded to 1st order in $\delta\theta$, the second order term dependent on $v_i$ would vanish in both the effective Lagrangian and the effective metric, and the latter would be formally identical to a flat Minkowski metric (up to conformal constant). This suggests an interpretation of the effective metric (4) as representing low-energy curvature fluctuations (due to the superfluid velocity) above a flat Minkowski background. This is formally similar to the linear approximation of solutions to the Einstein Equations in general relativity, which can likewise be approximated by low-energy fluctuations in curvature above a flat Minkowski background metric. This formal equivalence has been exploited to probe the physics of black holes and the nature of the cosmological constant.

(i) *Acoustic Black Holes*. The general idea is to identify the speed of light in the relativistic case with the speed of low-energy fluctuations, generically referred to as sound modes, in the condensed matter case; hence the terms "acoustic" spacetime and "acoustic" black hole. In general, acoustic black holes are regions in the background condensate from which low-energy fluctuations traveling at or less than the speed of sound cannot escape. This can be made more precise with the definitions of acoustic versions of ergosphere, trapped region, and event horizon, among others. A growing body of literature seeks to exploit such formal similarities between relativistic black hole physics and acoustic "dumb" hole physics (see, *e.g.*, Barceló, *et al.* 2005). The primary goal is to provide experimental settings in condensed matter systems for relativistic phenomena such as Hawking radiation associated with black holes.

(ii) *The Cosmological Constant*. Volovik (2003) has argued that the analogy between superfluid Helium and general relativity provides a solution to the cosmological constant problem. The latter he takes as the conflict between the theoretically predicted value of the vacuum energy density in quantum field theory (QFT), and the observationally predicted value: The QFT theoretical estimate is 120 orders of magnitude greater than the observational estimate. Volovik sees this as a dilemma for the marriage of QFT with general relativity. If the vacuum energy density contributes to the gravitational field, then the discrepancy between theory and observation must be addressed. If the vacuum energy density is not gravitating, then the discrepancy can be explained away, but at the cost of the equivalence principle. Volovik's preferred solution is to grab both horns by claiming that both QFT and general relativity are EFTs that emerge in the low-energy sector of a quantum liquid.

(a) The first horn is grasped by claiming that QFTs are EFTs of a quantum liquid. As such, the vacuum energy density of the QFT does not represent the true "trans-Planckian" vacuum energy density, which must be calculated from the microscopic theory of the underlying quantum liquid. At $T = 0$, the pressure of such a liquid is equal to the negative of its energy density (Volovik 2003, pg. 14, 26). This relation between pressure and vacuum energy density also arises in general relativity if the vacuum energy density is identified with the cosmological constant term. However, in the case of a quantum liquid in equilibrium, the pressure is identically zero; hence, so is the vacuum energy density. Moreover, if the liquid is in the form of a droplet, the pressure is not zero, but scales as an inverse power of the droplet size, and this models the cosmological constant term in the Einstein equations, which scales as the inverse square of the size of the universe.

(b) The second horn is grasped simply by claiming that general relativity is an EFT. Thus, we should not expect the equivalence principle to hold at the "trans-Planckian" level, and hence we should not expect the true vacuum energy density to be gravitating.

*Interpretation*

What do the acoustic spacetime program and Volovik's related solution to the cosmological constant problem have to say about the ontological status of spacetime? Note first that the acoustic metric arises in a *background-dependent* manner. The acoustic metric (4) is obtained ultimately by imposing particular constraints on *prior* spacetime structure; it is not obtained *ab initio.*[2] A natural question then is (i) *What should be identified as the background structure of acoustic spacetimes?* Second, the implicit claim associated with both the acoustic black hole program and Volovik's solution to the cosmological constant is that acoustic spacetimes can be considered analogues of general relativistic spacetimes. A second question then is (ii) *To what extent are acoustic spacetimes analogues of general relativistic spacetimes?* Evidently, the answer to the first question will have implications for the answer to the second.

In regard to the first question, one option is to identify Minkowski spacetime as the background structure of acoustic spacetimes. This might be motivated by the explicit form of the acoustic metric (4). As indicated above, it can be interpreted as describing low-energy curvature fluctuations, due to the superfluid velocity, above a flat Minkowski background metric. In particular, (4) can be written in the suggestive form $g_{\mu\nu}dx_\mu dx_\nu = \eta_{\mu\nu}dx_\mu dx_\nu + g'_{\mu\nu}dx_\mu dx_\nu$, where the first term on the right is independent of the superfluid velocity and is identical to a Minkowski metric, and the second term depends explicitly

[2] For the moment I will leave aside the question of how this structure can be interpreted. In particular, as will be made explicit below, background-dependence of a spacetime theory does not necessarily imply a substantivalist interpretation, any more than background-independence necessarily implies a relationalist interpretation.

on the superfluid velocity. (The issue of general covariance will be addressed in the subsequent discussion below.)

A second option, however, is to identify the background structure of acoustic spacetimes with (Galilei-invariant) Neo-Newtonian spacetime. This is motivated by paying attention to the procedure by which the acoustic metric was derived. This starts with the Galilei-invariant Lagrangian (1). Low energy fluctuations of the ground state to first order obey the Poincaré symmetries associated with Minkowski spacetime, and low energy fluctuations to second order obey the symmetries of the curved acoustic metric (4).[3] From this point of view, the relation between acoustic spacetimes and Minkowski spacetime is one in which both supervene over a background Neo-Newtonian spacetime. This second option seems the more appropriate: If acoustic metrics are to be interpreted as low-energy fluctuations above the ground state of a condensate, then the background structure of such spacetimes should be interpreted as the rest frame of the condensate ground state, which obeys Galilean symmetries.

This response has implications for the second question posed above; namely, to what extent are acoustic spacetimes analogues of general relativistic spacetimes? Note first that acoustic metrics are not obtained as solutions to the Einstein equations; they are derived *via* a low-energy approximation from the Lagrangian (1) (and similar Lagrangians for other types of condensed matter systems). As noted above, this approximation results schematically in the expansion $\mathcal{L}_{4He} = \mathcal{L}_0[\rho_0, \theta_0] + \mathcal{L}'_{4He}[\delta\theta]$. To make contact with the Lagrangian formulation of general relativity, Volovik (2003, pg. 38) interprets $\mathcal{L}_{4He}$ as comprised of a "gravitational" part $\mathcal{L}_0$ describing a background spacetime expressed in terms of the variables $\theta_0$, $\rho_0$, with gravity being simulated by the superfluid velocity, and a "matter" part $\mathcal{L}'_{4He}$, expressed in terms of the variable $\delta\theta$. To obtain the "gravitational" equations of motion, one can proceed in analogy with general relativity by extremizing $\mathcal{L}_{4He}$ with respect to $\theta_0$, $\rho_0$. This results in a set of equations that are quite different in form from the Einstein equations (Volovik 2003, pg. 41), and this indicates explicitly that the *dynamics* of acoustic spacetime EFTs does not reproduce general relativity. Hence acoustic spacetimes cannot be considered *dynamical* analogues of general relativistic spacetimes.

While acknowledging that acoustic spacetimes do not model the dynamics of general relativity, some authors have insisted, nonetheless, that acoustic spacetimes account for the *kinematics* of general relativity:

> ... the features of general relativity that one typically captures in an "analogue model" are the *kinematic* features that have to do with how fields (classical or

[3] Whether or not (4) exhibits non-trivial symmetries will depend on the explicit form of the superfluid velocity.

> quantum) are defined on curved spacetime, and the *sine qua non* of any analogue model is the existence of some "effective metric" that captures the notion of the curved spacetimes that arise in general relativity. (Barceló, *et al* 2005, pg. 10.)

> The acoustic analogue for black-hole physics accurately reflects half of general relativity -- the kinematics due to the fact that general relativity takes place in a Lorentzian spacetime. The aspect of general relativity that does not carry over to the acoustic model is the dynamics -- the Einstein equations. Thus the acoustic model provides a very concrete and specific model for separating the kinematic aspects of general relativity from the dynamic aspects. (Visser 1998, pg. 1790.)

Caution should be urged in evaluating claims like these. First, if the kinematics of general relativity is identified with Minkowski spacetime, as linear approximations to solutions to the Einstein equations might suggest, then arguably acoustic spacetimes cannot be considered kinematical analogues of general relativity. And this is because, as argued above, the background structure of acoustic spacetimes should be identified with Neo-Newtonian spacetime and not Minkowski spacetime. More importantly, just what the kinematics of general relativity consists of is open to debate. If we look beyond the linear approximation and consider solutions to the Einstein equations in their full generality, then just what the kinematics of such solutions amounts to is hard to identify, since what normally counts as the kinematics of a field theory (*i.e.*, those variables that describe the field in the absence of external forces), is dynamic in general solutions to the Einstein equations. This simply points to the fact that diffeomorphism invariance is essential for modeling general relativity (both dynamically and kinematically), and the low-energy EFT (2) is not diffeomorphism-invariant.[4] At this point it might be instructive to compare acoustic spacetimes as EFTs with the typical EFT that results from taking the low-energy limit of general relativity (see, *e.g.*, Donoghue 1995). The latter is constructed by explicitly imposing diffeomorphism invariance from the outset. One first notes that the Einstein-Hilbert Lagrangian density that produces the Einstein equations is proportional to the scalar curvature, and as such is the simplest diffeomorphism-invariant Lagrangian density that contains derivatives of the metric (which must be included for the metric to be a dynamical field in the theory). The effective Lagrangian density is constructed by including all other powers of the curvature, consistent with diffeomorphism invariance. These extra terms then serve to cancel infinities at all orders that arise in the quantization process.

The suggestion then is that acoustic spacetimes provide neither dynamical nor kinematical analogues of general relativity. In fact this sentiment has been expressed in the literature. Barceló, *et al* (2004) suggest that acoustic spacetimes simply

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[4] More precisely, the low-energy EFT (2) does not obey "substantive" (as opposed to "formal") general covariance in Earman's (2006) sense; *i.e.*, diffeomorphisms are not a local gauge symmetry of (2).

demonstrate that some phenomena typically associated with general relativity really have nothing to *do* with general relativity:

> Some features that one normally thinks of as intrinsically aspects of gravity, both at the classical and semiclassical levels (such as horizons and Hawking radiation), can in the context of acoustic manifolds be instead seen to be rather generic features of curved spacetimes and quantum field theory in curved spacetimes, that have nothing to do with gravity *per se*. (Barceló *et al* 2004, pg. 2.)

This takes some of the initial bite out of Volovik's solution to the cosmological constant problem. If acoustic spacetimes really have nothing to do with general relativity, their relevance to reconciling the latter with QFT is somewhat diminished. On the other hand, Volovik's solution to the cosmological constant problem is meant to carry over to other analogues of general relativity besides superfluid $^4He$. In particular, it can be run for the case of the superfluid $^3He\text{-}A$, which differs significantly from $^4He$ in that fields other than massless scalar fields arise in the low-energy limit. The fact that these fields model aspects of the dynamics of the Standard Model perhaps adds further plausibility to Volovik's solution. To investigate further, I now turn to $^3He$.

## 3.2. The Standard Model and Superfluid Helium 3

The second example of a spacetime analogue in a condensed matter system concerns the Standard Model of particle physics and the *A*-phase of superfluid Helium 3. Since $^3He$ atoms contain 3 nucleons and hence are fermions, in order for them to form a Bose condensate, pairs must be considered. These pairs are similar to the electron Cooper pairs described by the standard Bardeen-Cooper-Schrieffer (BCS) theory of conventional superconducters. Such electron Cooper pairs are all of one type, characterized by a spin singlet ($S = 0$) state with *s*-wave ($l = 0$) orbital symmetry. $^3He$ Cooper pairs have additional spin and orbital angular momentum degrees of freedom, characterized by spin triplet ($S = 1$) states with *p*-wave ($l = 1$) orbital symmetry. There are thus nine distinct types of $^3He$ Cooper pairs, characterized by 3 spin ($S_z = 0, \pm 1$) and 3 orbital ($l_z = 0, \pm 1$) momentum eigenvalues. This allows for a number of distinct superfluid phases. The *A*-phase is characterized by the absence of $S_z = 0$ substates, and by the zero spin axis $\hat{d}$ parallel or anti-parallel to the orbital momentum axis $\hat{l}$. One can thus think of a $^3He\text{-}A$ Cooper pair as consisting of two $^3He$ atoms spinning about anti-parallel axes that are perpendicular to the plane of their orbit.

An appropriate Hamiltonian that describes such $^3He\text{-}A$ Cooper pairs takes the following form (Volovik 2003, pg. 82),

$$H_{3HeA} = \sum_{k,\alpha,\beta} \chi^{\dagger}{}_{\alpha\beta}(\vec{k})\Big((\varepsilon_k - \mu)\sigma_3 + c_{\perp}(\vec{\sigma}\cdot\hat{d})(\hat{m}\cdot\vec{k}\sigma_1 - \hat{n}\cdot\vec{k}\sigma_2)\Big)\chi_{\alpha\beta}(\vec{k}) \ . \tag{5}$$

Here the $\chi$'s are non-relativistic $SU(2)$ 2-spinors consisting of creation and annihilation operators for $^3He$ atoms with momentum $\vec{k}$ ($\alpha$, $\beta$ being spin indices). The first term in the brackets describes the kinetic energy of a $^3He$ atom (with the chemical potential $\mu$ enforcing particle number conservation). The second term describes the particular interaction between two $^3He$ atoms that produces a $^3He$-$A$ Cooper pair. In this term, the unit vectors $\hat{m}$, $\hat{n}$ are such that $\hat{m}\times\hat{n} = \hat{l}$. Finally, the $\sigma$'s are Pauli matrices, and $c_{\perp} = \Delta_0/k_F$, where $k_F$ is the Fermi momentum and $\Delta_0$ is a constant.[5]

This Hamiltonian can be diagonalized to obtain the energy $E^2(\vec{k}) = (k^2/m - \mu)^2 + c_{\perp}{}^2(\vec{k}\times\hat{l})^2$. The energy can now be linearized about the two points $\vec{k} = qk_F\hat{l}$, $q = \pm 1$, where it vanishes.[6] Volovik refers to these points as "Fermi points". To second order, one obtains

$$\begin{aligned} E^2(k_i) &\approx 2\Big(c_{||}l_i(k_i - qA_i)\Big)^2 + 2\Big(c_{\perp}m_i(k_i - qA_i)\Big)^2 + 2\Big(c_{\perp}n_i(k_i - qA_i)\Big)^2 \\ &\equiv g^{ij}(k_i - q_aA_i)(k_j - qA_j) \ , \qquad\qquad i,\ j \ = \ 1,\ 2,\ 3, \end{aligned} \tag{6}$$

where $A_i = k_F l_i$ suggests a vector potential, and $c_{||} = k_F/m$. In the second line the notation has been simplified by the introduction of the quantity $g^{ij} = c_{||}{}^2\, l^i l^j + c_{\perp}{}^2(\delta^{ij} - l^i l^j) \equiv e_b{}^i e_b{}^j$ ($b = 1,\ 2,\ 3$) for the "dreibein" $e_1{}^i = 2c_{\perp}m_i$, $e_2{}^i = -2c_{\perp}n_i$, $e_3{}^i = q_a c_{||} l_i$ (Volovik 2003, pg. 106). Volovik interprets $g^{ij}$ as the spatial part of an effective metric $g^{\mu\nu}$ describing the $^3He$-$A$ superflow, with $g^{00} = -1$, $g^{0i} = -v^i$, and inverse given by

$$ds^2 = g_{\mu\nu}dx^{\mu}dx^{\nu} = -dt^2 + g_{ij}(dx^i - v^i dt)(dx^j - v^j dt) \ , \qquad \mu,\ \nu = 0,\ 1,\ 2,\ 3. \tag{7}$$

This is similar to the metric arising in $^4He$, except that it is anisotropic, depending on the direction $l_i$, and contains two velocity components, $c_{\perp}$ and $c_{||}$, representing the velocities of quasiparticles in motion transverse to, and parallel to $l_i$, respectively.

The low-energy Hamiltonian corresponding to (6) is given by (Volovik 2003, pg. 105),

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[5] (5) is a modification of the BCS Hamiltonian for conventional superconductors to take into account the extra degrees of freedom of $^3He$-$A$ Cooper pairs. The Fermi momentum $k_F$ is the value of the momentum at the Fermi surface in momentum space that separates occupied states from unoccupied states. In the BCS theory, $\Delta_0$ represents a constant gap in the energy spectrum for quasiparticle excitations above the Cooper pair condensate.

[6] More precisely, the energy vanishes at these points near the Fermi surface where $\mu = k_F{}^2/2m$.

$$H'_{3HeA} = \sum_{k,\alpha,\beta} \chi^{\dagger}{}_{\alpha\beta}(\vec{k})\Big(e_b{}^i(k_i - qA_i)\sigma_b\Big)\chi_{\alpha\beta}(\vec{k}) \ , \qquad b = 1, 2, 3. \tag{8}$$

and the corresponding Lagrangian can be written as,

$$\mathcal{L}'_{3HeA} = \bar{\Psi}\gamma^{\mu}(\partial_{\mu} - qA_{\mu})\Psi \ , \qquad \mu = 0, 1, 2, 3 \tag{9}$$

where $\gamma^{\mu} = e_b{}^{\mu} e_b{}^{\nu}(\sigma_{\nu} \otimes \sigma_3)$ ($\sigma_0$ being the $2 \times 2$ identity) are Dirac $\gamma$-matrices, the $\Psi$'s are relativistic Dirac 4-spinors (constructed from pairs of the $\chi$ 2-spinors), and the temporal component of the potential field is given by $A_0 = k_F l_i v_i$. This describes massless Dirac fermions interacting with a 4-vector potential $A_{\mu}$ in a curved spacetime with metric $g_{\mu\nu}$. Note that this effective Lagrangian is thoroughly relativistic. It is similar to the Lagrangian for massless quantum electrodynamics (QED), except for the fact that it does not have a term describing the Maxwell field (*i.e.*, the gauge field associated with the potential $A_{\mu}$).

It turns out that a Maxwell term arises naturally as a vacuum correction to the coupling between the quasiparticle matter field $\Psi$ and the potential field $A_{\mu}$. This is demonstrated by applying the low-energy approximation to the potential field variable: One expands (9) in small fluctuations in $A_{\mu}$ about its ground state value, and then integrates out the high-energy fluctuations. The result is a term that takes the form of the Maxwell Lagrangian in a curved spacetime $\mathcal{L}_{Max} = (4\beta)^{-1}\sqrt{-g}g^{\mu\nu}g^{\alpha\beta}F_{\mu\alpha}F_{\nu\beta}$, where $g^{\mu\nu}$ is the $^3$*He-A* effective metric, $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$, with $A_{\mu}$ the function of $l_i$ and $v_i$ given above, and $\beta$ is a constant that depends logarithmically on the cut-off energy.[7] Combining this with (9), the effective Lagrangian for $^3$*He-A* then is formally identical to the Lagrangian for (3+1)-dim QED in a curved spacetime.

Volovik (2003, pp. 114-115) now indicates how this can be extended to include $SU(2)$ gauge fields, and, in principle, the relevant gauge fields of the Standard Model. The trick is to exploit an additional degree of freedom associated with the quasiparticles described by (9). In addition to their charge $q$, such quasiparticles are also characterized by the value of their spin projection onto $\hat{d}$ given by $\vec{\sigma}\cdot\hat{d} = \pm 1$, which determines the spin orientation of the underlying $^3$*He* atoms. This can be interpreted as a quasiparticle $SU(2)$ isospin symmetry and incorporated explicitly into (9) by coupling $\Psi$ to a new effective field $W^i{}_{\mu}$ identified as an $SU(2)$ potential field (analogous to the potential for the weak force). Expanding this modified Lagrangian density in small fluctuations in the $W$-field about the ground state then produces to second order a

[7] See, *e.g.*, Volovik (2003, pg. 112). A detailed derivation is given in Dziarmaga (2002). This method of obtaining the Maxwell term as the second order vacuum correction to the coupling between fermions and a potential field was proposed by Zeldovich (1967).

Yang-Mills term. The general moral is that discrete degeneracies in the Fermi point structure of the energy spectrum induce local symmetries in the low-energy sector of the background liquid (Volovik 2003, pg. 116). For the discrete two-fold ($\mathbb{Z}_2$) symmetry associated with $\vec{\sigma} \cdot \hat{d}$, we obtain a low-energy $SU(2)$ local symmetry; and in principle for larger discrete symmetries $\mathbb{Z}_N$, we should obtain larger local $SU(N)$ symmetry groups. In this way the complete local symmetry structure of the Standard Model could be obtained in the low-energy limit of an appropriate condensed matter system.[8]

A similar low-energy treatment of the effective metric does not produce the Einstein-Hilbert Lagrangian of general relativity, however. Under this treatment, one expands the Lagrangian density in small fluctuations in the effective metric $g_{\mu\nu}$ about the ground state and then integrates out the high-energy terms. This follows the procedure of what is known as "induced gravity", after Sakharov's (1967) derivation of the Einstein-Hilbert Lagrangian density as a vacuum correction to the coupling between quantum matter fields and the spacetime metric. In Sakharov's original derivation, the metric was taken to be Lorentzian, and the result included terms proportional to the cosmological constant and the Einstein-Hilbert Lagrangian density (as well as higher-order terms). In the case of the $^3He\text{-}A$ effective metric, the result contains higher-order terms dependent on the superfluid velocity $v_i$, and these terms dominate the Einstein-Hilbert term.[9] These contaminating terms are not diffeomorphic invariant, which is understandable, stemming, as they do, ultimately from the non-relativistic Galilei-invariant superfluid Lagrangian density. Volovik (2003, pg. 130) indicates that such terms originate from integrating over quasiparticles far from the Fermi points. The mechanism that would enforce diffeomorphism invariance in the EFT would thus be one that constrains the integration over quasiparticles to regions close to the Fermi points, where the effective metric is Minkowskian. To investigate such a mechanism, Volovik (2003, pg. 132) considers the limit $m \to \infty$, $v_i \to 0$, interpreted as an "inert vacuum". In this limit, it turns out that vacuum fluctuations of the effective metric do induce the Einstein-Hilbert term without contamination. Since this limit involves no superfluidity, Volovik's (2003, pg. 113) conclusion is that our "physical vacuum" cannot be completely modeled by a superfluid.

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[8] There are complications to this program, however. The Standard Model has gauge symmetry $SU(3) \otimes SU(2) \otimes U(1)$ with electroweak sector given by $SU(2) \otimes U(1)$. The electroweak gauge fields belong to non-factorizable representations of $SU(2) \otimes U(1)$, and hence cannot be simply reconstructed from representations of the two separate groups. (Thanks to an anonymous referee for making this point explicit.)

[9] See, *e.g.*, Volovik (2003, pg. 113). Sakharov's original procedure results in a version of semiclassical quantum gravity, in so far as it describes quantum fields interacting with a classical, unquantized spacetime metric. In the condensed matter context, the background metric is not a classical background spacetime, but rather arises as low-energy degrees of freedom of a quantized non-relativistic system (the superfluid). Hence one could argue this condensed matter version of induced gravity is not semiclassical.

*Interpretation*
This approach to general relativity and the Standard Model views both as theories of low-energy phenomena induced by the ground state of a condensed matter system, although perhaps not a superfluid. It is a *background-dependent* approach, the background being the Galilei-invariant rest frame of the condensate. Under a literal interpretation, the relativistic matter fields and gauge potential fields of the Standard Model are interpreted as low-energy quasiparticle and collective bosonic excitations of the ground state of the condensate, with gauge fields interpreted as induced ground state corrections to the interactions between matter fields and potential fields. This interpretation is intended to extend to a treatment of the spacetime metric as described by general relativity. This metric, viewed as a gauge potential field, is interpreted as a low-energy collective bosonic excitation of the ground state of the condensate, with the Riemann curvature tensor (its associated gauge field) interpreted as an induced ground state correction to the interaction between matter fields and the metric field. The viability of this interpretation rests on the viability of Volovik's inert vacuum system. Given the nature of the $m \to \infty$ limit, it may appear doubtful that there are physical examples of condensed matter systems for which the Einstein-Hilbert term (that describes the Riemann curvature tensor) can be induced in the low-energy limit. Even apart from this problem, there is the question of whether all the symmetries of the Standard Model can be expressed in such systems. From a more constructive point of view however, Volovik's discussion indicates that any system purporting to reproduce general relativity and the Standard Model in the low-energy limit minimally must have Fermi points in its energy spectrum, and in order to avoid superfluidity, such Fermi points should not be the consequence of symmetry breaking.

In Section 4 below I will return to the issue of viability. There, I will indicate that this background-dependent condensed matter approach to general relativity and the Standard Model can be viewed as one approach to quantum gravity, and thus warrants further consideration. With this in mind, we may ask what it suggests about the ontological status of spacetime. Arguably, it is compatible with both relationalist and substantivalist interpretations of spacetime. A relationalist might claim that the fixed Neo-Newtonian background structure is embodied in the condensate itself in the form of Galilean symmetries, with no need to posit an independently existing Neo-Newtonian substantival spacetime. Relativistic spacetime symmetries (Poincaré and/or those associated with a dynamic spacetime metric) are then properties of low-energy phenomena; namely, matter fields and gauge potential fields. Alternatively, there are at least three possible substantivalist interpretations. The first would interpret spacetime as given once and for all by a fixed background Neo-Newtonian spacetime. This substantivalist would tell a story similar to the relationalist concerning the low-energy origins of relativistic matter and gauge fields, including the gravitational field. In particular, such a substantivalist would view relativistic spacetime structure as properties of physical fields, and not as properties of a substantival spacetime. A more intrepid substantivalist might attempt to treat relativistic spacetimes as low-energy phenomena in their own right, independent of both the fixed background and low-energy matter and gauge fields. This intrepid substantivalist might claim relativistic spacetimes are ontologically just as real as the fixed background Neo-Newtonian spacetime of the fundamental condensate, but are "generated" in a different manner; perhaps through a process of low-energy "emergence". Just what this might entail will be the topic of Section 4 below. A third type of substantivalist might interpret spacetime simply as the condensate itself. For this "super-substantivalist", matter fields and gauge fields would appear as low-energy aspects of spacetime itself. Arguably, such a super-substantivalist would be hard-pressed to distinguish herself from the relationalist in this context. Both make the same ontological claims, differing only on terminology.

## 3.3. Twistors and Quantum Hall Liquids

A final example of a spacetime analogue in a condensed matter system involves the twistor formalism and 4-dimensional quantum Hall liquids. Quantum Hall liquids initially arose in explanations of the 2-dimensional quantum Hall effect (QHE). In the following, I will first review the low-energy field theory of the 2-dim QHE and then indicate how a spacetime analogue can be associated with a twistorial formulation of a 4-dim extension of it.

*2-dim Quantum Hall Liquids*

The set-up for the 2-dim QHE consists of current flowing in a 2-dim conductor in the presence of an external magnetic field perpendicular to its surface. The classical Hall Effect occurs as the electrons in the current are deflected towards the edge by the magnetic field, thus inducing a transverse voltage. In the steady state, the force due to the magnetic field is balanced by the force due to the induced electric field and the *Hall conductivity* $\sigma_H$ is given by the ratio of current density to induced electric field. The quantum Hall effect occurs in the presence of a strong magnetic field, in which $\sigma_H$ becomes quantized in units of the ratio of the square of the electron charge $e$ to the Planck constant $h$:

$$\sigma_H = \nu \times (e^2/h) \ , \tag{10}$$

where $\nu$ is a constant. The Integer Quantum Hall Effect (IQHE) is characterized by integer values of $\nu$, and the Fractional Quantum Hall Effect (FQHE) is characterized by values of $\nu$ given by odd-denominator fractions.[10] Two properties experimentally characterize the system at such quantized values: The current flowing in the conductor becomes dissipationless, as in a superconductor; and the system becomes incompressible.

These effects can be encoded in a Lagrangian that initially describes non-relativistic 2-dim electrons coupled to magnetic and electric fields:

$$\mathcal{L}_{QHE} = \psi^\dagger(i\hbar\partial_0 - eA_0^{ext})\psi - \frac{1}{2m}\psi^\dagger(-i\hbar\partial_i - eA_i^{ext})^2\psi + V[\psi^\dagger\psi] + \mathcal{L}_{EM}\ ,\ i = 1, 2, \quad (11)$$

where $A_0^{ext}$, $A_i^{ext}$ are potentials for the magnetic and electric fields, $V$ is an appropriate interaction potential, and $\mathcal{L}_{EM}$ is the (non-relativistic) electromagnetic Lagrangian density. At low energies, this *fermionic* Lagrangian can be shown to be equivalent to a *bosonic* Lagrangian that describes bosons coupled to a magnetic, an electric, and a Chern-Simons field (see, *e.g.*, Zhang 1992, pg. 32):

$$\mathcal{L}_{QHE} = \varphi^\dagger(i\hbar\partial_0 - eA_0)\varphi - \frac{1}{2m}\varphi^\dagger(-\hbar i\partial_i - eA_i)^2\varphi + V[\varphi^\dagger\varphi] + \mathcal{L}_{CS}\ . \quad (12)$$

Here $A_0 \equiv a_0 + A_0^{ext}$, $A_i \equiv a_i + A_i^{ext}$, where $(a_0, a_i) \equiv a_\mu$ is a Chern-Simons (CS) potential field described by the term $\mathcal{L}_{CS} = (e/2p\phi_0)\epsilon^{\mu\nu\lambda}a_\mu\partial_\nu a_\lambda$, where $\phi_0$ is the quantum of magnetic flux (see footnote 10) and $p$ is an odd integer. At low energies, one can show that this term dominates the $\mathcal{L}_{EM}$ term in (11). The effect of coupling the bosons to the Chern-Simons field can be interpreted as attaching $p$ quanta of magnetic flux to each boson, and this effectively modifies their exchange statistics to mimic the Fermi-Dirac statistics of the electrons of (11).[11]

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[10] The constant $\nu$ is called the filling factor and is given by $\nu = \rho_e/\rho_B$, where $\rho_e$ is the electron density and $\rho_B = B^{ext}/\phi_0$ is the density of the external magnetic field flux. In the latter expression $\phi_0 = h/e$ is the quantum of magnetic flux (in units in which $c = 1$).

[11] This can be demonstrated explicitly by extremizing (12) with respect to $a_0$. The result gives the curl of the CS field, $\vec{\partial}\times\vec{a} = p\phi_0\rho_e(x)\hat{z}$, where $\hat{z}$ is the direction of the external magnetic field (Zhang 1992, pg. 35). Integrating this yields an expression $\oint \vec{a}\cdot d\vec{x} = p\phi_0$ for the flux of the CS field through the area containing a boson of unit charge located at the origin (Zhang 1992, pg. 28; one assumes that the density of composite bosons is identical to that of the original electrons). When two such bosons are exchanged, they pick up an Aharonov-Bohm phase equal to $\exp\left(ie/\hbar\int_0^\pi \vec{a}\cdot d\vec{x}\right) = -1$, mimicking Fermi-Dirac statistics. Note that this statistical transmutation for point particles only works in 2-dim: In 1-dim, point particles cannot be exchanged, and for dim $\geq 3$, any closed, continuous exchange path taken by two point particles can be continuously deformed into a point; hence such paths cannot be distinguished by means of winding numbers.

One can now show that the combined external and CS magnetic field felt by the composite bosons vanishes when the constant $\nu$ in (10) is given by $1/p$, corresponding to the FQHE.[12] At such values, the bosons feel no net magnetic field, and hence can form a condensate at zero temperature. This condensate, consisting of charged bosons, is referred to as a quantum Hall (QH) liquid, and can be considered to have the same properties as a superconductor; namely, dissipationless current flow and the expulsion of magnetic fields from its interior (what's called the Meissner effect). The latter entails there is no net internal magnetic field in the QH liquid; hence, in so far as the internal CS magnetic field is determined by the particle density (see footnote 11), this entails that the particle density is constant. Thus the QH liquid is incompressible.

The fact that the QH liquid is incompressible entails that there is a finite energy gap between the ground state of the condensate and the first allowable energy states. This means a low energy approximation about zero modes cannot be taken; hence a low energy EFT cannot be constructed for the bulk liquid. Such a low-energy EFT can, however, be constructed for the 1-dim edge of the liquid. Wenn (1990) assumed edge excitations take the form of low-energy surface waves obeying a linear dispersion relation (hence they can be made arbitrarily small, thus facilitating a low-energy approximation) and demonstrated that the effective Lagrangian for the edge states describes massless chiral fermion fields in (1+1)-dim Minkowski spacetime:

$$\mathcal{L}'_{edge} = i\psi^\dagger(\partial_t - v\partial_x)\psi \ , \tag{13}$$

where $v$ is the electron drift velocity.

*4-dim Quantum Hall Liquids*

The (1+1)-dim edge Lagrangian (13) tells us little about the ontology of (3+1)-dim spacetime. However, it suggests that (3+1)-dim massless relativistic fields may be obtainable from the edge states of a 4-dim QH liquid, and this is in fact borne out. Zhang and Hu (2001) provided the first extension of the 2-dimensional QHE to 4-dimensions. In rough outline, they replaced the 2-dim quantum Hall liquid with a 4-dim quantum Hall liquid and then demonstrated that the EFT of the 3-dim edge describes massless fields in (3+1)-dim Minkowski spacetime.

In slightly more detail, Zhang and Hu made use of a formulation of the 2-dim QHE in terms of spherical geometry first given by Haldane (1983). Haldane considered an electron gas on the surface of a 2-sphere $S^2$ with a $U(1)$ Dirac magnetic monopole at its

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[12] The magnitude of the combined external and CS magnetic field is given by $|\vec{\partial}\times(\vec{a}+\vec{A}^{ext})| = |p\phi_0\rho_e - B^{ext}|$, which vanishes when $\phi_0\rho_e/B^{ext} = \nu = 1/p$, where $\nu$ is the filling factor given in footnote 10.

center. The radial monopole field serves as the external magnetic field of the original setup. By taking an appropriate thermodynamic limit, the 2-dim QHE on the 2-plane is recovered. Zhang and Hu's extension to 4-dimensions is based on the geometric fact that a Dirac monopole can be formulated as a $U(1)$ connection on a principle fiber bundle $S^3 \to S^2$, consisting of base space $S^2$ and bundle space $S^3$ with typical fiber $S^1 \cong U(1)$ (see, *e.g.*, Nabor 1997). This fiber bundle is known as the 1st Hopf bundle and is essentially a way of mapping the 3-sphere onto the 2-sphere by viewing $S^3$ as a collection of "fibers", all isomorphic to a "typical fiber" $S^1$, and parameterized by the points of $S^2$. There is also a 2nd Hopf bundle $S^7 \to S^4$, consisting of the 4-sphere $S^4$ as base space, and the 7-sphere $S^7$ as bundle space with typical fiber $S^3 \cong SU(2)$. The $SU(2)$ connection on this bundle is referred to as a Yang monopole. Zhang and Hu's 4-dim QHE then consists of taking the appropriate thermodynamic limit of an electron gas on the surface of a 4-sphere with an $SU(2)$ Yang monopole at its center.[13]

Some authors have imbued the interplay between algebra and geometry in the 4-dim QHE extension with ontological significance. These authors note that there are only four normed division algebras: the real numbers $\mathbb{R}$, the complex numbers $\mathbb{C}$, the quaternions $\mathbb{H}$, and the octonions $\mathbb{O}$.[14] It is then observed that these may be associated with the four Hopf bundles, $S^1 \to S^1$, $S^3 \to S^2$, $S^7 \to S^4$, $S^{15} \to S^8$, in so far as the base spaces of these fiber bundles are the compactifications of the respective division algebra spaces $\mathbb{R}^1$, $\mathbb{R}^2$, $\mathbb{R}^4$, $\mathbb{R}^8$. Finally, one notes that the typical fibers of these Hopf bundles are $\mathbb{Z}_2$, $U(1) \cong S^1$, $SU(2) \cong S^3$, and $SO(8) \cong S^7$, respectively. These patterns are then linked with the existence of quantum Hall liquids:

> One , two, and four dimensional spaces have the unique mathematical property that boundaries of these spaces are isomorphic to mathematical groups, namely the groups $\mathbb{Z}_2$, $U(1)$ and $SU(2)$. No other spaces have this property. (Zhang and Hu 2001, pg. 827.)

> The four sets of numbers [*viz.*, $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$, $\mathbb{O}$] are mathematically known as division algebras. The octonions are the last division algebra, no further generalization being consistent with the laws of mathematics... Strikingly, in physics, some of the

[13] A low-energy Chern-Simons field theory for such 4-dim QH liquids analogous to (12) was constructed by Bernevig *et al.* (2002). This theory is based on statistical transmutations for extended objects ("branes"), as opposed to point particles, and thus side-steps the dimensional restrictions of the 2-dim Chern-Simons QHE theory (12) mentioned in footnote 11.

[14] A normed division algebra $A$ is a normed vector space, equipped with multiplication and unit element, such that, for all $a$, $b \in A$, if $ab = 0$, then $a = 0$ or $b = 0$. $\mathbb{R}$, $\mathbb{C}$, and $\mathbb{H}$ are associative, whereas $\mathbb{O}$ is non-associative (see, *e.g.*, Baez 2001, pg. 149).

division algebras are realized as fundamental structures of the quantum Hall effect. (Bernevig *et al.* 2003, pg. 236803-1.)

Our work shows that QH liquids work only in certain magic dimensions exactly related to the division algebras... (Zhang 2004, pg. 688.)

Before we see nature unfolding its secrets in the forms of division algebras and Hopf bundles, we should pause and take stock. Note first that Zhang and Hu's statement should be restricted to the spaces $S^1$, $S^2$, $S^4$, and should include $S^8$ as well, the boundary of $S^8$ being $S^7$. Furthermore, the statements of Bernevig *et al.* and Zhang should refer to *normed* division algebras. Baez (2001, pg. 149) carefully distinguishes between $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$, $\mathbb{O}$ as the only normed division algebras, and division algebras in general, of which there are other examples. Baez (2001, pp. 153-156) indicates how the sequence $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$, $\mathbb{O}$ can in principle be extended indefinitely by means of the Cayley-Dickson construction. Starting from an $n$-dim $*$-algebra $A$ (*i.e.*, an algebra $A$ equipped with a conjugation map $*$), the construction gives a new $2n$-dim $*$-algebra $A'$.[15] The next member of the sequence after $\mathbb{O}$ is a 16-dim $*$-algebra referred to as the "sedenions". The point here is that the sedenions and all subsequent higher-dimensional constructions do not form division algebras; in particular, they have zero divisors. The question therefore should be whether the absence of zero divisors in a normed $*$-algebra has physical significance when it comes to constructing QH liquids.

Zhang (2004, pg. 687) implicitly suggests it does. He identifies various quantum liquids with each Hopf bundle: 1-dim Luttinger liquids[16] with $S^1 \to S^1$, 2-dim QH liquids with $S^3 \to S^2$, and 4-dim QH liquids with $S^7 \to S^4$. Bernevig *et al.* (2003) complete the pattern by constructing an 8-dim QH liquid as a fermionic gas on $S^{15}$ with an $SO(8)$ monopole at its center. But whether this pattern is physically significant remains to be seen. It is not entirely clear, for example, how the bundle $S^1 \to S^1$ is essential in the construction of Luttinger liquids in general. In particular, while Luttinger liquids are necessarily 1-dim, it's not clear what role, if any, the trivial $\mathbb{Z}_2$ monopole associated with $S^1 \to S^1$ plays in their construction. Moreover, while Luttinger liquids arise at the edge of 2-dim QH liquids, this pattern does not carry over to higher dimensions: it is not the case that 2-dim QH liquids arise at the edge of 4-dim QH liquids, nor is it the case that 4-dim QH liquids arise at the edge of 8-dim QH liquids. Furthermore, and more

[15] The elements of $A'$ are defined as pairs of elements of $A$, with multiplication in $A'$ given by $(a, b)(c, d) = (ac - db^*, a^*d + cb)$, and conjugation in $A'$ given by $(a, b)^* = (a^*, -b)$, for $a, b \in A$.

[16] In Wenn's (1990) derivation of the EFT (13) of the edge of a 2-dim QH liquid, the ground state of the edge was identified as a Luttinger liquid. A Luttinger liquid is comprised of electrons, but differs from a standard Fermi liquid mathematically in the form of the electron propagator: a Luttinger liquid is characterized by a propagator with non-trivial exponents. See Wen (2004, pp. 314-315) for discussion.

importantly, Meng (2003) demonstrates that higher-dimensional QH liquids can in principle be constructed for any even dimension, and concludes that the existence of division algebras is not a crucial aspect of such constructions (see, also Karabali and Nair 2002). Hence, while the relation between Hopf bundles and normed division algebras on the one hand, and quantum liquids on the other, is suggestive, it perhaps should not be interpreted too literally.

So far in this discussion no mention of spacetime has been made. To see where spacetime comes in, we need to move to the edges.

*Edge States for 4-dim QH Liquids and Twistors*
The low-energy edge states of a 2-dim QH liquid take the form of (1+1)-dim relativistic massless fields described by (13). It turns out that edge excitations can be viewed as particle-hole dipoles formed by the removal of a fermion from the bulk to outside the QH droplet, leaving behind a hole (see, *e.g.*, Stone 1990). If the particle-hole separation remains small, such dipoles can be considered single localized bosonic particle states. The stability of such localized states is affected by the uncertainty principle: a stable separation distance entails a corresponding uncertainty in relative momentum, which presumably would disrupt the separation distance. In 1-dim it turns out that the kinetic energy of such dipoles is approximately independent of their relative momentum, hence they are stable. In the case of the 3-dim edge of the 4-dim QH liquid, Zhang and Hu (2001) determined that there is a subset of dipole states for which the isospin degrees of freedom associated with the $SU(2)$ monopole counteract the uncertainty principle. Their main result was to establish that these stable edge states satisfy the (3+1)-dim zero rest mass field equations for all helicities, and hence can be interpreted as zero rest mass relativistic fields (see, also, Hu and Zhang 2002). These include, for instance, spin-1 Maxwell fields and spin-2 graviton fields satisfying the vacuum linearized Einstein equations, as well as massless fields of all higher helicities.

By itself, this recovery of (3+1)-dim relativistic zero rest mass fields has limited applicability when it comes to questions concerning spacetime ontology. As with the examples in superfluid Helium, we would like to recover general relativity and the Standard Model in their full glory. This is where twistor theory makes its appearance, the goal of which is to recover general relativity and quantum field theory from the structure of zero rest mass fields. Sparling's (2002) insight was to see that Zhang and Hu's stable dipole states correspond to twistor representations of zero rest mass fields. In particular, Sparling demonstrated that the edge of a 4-dim QH liquid can be identified with a particular region of twistor space $\mathbb{T}$. $\mathbb{T}$ is the carrying space for matrix representations of $SU(2, 2)$ which is the double covering group of $SO(2, 4)$. Elements $Z^\alpha$ of $\mathbb{T}$ are called twistors and are thus spinor representations of $SO(2, 4)$. $\mathbb{T}$ contains a Hermitian 2-form $\Sigma_{\alpha\beta}$ (a "metric") of signature $(++--)$. This 2-form splits $\mathbb{T}$ into three regions $\mathbb{T}^+$, $\mathbb{T}^-$, $\mathbb{N}$, consisting of twistors $Z^\alpha$ satisfying $\Sigma_{\alpha\beta}Z^\alpha \bar{Z}^\beta > 0$, $\Sigma_{\alpha\beta}Z^\alpha \bar{Z}^\beta < 0$, and $\Sigma_{\alpha\beta}Z^\alpha \bar{Z}^\beta = 0$, respectively. The connection to spacetime is based on the fact that $SO(2, 4)$ is the double covering group of $C(1, 3)$, the conformal group of Minkowski spacetime. This allows a correspondence to be constructed under which elements of $\mathbb{N}$, "null" twistors, correspond to null geodesics in Minkowski spacetime, and 1-dim subspaces of $\mathbb{N}$ (*i.e.*, twistor "lines") correspond to Minkowski spacetime points.[17]

To make the identification of the edge of a 4-dim QH liquid with $\mathbb{N}$ plausible, note that the symmetry group of the edge is $SO(4) \cong S^3$ and that of the bulk is $SO(5) \cong S^4$. The twistor group $SO(2, 4)$ has $SO(4)$ in common with $SO(5)$. Intuitively, the restriction of $SO(2, 4)$ to $SO(4)$ can be induced by a restriction of twistor space $\mathbb{T}$ to $\mathbb{N}$.[18] With the edge identified as $\mathbb{N}$, edge excitations are identified as deformations of $\mathbb{N}$ (in analogy with Wenn's treatment of the edge in the 2-dim case). In twistor theory, such deformations take the form of elements of the first cohomology group of projective null twistor space $\mathbb{PN}$, and these are in fact solutions to the zero rest mass field equations of all helicities in Minkowski spacetime (Sparling 2002, pg. 25).

*Interpretation*

Sparling's twistorial formulation of the 4-dim QHE suggests an interpretation of twistors as low-energy excitations of the edge of a 4-dim quantum Hall liquid, with spacetime subsequently being derivative of twistors. This interpretation comes with two caveats. The first involves a technical question concerning the nature of the thermodynamic limit in the twistor formulation, which is still unknown at present (Sparling 2002, pg. 27). The second concerns the approach to spacetime in the twistor formalism in general. Even granted that the 4-dim QHE admits a thoroughly twistorial formulation down to the thermodynamic limit, there is still the question of whether spacetime as currently described by general relativity and quantum field theory can be recovered. It turns out that no consistent twistor descriptions have been given for massive quantum fields, or for field theories in generally curved spacetimes with matter content. In general, only conformally invariant field theory, and those general relativistic spacetimes that are conformally flat, can be completely recovered in the

[17] More precisely, the correspondence is between $\mathbb{PN}$, the space of null twistors up to a complex constant (*i.e.*, "projective" null twistors), and *compactified* Minkowski spacetime (*i.e.*, Minkowski spacetime with a null cone at infinity). This is a particular restriction of a general correspondence between projective twistor space $\mathbb{PT}$ and complex compactified Minkowski spacetime. For more details, and a general discussion of the relevance of the twistor formalism to philosophy of spacetime, see Bain (2006).

[18] Technically, this restriction corresponds to a foliation of the 4-sphere with the level surfaces of the $SO(4)$-invariant function $f(Z^\alpha) = \Sigma_{\alpha\beta}Z^\alpha \bar{Z}^\beta$. These surfaces are planes spanned by null twistors (Sparling 2002, pp. 18-19, 22).

twistor formalism. This is not to say that the twistor connection with the QHE is not a significant achievement. Advocates view twistors as a route to quantum gravity. As such, the twistor formulation of the 4-dim QHE points to similarities between two approaches to quantum gravity, *via* twistors and *via* condensed matter systems, that were previously seemingly unrelated.[19]

## 4. Low-Energy Emergence and Emergent Spacetime

The preceding examples have suggested to some authors that novel phenomena (fields, particles, symmetries, spacetime, twistors, *etc.*) "emerge" in the low-energy sector of certain condensed matter systems. In their review of models of analogue gravity, Barcelo, *et al.* (2005) speak of "emergent gravitational features in condensed matter systems" (pg. 84), and "emergent spacetime symmetries" (pg. 89); Dziarmaga (2002, pg. 274) describes how "... an effective electrodynamics emerges from an underlying fermionic condensed matter system"; Volovik (2003) in the preface to his text on low-energy properties of superfluid helium, lists "emergent relativistic quantum field theory and gravity" and "emergent non-trivial spacetimes" as topics to be discussed within; Zhang (2005) provides "examples of emergence in condensed matter physics", including the 4-dim quantum Hall effect; and Zhang and Hu (2001, pg. 825) speak of the "emergence of relativity" at the edge of 2-dim and 4-dim quantum Hall liquids. The purpose of this section is to indicate the nature of this notion of low-energy emergence and to situate it in discussions of emergence that have appeared in the philosophy of science literature.

Whatever else emergence is, minimally it is a relation. Coming to grips with it thus requires identifying the *relata*, and identifying the *relation* itself. In the philosophy of science literature, approaches to the first task divide into two camps: those who view the relata as properties (or entities, phenomena, *etc*; in general, *real world items*), versus those who view the relata as theories (or, in general, *representational items*) (see, *e.g.*, Silberstein 2002, pg. 90). Approaches to the second task typically appeal to two additional relations -- *reduction* and *supervenience* -- and describe emergence as the denial of either or both of them.[20] Silberstein (2002), for instance, considers ways of

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[19] Note further that the twistor emphasis on conformal invariance is not as restrictive as might first be thought, in so far as the verdict is still out on whether quantum field theory can be reformulated in a conformally invariant way. In general, the route to quantum gravity that stresses conformal structure over metrical structure should not be ignored by philosophers of spacetime.

[20] For the purposes of this essay, it will not be necessary to review all the myriad proposals for definitions of reduction and supervenience that have appeared in the philosophical literature. Traditionally, reduction has been taken as a nomic relation between theories, whereas supervenience has been taken as a weaker relation between properties. See, *e.g.*, Butterfield & Isham (1999, pp. 115-125).

denying both, while Howard (preprint) adopts the denial of supervenience. Note that these tasks are not unrelated. How one defines emergence will evidently depend, in this context, on one's prior convictions concerning reduction and/or supervenience, and such convictions may decide the issue of identifying the relata. Howard (preprint), for instance, takes supervenience to be an ontic relation between properties. Emergence as the denial of supervenience then entails that the relata of emergence are properties. To add further confusion to the topic, there is yet a third cut along which notions of emergence may fall: that between *ontological* emergence and *epistemological* emergence. Ontological emergence is the claim that the emergent relata are *ontologically distinct* from the "host" relata; perhaps in the sense of higher-level, collective phenomena. Epistemological emergence is the claim that the emergent relata are *epistemically distinct* from the host relata, in so far as knowledge of the latter does not suffice to ground knowledge of the former.[21]

Of particular relevance to the topic of this essay is Butterfield and Isham's (1999) analysis of emergence in the context of notions of (space)time in quantum gravity. They consider theories as the relata and suggest that to define emergence in terms of reduction and/or supervenience is simply to define one obscure concept in terms of other equally obscure ones. (For instance, they indicate that the standard definition of reduction in terms of *definitional extension* (see below) does not suffice to distinguish reduction from supervenience.) Ultimately, they suggest a heterogeneous approach to the concept of emergence as a relation between theories, given that there are many ways two theories can be related. The suggestion then is that, rather than engage in debate over the best way to formally define emergence, we should instead consider concrete examples of instantiations of the various ways theories can be related. On the surface this attitude meshes nicely with the above claims concerning low-energy emergence, for all of these claims minimally concern the relation between two theories -- a given theory $T$ and its low-energy EFT, call it $T'$. In the following I will suggest that, in so far as $T$ and $T'$ differ on their dynamical laws, low-energy emergence is minimally *epistemological* in nature. Whether in addition, it is an *ontological* claim will depend on how $T$ and $T'$ are interpreted.

[21] Silberstein (2002, pg. 90) equates ontological emergence just with the properties view of the First Task, and epistemological emergence with the theories view. On the surface, this seems a bit problematic: On the one hand, one could claim that properties are emergent in an epistemological sense; *i.e.*, such epistemologically emergent properties cannot be predicted on the basis of the host theory. On the other hand, one could also hold that one theory emerges from another in an ontological sense (in terms of the ontology of the former emerging from that of the latter). To be fair, Silberstein does explicitly warn the reader that the ontological and epistemological versions of emergence, as he construes them, are not independent of each other.

To further characterize low-energy emergence, it is perhaps best to say what it is *not*. In particular, it is not what might be called the "received view" of emergence in condensed matter systems. Under this view, emergent phenomena are associated with phase transitions. For example, the macroscopic correlations exhibited by superfluids and superconductors are due to a phase transition at which the correlation length (roughly, the measure of the correlation between spatially separated states) becomes infinite.[22] The highly correlated phenomena that result from such phase transitions have been called emergent (see, *e.g.*, Howard preprint, Humphreys 1997). Such phenomena should *not* be associated with low-energy emergence. To make this point clear, consider the example of superfluid Helium. Above a critical temperature, the system consists of a non-relativistic normal liquid. As the temperature is lowered below the critical value, a phase transition occurs (in this case accompanied by a spontaneously broken symmetry), and the system enters the superfluid phase. If the temperature is lowered further; *i.e.*, if we take a low-energy approximation, we obtain a relativistic system. It is this *latter* relativistic system that should be identified as low-energy emergent, and *not* the phenomena of superfluidity that result from the phase transition. Importantly, both the normal liquid and the superfluid, as well as the phase transition and the spontaneously broken symmetry, are all encoded in a single Lagrangian (see, *e.g.*, (1) for the case of superfluid $^4He$). In so far as distinct theories may be associated with distinct Lagrangians, all of these states and processes are described by a *single* theory. On the other hand, the low-energy relativistic system is encoded in the effective Lagrangian, which is formally distinct from the initial Lagrangian; hence, the low-energy system is described by a *different* theory than the highly correlated phenomena of the initial system (Figure 1).


Figure 1. The relation between the initial Lagrangian and the effective Lagrangian.

Importantly, the relation between the initial theory $T$ and its EFT $T'$ consists of a low-energy approximation. This relation involves a larger gap between $T$ and $T'$ than

[22] In some systems, such phase transitions are accompanied by spontaneously broken symmetries, but this is not always the case: topological phase transitions can occur between states of a system that share the same symmetries, but differ topologically. The standard example is the Kosterlitz-Thouless phase transition in which vortex fluctuations in a bosonic superfluid film are responsible for a discontinuous change in the correlation length (see, *e.g.*, Wenn 2004, pp. 102-104).

typical notions of reduction. On the standard (Nagelian) account of reduction, for instance, a necessary condition for $T'$ to reduce to $T$ is that $T'$ be a *definitional extension* of $T$ (see, *e.g.*, Butterfield and Isham 1999, pg. 115). This requires first that $T$ and $T'$ admit formulations as deductively closed sets of sentences in a formal language (*i.e.*, it assumes a *syntactic* conception of theories), and second that an extension $T^*$ of $T$ can be constructed such that the theorems of $T'$ are a sub-set of the theorems of $T^*$ (*i.e.*, $T'$ is a *sub-theory* of $T^*$). Formally, $T^*$ is constructed by adding to $T$ a definition of each of the non-logical symbols of $T'$. That this cannot be done in the case of a theory $T$ and its EFT should be clear from the examples above, in which the Lagrangian (or Hamiltonian) of the original theory differs formally from that of the EFT. In the Lagrangian formalism, a difference in the form of the Lagrangian entails a difference in dynamical laws; namely, a difference in the Euler-Lagrange equations for the various dynamical variables. And a difference in dynamical laws entails a difference in "theorems" derived from these laws. (The same holds true in the Hamiltonian formalism.) Hence the relation between a theory and its EFT cannot be described in terms of syntactic notions of reduction based on definitional extension.[23]

Arguably the distinction between $T$ and $T'$ is such that it cannot be made under more generous notions of reduction, either syntactic *or* semantic. For instance, under syntactic notions of reduction based on limit procedures (see, *e.g.*, Batterman's (2002, pg. 78) "physicists' sense" of reduction), $T'$ cannot be said to reduce to $T$. While condensed matter physicists like to talk about taking the low-energy "limit", mathematically, such a thing does not exist. The approximation scheme from which $T'$ is obtained from $T$ does not involve a formal limit. Moreover, under a semantic conception of theories, one generally claims that a theory reduces to another just when models of the first can be embedded in models of the second. However, this will not suffice to reduce $T'$ to $T$ so long as the embedding is required to preserve dynamical laws; and if it is not, then it is unclear whether the term "reduction" for such an embedding is appropriate (assuming, whatever else reduction amounts to, it is essentially nomic in nature).

These considerations suggest another way in which low-energy emergence and the "received view" are distinct. Under the latter, emergence is typically ontological in character. The highly correlated phenomena of superfluidity and superconductivity, for example, are typically interpreted as arising from ontologically emergent properties associated with entangled quantum states (see, *e.g.*, Howard preprint, Silberstein 2002, Humphreys 1997). But the received veiw is also an epistemological thesis about the

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[23] Butterfield and Isham (1999, pg. 122) observe that the standard defintion of supervenience can be characterized in terms of an *infinitistic* definitional extension; thus neither can it be said that an EFT *supervenes* (in this sense) on the original theory.

novelty of these phenomena. In particular, such phenomena, it is claimed, cannot be explained in terms of the "host" system, nor can they be reduced to the host, nor predicted from knowledge of the host. Of course this thesis depends on prior convictions concerning the concepts of explanation, reduction, and prediction, which are heady topics in philosophy of science in their own rights.. Such concepts will be all the more difficult to articulate in this particular context in which both host system and emergent system derive from the same theory. The situation is a bit more clear in the low-energy case, however, and herein lies the second way in which the latter case differs from the received view. In the low-energy case, as argued above, we have distinct theories, and this might provide the epistemological emergentist the where-with-all on which to base epistemic distinctions. For instance, given that the relation between a theory and its EFT cannot be described in terms of definitional extension, the low-energy emergent structure cannot be reduced (in a Nagelian sense) to the host, nor can it be predicted on the basis of the host; nor can it be explained (in a Deductive-Nomological sense) in terms of the host. Of course, concepts of reduction, prediction, and explanation based on definitional extension are certainly wanting; but the intuition hopefully stands: There is a wider epistemic "gap" between the relata of low-energy emergence than between the relata of the received view.

Minimally, then, low-energy emergence can be characterized as an epistemological claim. Again, what makes the relata of low-energy emergence distinct is that they subscribe to different dynamical frameworks, and since such dynamical frameworks prefigure the kinds of epistemic claims we can make about their constituents, distinct frameworks will prefigure distinct claims. Additional ontological theses may be draped over such frameworks, depending on one's proclivities (be they substantivalist or relationalist, for instance), however nothing essential to low-energy emergence dictates the form such theses must take. In the next section I will suggest one such thesis that, while not so-dictated, nevertheless seems particularly well-suited, given low-energy emergentism's minimalist epistemological trappings. In the remainder of this section, I will review the formal prospects for any such emergentist interpretation of spacetime.

*Emergent Spacetime: Prospects*

It is in a minimally epistemological sense of low-energy emergence that an interpretation of spacetime as an emergent phenomenon in condensed matter systems is to be initially understood. The viability of such a minimalist interpretation, and any more ontologically robust interpretation based on it, depends in particular on the type of condensed matter system, and on prior convictions on how best to model spacetime. For instance, an interpretation of spacetime as emergent in superfluid Helium 4 might be motivated by a desire to model spacetime as (some aspect of) the solutions to the Einstein equations in general relativity. In Section 3.1, we saw that the prospects for such an interpretation are limited: The effective Lagrangian for superfluid Helium 4 lacks both the dynamics associated with general relativity and, arguably, the kinematics. An interpretation of spacetime as emergent in superfluid Helium 3 might be motivated by a desire to model spacetime as the ground state for quantum field theories of matter, gauge, and metric fields. In Section 3.2, we saw that the prospects here are also limited: While the effective Lagrangian for superfluid $^3$*He-A* does reproduce relevant aspects of the Standard Model, it does not fully recover general relativity. Moreover, the verdict is still out on whether physical systems exist that could produce the Einstein-Hilbert term in a low-energy approximation. Finally, an interpretation of spacetime as emergent from the edge of a 4-dimensional quantum Hall liquid might be motivated by a desire to derive spacetime from twistors. Here the prospects as noted in Section 3.3 are limited primarily by the limitations of the twistor programme: Twistor formulations of general solutions to the Einstein equations, and massive interacting quantum fields, have yet to be constructed.

These results suggest that currently an interpretation of spacetime as a low-energy emergent phenomenon cannot be fully justified. However, such an interpretation should nevertheless still be of interest to philosophers of spacetime. Each of the examples above may be considered part of a general research programme in condensed matter physics; namely, to determine the appropriate condensed matter system that produces the relevant matter, gauge and metric fields in a low-energy approximation. This research programme may be seen as one path to quantum gravity in competition, for instance, with the background-independent canonical loop approach, and background-dependent approaches like string theory.[24] Thus to the extent that philosophers of spacetime should consider notions of spacetime associated with approaches to quantum gravity, they should be willing to consider low-energy emergentist interpretations of spacetime. The remainder of this section indicates how the condensed matter approach to quantum gravity compares conceptually with the two other primary approaches.

(a) The condensed matter approach is distinct from the canonical loop approach, in so far as it is a background-*dependent* approach, the background being the frame of the fundamental condensate. Moreover, while both the condensed matter approach and the loop approach predict violations of Lorentz invariance, these predictions differ in their details. First, the condensed matter approach predicts such violations at low energies, whereas the loop approach predicts violations at high energies (scales smaller than the Planck scale) at which it predicts spacetime becomes discrete. Second, the condensed matter approach explains the violation of Lorentz invariance in terms of the existence of a preferred frame; namely the condensate frame, whereas the loop approach explains the

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[24] See, *e.g.*, Smolin (2003, pp. 57-58). More precisely, the condensed matter programme is an approach to reconciling general relativity and quantum theory. Ultimately it suggests gravity need not be quantized, since it claims that gravity emerges in the low energy limit of an already quantized system.

violation in terms of background-independence: at the Planck scale, there are no preferred frames, whether Lorentzian or otherwise.[25]

(b) The condensed matter approach differs from background-dependent approaches like string theory in three general respects. First, as is evident in the previous sections, the condensed matter approach differs from string theory in that the structure it attributes to the background is not Minkowskian: Given that the fundamental condensate is a *non-relativistic* quantum liquid, the background will be Neo-Newtonian. Second, while background-dependent approaches that are ultimately motivated by quantum field theory (as string theory is) typically view QFTs as low-energy EFTs of a more fundamental theory, such approaches view the latter as a theory of *high-energy* phenomena (strings, for example). The phenomena of experience, as described by current QFTs, are then interpreted as emerging *via* a process of symmetry breaking. The condensed matter approach, on the other hand, views QFTs and general relativity as EFTs of a more fundamental *low-energy* theory, and the process by which the former arise is a low-energy emergent process that, again, is not to be associated with symmetry breaking. Finally, in general, the condensed matter approach can be characterized by placing less ontological significance on the notion of symmetry than background-dependent approaches in at least three ways.

• First, background-dependent approaches that view QFTs as EFTs describe the phenomena of experience as obeying "imperfect" (gauge) symmetries that result from a process of symmetry breaking of a "more perfect" fundamental symmetry. Mathematically, the more perfect fundamental symmetry is hypothesized as having the structure of a single compact Lie group with a minimum of parameters. This is then broken into imperfect symmetries that are characterized by product group structure and relatively many parameters. In particular, the gauge field group structure of the Standard Model, below electroweak symmetry breaking, is given by $U(1) \otimes SU(2) \otimes SU(3)$. In the condensed matter approach, the fundamental condensate is *not* expected to have symmetries described by a single compact Lie group. In the case of superfluid Helium 3, for instance, the "fundamental" symmetries *already* have a "messy" product group structure $U(1) \otimes SO(3) \otimes SO(3)$, reflecting the spin and orbital angular momentum degrees of freedom of the $^3He$ Cooper pairs. Moreover, in terms of spacetime symmetries in the condensed matter approach, there are also senses in which the emergent relativistic (*viz.*, Poincaré) symmetries are *more perfect* than the fundamental Galilean spacetime symmetries. Note first that the Poincaré group can be characterized as leaving invariant a *single* Lorentzian spacetime metric, whereas the

[25] Smolin (2003, pg. 20) indicates that current experimental data on the violation of Lorentz invariance place very restrictive bounds on preferred frame approaches. Nevertheless he suggests the condensed matter approach may provide key information on the way spacetime might emerge in other scenarios; spin foams, for instance.

Galilei group cannot; the latter leaves *separate* spatial and temporal metrics invariant. Moreover, the Galilei group does not admit unitary representations, whereas the Poincaré group does.[26]

• The second way in which the condensed matter approach de-emphasizes the ontological status of symmetries involves viewing it as an alternative logic of nature to the logic of the Gauge Argument, which typically finds adherents in quantum field theory. According to the Gauge Argument, matter fields are fundamental and imposing local gauge invariance on a matter Lagrangian requires the introduction of interactions with potential gauge fields. The emphasis here is on the fundamental role of local symmetries in explaining the origins of gauge fields (see Martin 2002 for a critique of this argument). According to the condensed matter approach, symmetries, both local and global, as well as matter and potential fields, are low-energy emergent phenomena of the fundamental condensate. In particular, local symmetries do not play a fundamental role in the origin of gauge fields.

• Finally, as will be discussed in the next section, the condensed matter approach can be associated with a notion of structure that is defined in terms of *topology* as opposed to symmetry.

## 5. Universality, Dynamical Structure, and Structural Realism

There is a notion of universality associated with the examples discussed above that can be used to inform a concept of dynamical structure, which subsequently might provide a basis for a structural realist interpretation of spacetime. In this last section, I will indicate how this goes. Ultimately, given the minimal *epistemological* nature of low-energy emergence, I will suggest that this structural realist interpretation is an appropriate way to understand the notion of spacetime as a low-energy emergent phenomenon from an *ontological* point of view.

*Universality and Topology*

To motivate the notion of universality, consider the question of why superfluid $^3$*He-A* reproduces relevant aspects of the Standard Model in the low-energy approximation. Briefly, it turns out that the ground states of $^3$*He-A* and the Standard Model belong to

[26] Of course these senses depend on a more nuanced characterization of "perfection" in group-theoretic terms than in the case of gauge symmetries. Technically, the second sense is based on the fact that the Galilei group has non-trivial exponents, whereas the Poincaré group does not. Unitary representations of the Galilei group up to a phase factor *can* be constructed (so-called projective representations). The importance of unitary representations comes with implementing spacetime symmetries in the context of quantum theory.

the same universality class. This class characterizes the ground state of all fermionic systems with momentum spaces that contain *Fermi points*; *i.e.*, topologically stable point defects where the quasiparticle energies vanish. This is a topological property of momentum space; hence it provides a topological characterization of the universality class. Now the same ground state universality class entails the same low-energy dynamics; hence we have a topological characterization of low-energy dynamics that is *independent* of the "phenomenological" makeup of a system. This suggests a topological notion of dynamical structure.

In a bit more detail, the notion of a universality class of ground states is given a formal treatment in renormalization group (RG) theory. In RG theory, the low-energy behavior of a system can be exhibited by imposing an energy (or momentum) cutoff and then observing how the parameters of the theory evolve as the cutoff is reduced.[27] This involves a two-step procedure in which one first integrates out high-energy modes of the dynamical variables (*i.e.*, modes with energies greater than the cutoff), and then masks the result by rescaling the theory's parameters. Doing this successively generates a flow in parameter (or coupling constant) space. Such a flow may be characterized by a fixed point: a point at which the energy scales smoothly to zero and that is invariant under further rescaling. Such a fixed point may be associated with an EFT.[28] Furthermore, in the space of all coupling constants, more than one flow line may terminate in the same fixed point. A fixed point thus also represents a universality class; namely, it characterizes the low-energy properties of all systems that flow into it. All systems that flow into the same fixed point are thus characterized by the same low-energy EFT.

It turns out that universality classes of *fermionic* systems can be characterized by their momentum space topology (Horava 2005, Volovik 2003). For such systems, an RG analysis requires that the momentum cutoff be reduced towards the Fermi surface, as opposed to the origin.[29] Technically this rescaling of momenta is only done for

[27] Technically, this is Wilson's version of RG theory as applied to condensed matter systems. RG theory can also be applied to quantum field theories in particle physics with slight modification of the terminology above. See, *e.g.*, Polchinski (1992) and Shankar (1994).

[28] A necessary condition for the existence of an EFT, so characterized, is that the associated system exhibit *gapless* excitations; *i.e.*, low-energy excitations arbitrarily close to the ground state. This makes possible a low-energy linear approximation. This notion of an EFT is that described by Polchinski (1992) and Weinberg (1996, pg. 145). For Polchinski, an EFT must be "natural" in the sense that all mass terms should be forbidden by symmetries. Mass terms correspond to *gaps* in the energy spectrum in so far as such terms describe excitations with finite rest energies that cannot be made arbitrarily small. For Weinberg, RG theory should only be applied to EFTs that are massless or nearly massless. (Note that this does not entail that massive theories have no EFTs in so far as mass terms that may appear in the high-energy theory may be encoded as interactions between massless effective fields).

[29] For interacting fermionic systems, the Fermi surface separates the energies of bound (*viz.*, interacting) states from unbound states. In the corresponding EFT (when it exists), the Fermi surface becomes the surface on which quasiparticle energies vanish. An example of a fermionic EFT is Landau's theory of

components of momenta perpendicular to the Fermi surface, allowing transverse components to remain arbitrarily large. This feature makes possible topolgoically interesting regions in momentum space where the quasiparticle energy vanishes, *viz.*, zero modes. To investigate the stability of such modes, one perturbs the system about its fixed point EFT. If the perturbed system maps back to the fixed point under the RG flow, the zero modes are dynamically stable. This dynamical stability can be characterized topologically by noting that a perturbation of the system corresponds to a continuous deformation of its momentum space. Hence dynamically stable zero modes correspond to zero modes in momentum space that are stable under continuous deformations. Such zero modes are said to be topologically stable and can be characterized by homotopy groups in the same way that topologically stable defects in coordinate (*i.e.*, real) space can be.

We now have the resources to answer the question posed at the beginning of this section; namely, Why does superfluid $^3$*He-A* reproduce relevant aspects of the Standard Model in the low-energy limit? The short answer is that superfluid $^3$*He-A* and the Standard Model have ground states characterized by the same momentum space topology; hence these ground states belong to the same universality class, which implies that the low-energy dynamics of both systems is characterized by the same EFT. This short answer needs three qualifications. First, the common momentum space topology involves the existence of stable point defects, *viz.*, Fermi points. Second, strictly speaking, it is only the sector of the Standard Model above electroweak symmetry breaking, characterized by massless Weyl fermion fields, that has such Fermi point topology. Finally, the relation between this sector and superfluid $^3$*He-A* is that the latter is the high-energy, short-distance fundamental theory with respect to which the former is a low-energy approximation. In other words, the sector of the Standard Model above electroweak symmetry-breaking is an EFT of superfluid $^3$*He-A*.[30]

*Dynamical Structure and Structural Realism*

This suggests the following notion of dynamical structure. For fermionic systems, the same momentum space topology entails the same low-energy dynamics (*i.e.*, the same EFT), and this is *regardless* of the microscopic details of the systems. Such details include the values of parameters that appear in the theories describing the systems, for instance, such as the speed of propagation, mass values, and coupling constants. These parameters are theory-specific but, if the given theory belongs to a universality class of

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Fermi liquids (Polchinski 1992, Shankar 1994). This theory describes the strongly interacting electrons in conductors in terms of an EFT that describes a sea of free "dressed" electrons (quasiparticles).

[30] Interpreting the Standard Model as an EFT is arguably justifiable in so far as its "quasiparticle" energies (the energies of the fermion fields that appear in it) are extremely small compared to its cut-off energy (*viz.*, the Planck energy).

ground states, can always be rescaled without affecting the low-energy dynamics. Hence (a particular sector of) the Standard Model, superfluid $^3$*He-A*, and *any* condensed matter system with ground state momentum space topology characterized by Fermi points, all possess the same low-energy dynamics. One might say they all possess the same low-energy dynamical structure. This structure is independent of the values of parameters specific to any member of the universality class. Moreover, it is characterized in terms of topology, as opposed to symmetry.

This notion of low-energy dynamical structure suggests a structural realist interpretation of the condensed matter approach to general relativity and the Standard Model. In general, structural realists interpret theories as referring to structure, but just how they do this has been a matter of some discussion in the recent philosophy of science literature. *Epistemological* structural realists (ESRers) claim that theories provide knowledge only of the structural features of the theoretical entities they posit, whereas *ontological* structural realists claim that theories refer directly to structure and only to structure (and in particular, not to "individual-based ontologies").[31] ESRers typically have faced difficulties in articulating how knowledge of structure can be made non-trivially distinct from (*i.e.*, independent of) knowledge of "individuals". OSRers typically have faced difficulties in articulating exactly *what* structure amounts to, and how a given theory can be said to refer to it.[32] This essay will make no attempt to plumb the depths of these debates. However, the suggestion below will be that the condensed matter approach, with the associated notions of low-energy EFTs and dynamical structure, provides an example of a theory (scheme) that is eminently amenable to a structural realist interpretation that addresses both the concerns of the ESRer and the OSRer.

A structural realist interpretation of the condensed matter approach can be based on the following claims:

1. *The phenomena of experience, as described by general relativity and the Standard Model, are low-energy emergent.*
2. *Theories of such phenomena are EFTs of a "fundamental" theory T that describes a fundamental condensate.*
3. *As EFTs such theories only provide us with knowledge of the low-energy structure of the fundamental condensate.* They only provide us with knowledge of the universality class of which *T* is a member.

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[31] Ladyman (1998) provides a general introduction into the literature on this distinction. In both versions, the claims concerning structure are restricted to in-principle unobservable aspects of a theory's ontology. This can, but does not have to, be based on a distinction in a theory's vocabulary between observational and theoretical predicates, with the referants of the latter being in-principle unobservable.
[32] For a recent review of these problems, see Pooley (preprint).

Given Volovik's analysis of the condensed matter approach (Section 3.2 above), any condensate that purports to reproduce general relativity and the Standard Model must be fermionic (in particular, it must possess Fermi points), hence the dynamical structure of the fundamental condensate will be characterized topologically. Moreover, it is well-defined in the sense of being a universality class. And the relation between a universality class of systems and the systems themselves is also well-defined from a renormalization group-theoretic point of view. Hence this notion of structure arguably addresses the concerns that have been directed towards ontological structural realist interpretations of theories. Given claim (2) above, the dynamical structure associated with a theory just is the universality class to which it belongs.[33]

On the other hand, the above structural realist interpretation also expressly addresses concerns with epistemological structural realist interpretations of theories. In particular, claim 3 allows that we are directly acquainted with the phenomena of experience. However, in so far as these phenomena are low-energy emergent from a fundamental condensate, we have knowledge only of the structure of the fundamental nature of the world as explicitly exhibited by the universality class of the fundamental condensate. The epistemological nature of low-energy emergence plays an essential role here. Again, in so far as the phenomena of experience are low-energy emergent in the epistemological sense, we cannot make inferences from them to the phenomenological properties of the fundamental theory (properties like exact values of parameters, *etc.*). At most, we can only make inferences to structural properties of the fundamental theory; namely, those universal properties that characterize the universality class it belongs to. Note that this is not to say that the nature of the fundamental condensate is purely structural, as an ontological structural realist might claim; rather, it is compatible with the claim that the fundamental condensate is describable by means of an "individuals-based" ontology; *i.e.*, it is some *particular* condensate of particles, say. Hence, while it does provide an explicit notion of structure and how a theory relates to the structure it exhibits, the above structural realist interpretation is minimally *epistemological* in nature.

Note that, as such, it is distinct from typical versions of epistemological structural realism that claim that a theory refers to structure by means of its Ramsey sentence.[34]

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[33] Saunders (2003) also suggests that dynamical structure can be associated with universality classes, but in a more general setting.

[34] See, *e.g.*, Pooley (preprint). Recall that the Ramsey sentence of a theory $T$ formulated as a sentence $S$ in a formal language $L$ is obtained by replacing some subset of $L$-terms that occur in $S$ with variables and then existentially quantifying over these variables. Suppose these $L$-terms are given by $R_1, ..., R_k$. Then the Ramsey sentence of $S(R_1, ..., R_k)$ is given by $\exists x_1 ... \exists x_k S(x_1 , ... x_k)$.

Such versions are hard pressed to avoid the following dilemma:[35] On the one hand, if all predicates of a theory are Ramseyfied, then the Ramsey sentence is trivially true up to a cardinality claim about the domain of individuals, and hence has no empirical content beyond the latter. On the other hand, if some subset of predicates is deemed *observational* and exempted from Ramseyfication, then the Ramsey sentence will be true *if and only if* the statements involving these observational predicates are true. Thus the claim that the Ramsey sentence provides knowledge only of the structure of the referants of *theoretical* predicates appears to be unsubstantiated, and this variant of structural realism risks becoming indistinguishable from constructive empiricism. This dilemma is easily avoided by the above emergentist epistemological structural realism. Simply put, the notion of low-energy emergent dynamical structure based on universality classes cannot be articulated in terms of a Ramseyfication of a *single* theory. Rather, such structure is obtained by means of a low-energy approximation relation between two *different* theories. Now this would be beside the point if this relation could be fleshed out in terms of the EFT being embeddable in the original theory, which would allow the construction of a single Ramsey sentence encompassing both. However, from Section 4, this is not the case: the relation of low-energy approximation cannot be described in terms of definitional extension (either finitistic or infinitistic). Hence there is a "larger gap" between the structure of the fundamental theory and phenomenological EFTs than can be described by notions of structure based on Ramsey sentences.

The above structural realist interpretation of the condensed matter approach suggests the following structural realist interpretation of spacetime.

1. *The spatiotemporal aspects of the phenomena of experience are low-energy emergent.* Such aspects in particular include the spacetime symmetries of the Standard Model and general relativity, our current best theories of matter and gravity.
2. *The spatiotemporal aspects of the fundamental condensate are structural.* Since the fundamental condensate can only be known structurally, through its low-energy EFTs, its spatiotemporal aspects can only be known structurally. And these aspects are just those spatiotemporal properties of the universality class to which the fundamental condensate belongs.

Such a structural realist interpretation of spacetime, as we've seen, comes with a major qualification; namely, that the universality class that best describes spacetime structure is unknown at present. It certainly cannot be identified with the universality class of superfluid $^3$*He-A* and the Standard Model, in so far as neither can recover the dynamics of general relativity. Again, however, on a positive note, such a structural realist

[35] In it's original form, this is due to an objection given by Newman in 1928 to a version of epistemological structural realism briefly advocated by Russell.

interpretation can be linked with a definite research programme in condensed matter physics, and for this reason alone should be given due consideration.

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