# Tense Logic in Einstein-Minkowski Space-time

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**Abstract**

This paper argues that the Einstein-Minkowski space-time of special relativity provides an adequate model for classical tense logic, including rigorous definitions of tensed becoming and of the logical priority of *proper time*. In addition, the extension of classical tense logic with an operator for predicate-term negation provides us with a framework for interpreting and defending the significance of future contingency in special relativity. The framework for future contingents developed here involves the dual falsehood of non-logical contraries, only one of which becomes true. This has several methodological, metaphysical and physical advantages over the alternative traditional frameworks for handling future contingents.

## 1 Introduction

Elsewhere(Forthcoming) I have argued for the existence of a distinction between the past and the future drawn using only the standard resources of special relativity and the structure of Einstein-Minkowski space-time. However, the distinction between an indeterminate future and a determinate past leaves us with two significant semantic problems. First, given the usefulness of the tense logical formulation of natural language tenses, it would be useful to demonstrate that it carries over on the transition from a global temporal structure in a universe whose spatio-temporal structure consists of distinct 3-dimensional space and 1-dimensional time to a local structure in a 4-dimensional space-time universe. Second, the formulation of relational indeterminacy in terms of the probability structure on the possible states of space-time leaves the traditional problem of future contingents somewhat obscure.

The general consensus seems to have become that no model constructed only from resources of modern space-time physics plus standard model theory can offer a solution to either of these problems. More precisely, it has generally been taken to be the case that no model making use only of the standard space-time of special relativity and the associated causal relations can account for either tensed becoming or the existence of future contingency. First, the claim is that any explication of tensed becoming must amount to a reduction of tense since nothing in the structure of space-time uniquely distinguishes a present time. Second, that the ontological commitments attendant upon the space-time conception, in particular the commitment to the existence of the entire space-time, rules out future contingency.

The semantic structure developed in this essay addresses both of these concerns. Using only the resources of standard model theory and the structure of Einstein-Minkowski space-time, I demonstrate that there are two natural senses in which a four-dimensional universe can be a model of standard tense. First, if we are willing to make use of the present tense only locally, the entire space-time constitutes a model for the standard tense logic. Second, I show that *proper time along any world-line* in Einstein-Minkowski space-time is a standard model for tense. Finally, since the objects in the domain of the model developed in §2 have both space-like and time-like separated elements, it demonstrates that our use of tense logical formalism does not commit us to any particular model of the persistence of objects.

Next, I further refine the structure introduced in §2 to deal with future contingents. To do so I introduce two refinements to the structure. First, I introduce an operator for predicate-negation into the tense logical language. Second, I modify the valuation functions of the models to make use of a double indexing structure. Given the existence of a predicate-negation operator there is a precise sense in which both the assertion and the predicative denial that a particular object possesses a particular property or stands in a particular relation in the future can be false. In the following section, I argue that this account of future contingents as future falsehoods better accords with our intuitions about the concept of the occurrence of an event. In addition, since it need not postulate either truth-value gaps, three-valued logic, or branching space-times it has significant theoretical advantages as well. Finally, I will argue that the double indexing structure introduced allows us to capture the semantic content of *relational indeterminacy.*

## 2 The Language and the Model Structure

In order to demonstrate the compatibility of tense logic with Einstein-Minkowski space-time, we consider a basic tense logical language, $\mathscr{L}_{\mathrm{T}}$, consisting of singular terms $\mathrm{t}_1 \cdots \mathrm{t}_\mathrm{n}$ and n-ary predicates, $\mathrm{P}^\mathrm{n}_1 \cdots \mathrm{P}^\mathrm{n}_\mathrm{i}$ the universal quantifier, $\forall$, negation,$\sim$, conjunction, &, and the tense operators, $\boldsymbol{F}$ and $\boldsymbol{P}$. The formation rules for $\mathscr{L}_{\mathrm{T}}$ are completely standard.

Now consider a possible model structure for $\mathscr{L}_{\mathrm{T}}$, $\mathscr{M}$. The model is an ordered sextuple, $<\mathbb{W}, >, \mathcal{T}(\mathbb{W}), \mathbb{D}, f, \mathbf{v}>$. $\mathbb{W}$ is time-oriented Einstein-Minkowski space-time of special relativity. $\mathcal{T}(\mathbb{W})$ is the ordinary topology on $\mathbb{W}$, and $>$ is the relation of time-like separation between points of $\mathbb{W}$. $\mathbb{D}$ is a (generally non-empty) set–the objects. $f$ is the occupation function for the objects assigning elements of $\mathbb{D}$ into $\mathcal{T}(\mathbb{W})$, open regions in $\mathbb{W}$

$$f : \mathbb{D} \to \mathcal{T}(\mathbb{W})$$

.

## 2.1 Explicating Einstein-Minkowski Space-Time

Technically, time-oriented Einstein-Minkowski space-time is a four-dimensional real manifold with the topology of $\mathbb{R}^4$ with a flat semi-Riemannian, Lorentz signature metric and a time-orientation. A real n-dimensional manifold is one that can be mapped smoothly into $\mathbb{R}^n$. In Einstein-Minkowski space-time the maps are bijections, and thus every open set on $\mathbb{W}$ maps to an open set on $\mathbb{R}^4$ and induces the standard topology. Given these maps the concept of a smooth curve on $\mathbb{W}$ is well-defined and induces a vector space structure, the tangent space, at every point of $\mathbb{W}$. A semi-Riemannian, Lorentz signature metric is a tensor field of signature $<3,1>$ on $\mathbb{W}$, that defines an inner product on the tangent space at each point. Since Einstein-Minkowski space-time is flat, the metric tensor immediately induces a (pseudo-)distance function between points in the space-time.

The essential point about such metrics is that they partition the tangent space at each point of the manifold into three classes. Consider the inner product of a tangent vector, v, with itself–its norm, $\mathbf{g}(\mathbf{v},\mathbf{v})$. If $\mathbf{g}(\mathbf{v},\mathbf{v})= 0$, we call it null; if $\mathbf{g}(\mathbf{v},\mathbf{v})>0$, it is spacelike; if $\mathbf{g}(\mathbf{v},\mathbf{v})<0$, it is timelike. A vector is causal if it is either timelike or null. This induces a (non-exhaustive) classification of curves on the manifold as null (timelike, causal, or spacelike) curves if, and only if, their tangents are everywhere null (timelike, causal, or spacelike). Finally, we say that two points are null (timelike, causal, or spacelike) separated if they are connectable by a null (timelike, causal, or spacelike) curve.

In addition, the metric defines the length of each curve in the manifold via integration. This serves to distinguish a special class of curves, the metric geodesics. For a positive definite metric, the geodesics are simply the curves of extremal (maximal or minimal) length between any two points. Unfortunately, the definition of a metric geodesic in mixed signature metrics is considerably more subtle because there need not be any extremal length curve between two points. Technically, the metric geodesics of a Lorentz signature metric have critical length. However, there is a more intuitive characterization for Minkowski spacetime. Using the formal characterization of geodesics as critical curves, we can prove that if the tangent to a geodesic is anywhere null (timelike, causal, or spacelike) then it is everywhere null (timelike, causal, or spacelike). Thus, the metric does provide an exhaustive classification of geodesics. Given this classification of metrics, we can independently characterize timelike and spacelike geodesics. Thus, a timelike geodesic is the most negative length timelike curve connecting any two points. (Remember that timelike curves have negative length.) Spacelike geodesics restricted to a spacelike hyperplane of the spacetime are the shortest spacelike curves between any two points in the hyperplane. Next, it is possible to prove that for a given metric, there is one, and only one, affine structure compatible with that metric. An affine structure and a metric are compatible if and only if all and only the metric geodesics are affine geodesics. Thus, the metric structure induces an affine structure, and therefore an inertial structure, on the spacetime.

The class of null vectors at a point forms a double-lobed hypercone in the tangent space at the point, while the class of null geodesics through a point forms a double-lobed hypercone on the manifold with its vertex at the point. Both of these structures are often called the *null cone*, or the *light cone*. In the flat spacetime of special relativity, this ambiguity does not matter much, since the null cone of vectors at a point uniquely determines the null cone of curves through a point. It should be obvious from the definition of null geodesics that every null vector is tangent to at least one such geodesic. In order to show that the geodesic is unique we appeal to the flatness of the space. Assume that there are two distinct null geodesics, $\gamma$ and $\gamma'$, through a point $p$ which have the same tangent vector. The requirement that the geometry of geodesics be Euclidean implies that any two parallel geodesics remain parallel and thus share a tangent vector at every point. However, its tangents at every point define a curve on a manifold. Therefore, $\gamma$ and $\gamma\prime$ must be the same curve, contrary to our assumption. However, note that the proof of this fact depends essentially on the assumption of flatness, and the null cone at a point clearly does not determine the null cone through a point in general. Because of this, I will maintain the distinction between the null cone at a point and the null cone through a point. Thus, both the relation of time-like separation and the topology in the $\mathscr{M}$ are induced by the space-time rather than separately specified as that notation might imply. Finally, a time-orientation on such a space-time is an everywhere time-like vector field on $\mathbb{W}$ that determines at each point of $\mathbb{W}$ a distinguished lobe of the tangent space null cone and thus of the manifold light cone as well.

## 2.2 Explicating the Valuation Function

The valuation function of $\mathscr{M}$ takes the following form:

**Definition 2.1.** $\mathbf{v}$ is a valuation function that assigns to each $\mathrm{t_i}$ in $\mathscr{L}_\mathrm{T}$ an element $\delta_\mathrm{i} \in \mathbb{D}$ and to each $\mathrm{P^n_i}$ a function $g_{\mathrm{P}_i} : \mathbb{W} \to \mathscr{P}(\mathbb{D}^\mathrm{n})$.

The first clause of $\mathbf{v}$ assigns to each name in $\mathscr{L}_\mathrm{T}$ a single object within the space-time, independently of the location of evaluation or of the location of the assigned object. The second assigns an extension to every predicate at each point of $\mathbb{W}$.

## 2.3 Interpretation and Consequences

We can now state the interpretation of $\mathscr{L}_\mathrm{T}$ in $\mathscr{M}$. $\mathbf{V}$ is an interpretation function, such that:

1. *Atomic Sentences* $\mathbf{V}[\mathscr{W}, w \in \mathbb{W}, P^\mathrm{n}(t_1 \cdots t_\mathrm{n})] = 1$ if and only if $\delta_1 \cdots \delta_\mathrm{n} \in \mathbb{D}$
   (a) $\mathbf{v}(t_1) = \delta_1 \cdots \mathbf{v}(t_\mathrm{n}) = \delta_\mathrm{n}$
   (b) $w \in \bigcap_{i=1}^{n} f(\delta_\mathrm{i})$
   (c) $< \delta_1 \cdots \delta_\mathrm{n} > \in \mathbf{v}[\mathrm{Pn}](w)$

2. $\mathbf{V}[\mathscr{W}, w \in \mathbb{W}, \sim \phi)] = 1$ if and only if $\mathbf{V}[\mathscr{W}, w \in \mathbb{W}, \phi)] = 0$

3. $\mathbf{V}[\mathscr{W}, w \in \mathbb{W}, \phi \& \psi)] = 1$ if and only if $\mathbf{V}[\mathscr{W}, w \in \mathbf{W}, \phi)] = \mathbf{V}[\mathscr{W}, w \in \mathbb{W}, \psi)] = 1$

4. $\mathbf{V}[\mathscr{W}, w \in \mathbb{W}, \forall x \phi)]$ if and only if $\mathcal{M}\delta/x$ where $w \in f(\delta)$ $\mathbf{V}[\mathscr{W}\delta/x, w \in \mathbb{W}, \phi)] = 1$

5. $\mathbf{V}[\mathscr{W}, w \in \mathbb{W}, \mathrm{F}\phi] = 1$ if and only if $\mathbf{V}[\mathscr{W}, w' \in \mathbb{W}, \phi] = 1$ where $w' > w$ and $w' \in f(\mathbf{v}[t_\mathrm{i}])$ for all $\mathrm{t_i}$ terms in $\phi$

6. $\mathbf{V}[\mathscr{W}, w \in \mathbb{W}, \mathrm{P}\phi)] = 1$ if and only if $\mathbf{V}[\mathscr{W}, w' \in \mathbb{W}, \phi)] = 1$ where $w > w'$ and $w' \in f(\mathbf{v}[t_\mathrm{i}])$ for all $t_{\mathrm{i}}$ terms in $\phi$ .

This is clearly a model for standard tense logic barring only that the index set is merely partly ordered, but see below.

Given that the concept of an object being sequentially present along the time-like separated points of its world-line (world-worm) is a coherent notion, whether analyzed indexically or taken as a primitive concept, this model answers most of the fundamental challenges to the compatibility of space-time formulations of special relativity and becoming. First, we have a clear cut sense of tensed becoming, even in what is paradigmatically taken to be a tenseless universe. Consider some object $\delta$. For simplicity assume that $\delta$ is point-like in that every point in $f(\delta)$ is time-like separated from every other point. As long as $f(\delta)$ contains at least two points, p>q, there will be some well-formed formula, $\phi$, in $\mathscr{L}_\mathrm{T}$ such that $\boldsymbol{F}\phi$ is true at q and $\boldsymbol{P}\phi$ is true at p.

Next, the model provides us with a particularly neat argument that *proper time along any worldline* just is how time is defined for an entity occupying that worldline. Consider any arbitrary time-like curve, $\gamma$ in $\mathbb{W}$. $[\gamma]$, the image of $\gamma$, is a one-dimensional sub-manifold of $\mathbb{W}$. Consider $\mathbb{D}' \subseteq \mathbb{D}$, where $\delta \in \mathbb{D}'$ iff $f(\delta) \cap [\gamma] \neq \emptyset$. Finally, consider $f'$, the restriction of $f(\mathbb{D})$ to a domain of $\mathbb{D}'$ and the range restricted to $[\gamma]$. Then, $< [\gamma], >, \mathbb{D}', f', \mathbf{v} >$ is a perfectly ordinary dense, linear time flow.

Finally, note that these consequences flow from a model whose *prima facie* metaphysical committments are both strongly eternalist and strongly perdurantist. Eternalist, because the model commits us to the existence of all of the objects in the domain wherever they exist in the space-time. Perdurantist, because the basic elements of the model are both spatially and temporally extended and clearly have both spatial and temporal parts.

## 3 Extending the Language and the Model to Account for Future Contingents

While the basic model of §2 addresses the compatibility of a space-time ontology with tense logic and demonstrates the reality of tensed becoming in Einstein-Minkowski space-time, nothing has yet been said about "the problem of future contingents." The next three sections rectify that lapse. In this section, I introduce a formalism that allows us to represent the status of future contingents in Einstein-Minkowski space-time. This requires us to introduce a predicate-negation operator and to modify the valuation function of the previous section. In Section 4, I argue that the representation of the joint falsehood of what would otherwise be contrary statements is the best available conception of future contingents. Then Section 5 argues that given special relativity such indeterminate predicates in fact exist.

I claim that one of the most basic intuitions about the indeterminate future is that there is something "fuzzy" about entities to the future. Even if they exist, it is far from clear what properties they possess or what events they are involved in–what Adolf Grünbaum called "attribute indefiniteness."(Grünbaum, 1963)What then would it mean to claim that an object neither truly possesses nor truly lacks a particular attribute. Consider the following two predicates: "...is red." and "...is not red." In standard first-order logic we interpret a sentence involving the second predicate as the truth-functional negation of one involving the second sentence. But, we need not commit ourselves to that interpretation. Let us introduce an additional operator into $\mathscr{L}_{\mathrm{T}}$: the predicate-negation operator, **N**. **N** obeys the following two formation rules:

$$\mathbf{N}\mathrm{P}_i^n \text{is an n-ary predicate of } \mathscr{L}_{\mathrm{TN}} \text{ if and only if } \mathrm{P}_i^n \text{ is.} \tag{1}$$

$$\mathbf{N}\mathbf{N}\mathrm{P}_i^n = \mathrm{P}_i^n \tag{2}$$

Thus, the operator '**N**' operates as, in the language of neo-Aristotelian term logic, a variety of *predicate-term* negation.[1] Now consider the difference between '∼Pt' and '**N**Pt' for some one place predicate. These are logically equivalent if and only if '**N**' designates the logical contrary to a predicate at some w$\in \mathbb{W}$. Let us say that a predicate such that this is the case, according to a particular model, is *determinate*. That is if:

**Definition 3.1.**

$$\mathrm{P}_i^n \text{is } \textbf{determinate} \text{ if and only if } \mathbf{v}[\mathscr{M}, w, \mathrm{P}_i^n] \cup \mathbf{v}[\mathscr{M}, w, \mathbf{N}\mathrm{P}_i^n] = \mathbb{D}^n$$

However, nothing in the concept of predicate negation seems to require this constraint. First, because most predicates do possess a range of non-logical contraries, e.g. colors. Second, given a collection of predicates representing what we take to be intrinsic or "projectible" properties, there does seem to be a "natural" contrary to each such predicate read as "definitely does not possess that property or relation." The basic intuition about "attribute-indefiniteness" seems to be captured precisely by the idea that an object neither definitely possesses nor definitely lacks a property or relation. Finally, when we relax this condition, we obtain a natural sense of what it is for an event to occur (see §4). Thus, on this account, if, for simplicity, $\mathrm{P}_i^n$ is a monadic predicate and is not determinate then there is some $\delta \in \mathbb{D}$ such that both $P_i$(t) and $\mathbf{N}\mathrm{P}_i$(t) are false. In the next section, I will

[1] Thanks to Heinrich Wansing for pointing this out to me at Logica 2007 and directing me to his useful article, "Negation" in (Goble, 2001)

argue that this is the most plausible way to understand the status of future contingents. However, we first need some additional technical machinery.

In particular, we must re-define the valuation function for $\mathscr{M}$ so as to account for the difference between evaluating a sentence at a point in space-time and evaluating it from, or relative to, a point in space-time. I do this by introducing double-indexing of the valuation function.

**v**:
1. $\{t_{\rm i}\} \otimes \mathbf{W} \rightarrow \mathbb{D}$, $\mathbf{v}(t_{\rm i}, w_{\rm j}) = \mathbf{v}(t_{\rm i}, w_{\rm k})$
2. $\{{\rm P}^n_i\} \otimes \mathbf{W} \rightarrow \{g_P : \mathbf{W} \rightarrow \mathbb{P}(\mathbb{D}^n)\}$

The first clause, despite the restriction on the extension of the terms, is fairly standard. The restriction simply guarantees that the extensions of the terms do not vary over "time." The second clause is to be read as assigning an extension to ${\rm P}^n_i$ at a point $w$ only relative to another point $w'$, $\mathbf{v}[{\rm P}^n_i, w'](w) = \Delta$, where $\Delta \in \mathscr{P}(\mathbb{D}^n)$. This does allow the the extensions of the predicates to vary over "time," in particular relative to different points along the world-lines of objects in the space-time. The final constraint, to capture the notion that everything is determinate when evaluated relative to itself is that:

$$\text{For all } {\rm P}^n_i\text{, if w=w', then } {\rm P}^n_i \text{ is determinate.} \tag{3}$$

However, now we have access to a specific way to capture the status of future contingents. The claim that a particular entity is indeterminate with respect to a particular property or relation results from the indeterminacy of the relevant predicate as to the particular entity–the entity falls into the "extension gap" of the predicate. In such a case, both the assertion of the relation and the predicate-denial of the negation are false, and the sentential-negation of both the assertion and the predicate-denial are true. I argue in the next section that this is the correct interpretation of future contingents. When the predicate becomes determinate, then one of the assertion or the predicate-denial becomes true and the other remains false.

## 4 Are Future Contingents Actually False?

The claim at the end of the previous section that future contingency reveals itself at the semantic level as the joint falseness of the assertion and the predicate-denial that an object instantiates a particular property or relation runs counter to the standard attempts to deal with the semantics of future contingents. The standard accounts of the semantics of future contingents seem to be variations or combinations of three distinct positions–three, or more, valued logics;(See, e.g. Prior, 1953) truth-value gaps(See, e.g. Thomason, 1970); or branching space-times(See, e.g., Belnap, 1992; McCall, 1976, 1994). The argument that "predicate-denial theory of future contingency" best captures our usual conception will proceed in four phases. First, I characterize the four possibilities in terms of their account of Aristotle's classic sea-battle example. Next, I argue that this theory better accounts for a significant grammatical distinction between two modes of denying that the sea-battle will occur tomorrow. Third, that it best accounts for our ordinary intuitions about what it means for the sea-battle, or any other event to occur. And, finally that it has the significant advantage of doing so without "messing about" with the semantic values of our language, the structure of our logic, or the structure of space-time.

Now consider the classical problem of future contingents from Aristotle's *de Interpretatione*. Assume that it is not now determined whether a sea-battle will occur tomorrow. It still seems to be the case that:

Either a sea-battle will occur tomorrow or a sea-battle will not occur tomorrow. (4)

The problem arise because (4) seems to be a tautology, and thus true at all times, including the present. However, this seems to require that one its two disjuncts must also be true *now*. But, that seems to imply that it is, in fact, *now* determined whether a sea-battle will happen tomorrow, contrary to the initial assumption.

On a 3-valued account of logic, the interpretation of Aristotle's sea-battle depends on the precise version of "disjunction" one chooses to use to interpret (4). Since the first disjunct is to be assigned the middle truth-value, so must the second. Then, if one chooses the "weak" definition, such that the value of the disjunction is the maximum of the disjuncts, (4) is also indeterminate. The "weak" definition, thus, "solves" the problem by diagnosing a previously unrecognized ambiguity in the logical connectives. On the "strong," or additive, definition, the disjunction is true. Thus, it solves the puzzle only by denying excluded middle. On an account such that neither disjunct currently possesses a truth-value, that they occupy a truth-value gap, the tautology is true because the truth of the disjunction holds on every possible consistent assignment of truth values to the disjuncts. On the branching space-time account (4) is true because, while it is not now determined which branch will become actual, whatever branch becomes actual will have one disjunct true and one false. Thus, both of these "solutions" solve the problem by denying excluded middle. The second, where the branching allegedly takes place in physical space-time, also poses massive problems for the physics of space-time.

On the theory defended here, (4) is true, if it is true, in precisely the way that all disjunctions are true, by having one true disjunct. Consider two readings of the second disjunct of (4).

It is not the case that a sea-battle will occur tomorrow. (5)

A sea-battle will not occur tomorrow. (6)

On this account (4) is only a tautology when we read the second disjunct as (5). In that case it is true because (5) is. But, if we read the second disjunct as (6), then (4) is not a tautology, and if the occurrence of the sea-battle is indeterminate, it is in fact false. Consider a name, t, and the predicate

$$\text{S}=\text{`}\ldots\text{is a sea-battle in the Mediteranean.'} \tag{7}$$

Then, (5) would be translated into $\mathscr{L}_{\text{TN}}$ as:

$$\sim \boldsymbol{F}\text{St} \tag{8}$$

and (6) as:

$$\boldsymbol{F}\mathbf{N}\text{St} \tag{9}$$

However, notice the suggestive difference between these two readings. (8) is quite plausibly read as the *present* denial that a sea-battle takes place in the future. But, since the sea-battle has not actually occurred, it does seem plausible that the denial that it has occurred is true. (9), on the other hand, *now*asserts, about the future, that it will not be occupied by a sea-battle. But, since we nothing *now* determines whether that will be the case, it seems equally plausible that such an assertion be presently false. Such a grammatical distinction can never be anything but suggestive. However, I claim that the model introduced here has additional advantages.

First, consider what we ordinarily mean when we claim that an event occurs. As a general rule, this amounts to a claim that some entity comes to possess some property or to stand in some relation that it previously did not. On this reading, we can give a precise content to this notion. Consider some property, represented by the predicate $\text{P}_i$, that is indeterminate with respect to an object $\delta$. When the object comes to possess the property, it *literally* is added to the extension of $\text{P}_i$. The availability of this straightforward account of occurrence confers a significant theoretical advantage on this approach.

Finally, it handles future contingents without any of the messiness of the other three accounts, although certainly with its own kind. We need not become involved in the messy debates over the plausibility of multiply valued logics and of the need to formulate plausible consequence relations for them. We need not deny the law of excluded middle. And, we can continue to use our ordinary physics on ordinary space-time. However, do we have any reason to believe that ordinary space-time contains such future contingents? In the next section, I argue that we do.

## 5 Does the World Contain Future Contingents?

We have reason to believe that every space-time point not past causally separated from a given point is *relationally indeterminate* to that point. That relation is defined in terms of an abstract characterization of the state of space-time as follows.

Start with space-time theories. Let's say that, abstractly, such a theory consists of a space-time and the assignment of possible values to every point of space-time. In the standard cases, assignments of scalar, vector or tensor fields to the space-time. A specification of all of the available types of values to each point of the space-time, I will call the state of the space-time. Now, let us consider topologically open regions of space-time, down to points. Obviously, given the state of space-time, the relevant values are also determined for each such region. In the absence of a dynamics, there is a meaningful sense in which all possible assignments of such values are equally allowable. But, of course, this is not what we want to know. We do not deal in God's eye views of space-time, except at the most abstract levels. We deal with regions of space-time.

What we really want to know is given the (full or partial) specification of the state of a region of space-time, how does that, given a relevant theory, constrain the states of other regions of space-time? Obviously, to do this we need a dynamics which connects the states of different regions of space-time and connects the various "elements" of the state. Assign equal probability to all dynamically consistent states of the entire space-time. Now consider an arbitrary region, an open set in the usual topology, R of the space-time. Some of the dynamically possible states of the space-time assign the same state to that region, some of them assign different states. Thus, the states of regions inherit the probability that they will be in a given state from the initial probability assignment to states of the space-time. That is, for all possible states r of R, we can derive the probability that R is in state r, prob(r≡R). But, now consider another region, S. Via the usual definition of conditional probability, we can define for each state s of S, the probability that S is in s given that R is in r, prob(s≡S|r≡R) for each of possible state r of R.

Finally, given that our goal is to investigate what regions of space-time are indeterminate relative to certain other regions in various space-times, the minimal constraints on the dynamics are the relations of causal connection appropriate to the space-time under consideration.

**Definition 5.1.** **caus**(S,R) if and only if the state of S could causally influence the state of R.

First, note that in these definitions regions can be replaced with points as the degenerate case of regions in the usual topology. Second, Definition 5.1 allows us to define a partition of the space-time, relative to any given region R, into the causal past, $\mathbb{C}$(R); the causal present, $\mathbb{P}$(R); and the causal future, $\mathbb{F}$(R), as follows:

**Definition 5.2.** $\forall q, R\ \exists p\{(q \in \mathbb{C}(\mathrm{R})) \Leftrightarrow (p \in R\ \&\mathbf{caus}qp\ \&\sim \mathbf{caus}pq)\}$

**Definition 5.3.** $\forall q, R\ \exists p\{(q \in \mathbb{P}(\mathrm{R})) \Leftrightarrow (p \in R\ \&\mathbf{caus}qp\ \&\ \mathbf{caus}pq)\}$

**Definition 5.4.** $\forall q, R\ \exists p\{(q \in \mathbb{F}(\mathrm{R})) \Leftrightarrow (p \in R\ \&\sim \mathbf{caus}qp\ \&\ \mathbf{caus}pq)\}$

It will also be useful to have a name for the entire region from which R is causally accessible, i.e. the union of $\mathbb{C}$(R) and $\mathbb{P}$(R): call it, $\mathbb{A}$(R). We are now ready to say what it is for the state of one region of space-time to determine the state of another region.

**Definition 5.5.** S **determines** R [**det**(S,R)] if and only if for each possible state s of S, there exists a state r of R, such that for some Q⊆S, Q⊆ $\mathbb{A}$(R) and for the state q of Q induced by s, prob(r≡R|q≡Q)=1.

Finally, we can return to the question that opened this section, what would it be for the future to be open? Given the above it must be that the future is open if it is not determined, in the sense of Def. 5.5, by the past. Which past? Given that the future is defined relative to our changing space-time location, it must be relative to *our* past. Thus, define the following two concepts:

**Definition 5.6.** A point $q_o$ is **Relationally Indeterminate** (RI) relative to a point $q_1$ if and only if there is no R⊆ $\mathbb{A}(q_1)$ such that **det**(R,$q_0$)

**Definition 5.7.** A point $q_o$ is **determinate** relative to a point $q_1$ if and only if there is an R⊆ $\mathbb{A}(q_1)$ such that **det**(R,$q_0$)

Therefore, we can finally state that the future is open, in the only sense that seems to matter, if and only if it is relationally indeterminate relative to our changing location. Or, alternatively, that an event happens at $q_o$ relative to $q_1$ only when the state of $q_o$ is determinate relative to $q_1$. Given a standard and natural reading of special relativity, future regions of space-time are, in fact, relationally indeterminate.

There are serious problems with representing this concept of the state of space-time and, thus, of relational indeterminacy in a basically first-order language. However, we can make a first pass at it as follows. Consider a toy space-time theory in which the only values assigned to space-time are two scalar fields. Such a scalar field assigns two real number values to each point of space-time. We can then define two sets of monadic predicates, $Q_i$ and $R_i$, as follows. Divide the possible values of each of the two fields into a collection of half-open sets, {[0,1), [1,2)...[n,+)}. Associate a predicate with each element of the partition so that:

**Definition 5.8.** $Q_n w$ iff the value of the Q-field is such that n≤ $Q(w)<(n+1)$

And, similarly for P. Take the domain to be open sets on the space-time and the occupation function to be identity. Now, assume that we have a dynamics for P and Q given by ordinary partial-differential equations with a well-formed initial value problem(IVP). Given the existence of a well-formed IVP, then the specifications of the fields on any single space-like hypersurface (Cauchy surface) have a single *dynamically acceptable* extension to the remainder of space-time. But, for any region less than a complete Cauchy surface we have the same only for some spatio-temporal volume, less than the entire space-time, determined by the volume of the initial region. However, outside of that determinate region, P and Q can take pretty much any values depending on the state of the remainder of space-time.

Define a state description for a point of space-time,

**Definition 5.9.** $\Omega_w$, is the set$\{\sim\mathrm{P}_1 w \ldots \sim\mathrm{P}_{(i-1)}w$, $\mathrm{P}_i w$,$\sim\mathrm{P}_{(i+1)}w \ldots \sim\mathrm{P}_n w$,
$\sim\mathrm{Q}_1 w \ldots \sim\mathrm{Q}_{(i-1)}w$, $\mathrm{Q}_i w$,$\sim\mathrm{Q}_{(i+1)}w \ldots \sim\mathrm{Q}_n w\}$
where $i \leq P(w) < (i+1)$ and $j \leq Q(w) < (j+1)$.

Clearly a specification of the state of space-time, as above, determines a state description in this sense for every point and region of space-time. Just as clearly, the state description, $\Omega_C$ of a Cauchy surface possesses a single dynamically acceptable extension to a state-description, $\Omega_{\mathbb{W}}$ for the entire space-time. However, just as clearly there will be more than one dynamically acceptable extension of any region less than a complete Cauchy surface. Certainly if we make n sufficiently large and the partition used to define the predicates fine enough. But, I do not see any principled bar to defining such a language for any physical theory defined as above.[2]

Thus, we can now define *semantic relational indeterminacy* as follows. Now, considering arbitrary predicates and domains again.

**Definition 5.10.** $< \delta_1 \cdots \delta_n > \in \mathbf{v}[\mathrm{P}^n, w'](w)$ if and only if for R, the causal past of w′, $\mathrm{P^n(t_1 \cdots t_n)}$ belongs to every dynamically acceptable extension of $\Omega_\mathrm{R}$ for some $< t_1 \cdots t_n >$ such that $\mathbf{v}(\mathrm{t_1}) = \delta_1 \cdots \mathbf{v}(t_n) = \delta_n$.

and

**Definition 5.11.** $< \delta_1 \cdots \delta_n > \in \mathbf{v}[\mathbf{N}\mathrm{P}^n, w'](w)$ if and only if for R, the causal past of w′, $\mathbf{N}\mathrm{P^n(t_1 \cdots t_n)}$ belongs to every dynamically acceptable extension of $\Omega_\mathrm{R}$ for some $< t_1 \cdots t_n >$ such that $\mathbf{v}(\mathrm{t_1}) = \delta_1 \cdots \mathbf{v}(t_n) = \delta_n$.

Then, given the existence of relational indeterminacy and a sufficiently carefully designed language, there will be at least some indeterminate predicates of $\mathscr{L}_{\mathrm{TN}}$.

## 6 Conclusion

To conclude, I have argued, first, that Einstein-Minkowski space-time with the standard topology and a time ordering provides a model structure for standard tense logic when we restrict the use of the present tense to local regions of space-time. In addition, when we consider *proper time* along a world-line within Einstein-Minkowski space-time, it constitutes a completely standard dense linear time flow. Given this we have a full conception of tensed becoming within Einstein-Minkowski space-time when we postulate that objects are sequentially "present" along their world-lines, whether we explain that sequential presence indexically or take it as a metaphysical primitive.

[2] Actually, I am eliding a whole class of problems in first-order representations of physical values, here. Including problems about values that vary along multiple dimensions; the nature of vector and tensor fields; and zero-value physical quantities, just to name a few. For an interesting discussion of these issues, focused on the last problem, see Balashov (1999)

Second, I have argued that when we extend standard first-order tense logic to include a predicate-negation operator, we can formulate a precise definition of the status of future-contingents as the joint falsehood of the assertion and of the predicate-denial that an object within the model possesses a certain property or stands in a particular relation. This provides a rigorous account of the concept of the "attribute indefiniteness" of future entities. Finally, even in a deterministic universe, the causal structure of special relativity guarantees that at least some indeterminacy will occur to the future of any region of space-time that does not constitute a Cauchy surface of the dynamics.

Finally, let me indicate two additional features of the theory developed here. First, at least for worlds with a fixed background space-time, the model structure can easily be adapted to other possible worlds, and probably to the actual world, where space-time is at best locally Minkowskian. Second, it also allows us to formulate a rigorous notion of what it means to say that time is or is not real within a given world. That is, we should accept that time is real in a given possible world just in case the space-time of that world supports a model for tense logic. Even more so, we might have worlds where time is, in a sense, partly real. Thus, suppose that a given space-time is not globally orientable and, thus does not possess a global time orientation. In such a world, for any time-orientation$<$, there are points p, q such that both $p<q$ and $q<p$. It might still be locally orientable and particular curves within the space-time might possess an orientation. Such curves, when viewed as the world-lines of objects in the space-time, constitute models of tense logic for objects occupying those world-lines. It would then be reasonable to claim that time is real for those objects, but not for objects in other regions of the space-time. And, this would be a perfectly objective fact about the entities and their histories within space-time. Not a subjective fact about time-perception.

## References


Yuri Balashov. Zero-value physical quantities. *Synthese: An International Journal for Epistemology, Methodology and Philosophy of Science*, 119(3):253–286, 1999.

Nuel Belnap. Branching space-time. *Synthese: An International Journal for Epistemology, Methodology and Philosophy of Science*, 92(3):385–434, 1992.

Lou Goble. *The Blackwell guide to philosophical logic.* Malden, 2001.

Adolf Grünbaum. *Philosophical Problems of Space and Time.* Alfred A. Knopf, New York, 1963.

James Harrington. Special relativity and the future: A defense of the point-present. *Studies in the History and Philosophy of Modern Physics*, forthcoming.

Storrs McCall. Objective time flow. *Philosophy of Science*, 43:337–362, 1976.

Storrs McCall. *A Model of the Universe : Space-time, Probability, and Decision.* Clarendon Library of Logic and Philosophy. Clarendon Press, Oxford and New York, 1994.

A. N. Prior. Three-valued logic and future contingents. *Philosophical Quarterly*, 3:317–326, 1953.

Richmond H. Thomason. Indeterminist time and truth-value gaps. *Theoria: A Swedish Journal of Philosophy*, 36:264–281, 1970.