Common-causes are not common common-causes

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Abstract

A condition is formulated in terms of the probabilities of two pairs of correlated events in a classical probability space which is necessary for the two correlations to have a single (Reichenbachian) common-cause and it is shown that there exists pairs of correlated events probabilities of which violate the necessary condition. It is concluded that different correlations do not in general have a common common-cause. It is also shown that this conclusion remains valid even if one weakens slightly Reichenbach's definition of common-cause. The significance of the difference between common-causes and common common-causes is emphasized from the perspective of Reichenbach's Common Cause Principle.

1 Motivation of the problem

Suppose that within a hypothetical population one finds the following probabilities:

1. the probability $p(A_1) = \frac{7}{8}$ of a citizen's causing a traffic accident,
   the probability $p(B_1) = \frac{7}{8}$ of a citizen's suffering from advanced ulcer,
   the probability $p(A_1 \wedge B_1) = \frac{13}{16}$ of a citizen having advanced ulcer causing a traffic accident;
2. the probability $p(A_2) = \frac{1}{8}$ of having a criminal record of domestic violence,
   the probability $p(B_2) = \frac{1}{8}$ of a citizen having liver disease,
   the probability $p(A_2 \wedge B_2) = \frac{1}{16}$ of a citizen having liver disease abusing his/here spouse.

Can one entertain the idea that the positive correlations betwen having advanced ulcer and causing a traffic accident on one hand and between having a criminal record of domestic violence and liver disease on the other have the *same* cause, alcoholism, say? Clearly, this question cannot be answered before one has specified a notion of "cause" of a correlation. One such notion is Reichenbach's concept of common-cause of a correlation. Having Reichenbach's concept, one just has to check whether Reichenbach's conditions are satisfied for both correlations by the event "alcoholism" in the statistical ensemble of the population, and, if they are, the question is answered in the affirmative. But suppose Reichenbach's conditions are *not* satisfied for both correlations by the event "alcoholism". Can one maintain nevertheless that the two correlations are explainable by the *same* common-cause, although not by alcoholism? To be more precise, the general problem we wish to raise and investigate in this paper is whether one can decide unambigiously and exclusively on the basis of the given probabilities whether the two correlations under consideration can possibly have the *same* (Reichenbachian common) cause – whatever that common- cause may be: *any* attribute in the population, or *any* attribute in *any* population extending that of our hypothetical population – assuming that the probabilites of the correlated events remain the same in the extended population.

In view of the generality of this question one may surmise that the answer to it is "yes"; that is to say, one may conjecture that given any two correlations there can always exist a Reichenbachian common-cause which is a common-cause for *both* correlations, since, one may reason, we just have to refine our picture of the world by adding more and more events to the original event structure in a consistent manner, and finally we shall find a single common-cause that explains both correlations. In this paper we prove that, appealing as this conjecture may appear on some grounds, it is false: We formulate a simple necessary condition for two correlations to have the same Reichenbachian common-cause and show that this condition can be violated. In particular, the numbers given above violate the condition; hence, there exists no single event whatsoever that could be the common-cause of both correlations.

The structure of the paper is the following. First we recall Reichenbach's notion of common-cause of a (positive) correlation (Definition 1); this is followed by a precise formulation of the problem of existence of a common common-cause of two correlations (Problem). Proposition 4 gives a necessary condition for two correlations to have a common common-cause; Problem is solved then in the negative (Proposition 3) by showing that the necessary condition can easily be violated by correlations in simple probability spaces (Proposition 5 and 6). We discuss some refinements of the argument in Section 3 by considering a natural weakening of Reichenbach's definition of common-cause and by allowing negative correlations also enter the scene. It will be seen that the conclusion spelled out in Proposition 3 remains valid under the weakening of the definition of common-cause (Proposition 7), and it also turns out that the situation does not change if one of the correlations are allowed to be negative. We close by commenting in Section 4 on the philosophical significance of the fact that common-causes are very different from common common-causes.

2 Definitions of common-cause and of common common-cause

Let $(\mathcal{S}, p)$ be a classical probability space, where $\mathcal{S}$ is the Boolean algebra of random events and p is the probability measure on $\mathcal{S}$. Events A and B are said to be (positively) *correlated* if

$$Corr(A,B) \equiv p(A \wedge B) - p(A)p(B) > 0 \tag{1}$$

According to Reichenbach ([8], Section 19), a probabilistic common-cause type explanation of a correlation like (1) means finding an event C (common-cause) that satisfies the conditions specified in the next definition.

Definition 1 *C is a* common-cause *of the correlation (1) if the following (independent) conditions hold:*

$$\begin{aligned} p(A \wedge B|C) &= p(A|C)p(B|C) & (2) \\ p(A \wedge B|C^{\perp}) &= p(A|C^{\perp})p(B|C^{\perp}) & (3) \\ p(A|C) &> p(A|C^{\perp}) & (4) \\ p(B|C) &> p(B|C^{\perp}) & (5) \end{aligned}$$

where $p(X|Y) = p(X \wedge Y)/p(Y)$ denotes the conditional probability of X on condition Y, $C^{\perp}$ denotes the complement of C and it is assumed that none of the probabilities $p(X)$, $(X = A, B, C, C^{\perp})$ is equal to zero.

We shall occasionally refer to conditions (2)-(5) as "Reichenbach(ian) conditions". To exclude trivial common-causes we call a common-cause C *proper* if it differs from both A and B by more than a measure zero event. In what follows "common-cause" will always mean a proper common-cause.

By saying that displaying a common-cause C for the correlation (1) one *explains* the correlation Reichenbach means that the following proposition holds.

Proposition 1 *Let A, B and C be three elements in the Boolean algebra S of the measure space (S, p), and assume that (2)-(5) hold. Then A, B are positively correlated.*

The above proposition is based on the following observation: If (2) and (3) hold for A, B and C. Then we have

$$p(A \wedge B) - p(A)p(B) = p(C)p(C^{\perp})[p(A|C) - p(A|C^{\perp})][p(B|C) - p(B|C^{\perp})] \tag{6}$$

Next we define the *extension* of a probability space. Before giving the definition recall that the map $h: \mathcal{S}_1 \to \mathcal{S}_2$ between two Boolean algebras $\mathcal{S}_1$ and $\mathcal{S}_2$ is a *Boolean algebra homomorphism* if it preserves all lattice operations (including orthocomplementation). A Boolean algebra homomorphism h is an *embedding* if $X \neq Y$ implies $h(X) \neq h(Y)$.

Definition 2 *The probability space $(\mathcal{S}', p')$ is called an* extension *of $(\mathcal{S}, p)$ if there exists a Boolean algebra embedding h of $\mathcal{S}$ into $\mathcal{S}'$ such that*

$$p(X) = p'(h(X)) \qquad \textit{for all } X \in \mathcal{S} \tag{7}$$

As we indicated in Section 1. the problem we wish to investigate in this paper is whether two correlations $Corr(A_1, B_1) > 0$ and $Corr(A_2, B_2) > 0$ can have a *common* common-cause.

Definition 3 *Given two correlations $Corr(A_1,B_1)$ and $Corr(A_2,B_2)$ in the same probability space $(\mathcal{S},p)$ the element $C\in\mathcal{S}$ is called a* common common-cause *if C is a common-cause (in the sense of Definition 1) of both $Corr(A_1,B_1)$ and $Corr(A_2,B_2)$.*

By the definition of common common-cause above the question of whether two correlations can have a common common-cause presupposes that the two correlations are in the same probability space $(\mathcal{S},p)$. Given two correlations in a probability space it may happen very well, however, that one of the correlations does not have a common-cause in the given probability space (it also is possible that none does). If this is the case then we call $(\mathcal{S},p)$ *common-cause incomplete.* Absence in $(\mathcal{S},p)$ of common-causes of the two correlations should not, however, be taken reason enough to conclude that the two correlations cannot have a common common-cause, because common-cause incomplete probability spaces can always be enlarged so that the larger probability space contains a proper common-cause of the given correlation. To be more precise, Proposition 2 below is true. Before spelling out the proposition recall that a common-cause C of a correlation $Corr(A,B)$ is said to have the type

$$Type_p(A,B,C)=(r_C,r_{A|C},r_{A|C^\perp},r_{B|C},r_{B|C^\perp}) \tag{8}$$

if these five real numbers are equal to the probabilities indicated by their indices, i.e. if it holds that

$$\begin{aligned}
r_C &= p(C)\\
r_{A|C} &= p(A|C)\\
r_{A|C^\perp} &= p(A|C^\perp)\\
r_{B|C} &= p(B|C)\\
r_{B|C^\perp} &= p(B|C^\perp)
\end{aligned}$$

Given a correlation $Corr(A,B)>0$, a set of five real numbers (8) is called an *admissible type* if the assumption of a common-cause of $Corr(A,B)>0$ having this type does not lead to contradiction.

Proposition 2 *Given any* finite *set of correlations $Corr(A_i,B_i)>0$ $(i=1,\ldots,n)$ in a probability space $(\mathcal{S},p)$, and given any set of types $Type_p(A_i,B_i,C_i)$ $(i=1,2\ldots n)$ such that $Type_p(A_i,b_i,C_i)$ is admissible for $Corr(A_i,B_i)$ for every i, there exists an extension $(\mathcal{S}',p')$ of $(\mathcal{S},p)$ such that each correlation $Corr(A_i,B_i)$ $(i=1,2\ldots n)$ has a common-cause $C_i\in\mathcal{S}'$ of $Type_p(A_i,B_i,C_i)$.*

(See Proposition 2 and its proof in [2].)

Note that what Proposition 2 says is not that there exists a *common* common-cause C of the whole set of correlations $Corr(A_i,B_i)$ $(i=1,2\ldots n)$; in fact, the common-causes C_i constructed explicitly in the proof of Proposition 2. in [2] are all different: $C_i\neq C_j$ $(i\neq j)$. In view of Proposition 2 we may always assume that a finite set of correlations in a given probability space is such that each correlation has a common-cause in the given probability space; hence, the general and precise formulation of our problem is the following.

Problem : Let $(\mathcal{S},p)$ be a probability space and $Corr(A_i,B_i)$ $(i=1,2\ldots n)$ be a finite set of correlations in $(\mathcal{S},p)$. Does there exist an extension $(\mathcal{S}',p')$ of $(\mathcal{S},p)$ such that there exists a $C\in\mathcal{S}'$ which is a *common* common-cause of all the correlations $Corr(A_i,B_i)$ $(i=1,2\ldots n)$?

We solve this problem by proving the following proposition.

Proposition 3 *There exists a probability space $(\mathcal{S},p)$ and two correlations*

$$Corr(A_1,B_1) > 0 \tag{9}$$
$$Corr(A_2,B_2) > 0 \tag{10}$$

in $(\mathcal{S}, p)$ such that there cannot exists an extension $(\mathcal{S}', p')$ of $(\mathcal{S}, p)$ that contains a common common-cause of these two correlations.

We prove the above proposition in two steps. First we give a necessary condition for two correlations to have a common common cause, and then we show that the condition can be easily violated in a simple probability space.

Proposition 4 *If two correlations (9)-(10) have a common common-cause in $(\mathcal{S}, p)$ then the following inequality holds.*

$$max\{r_1, r_2\} \leq min\{R_1, R_2\} \tag{11}$$

where R_1, R_2, r_1, r_2 are the following real numbers

$$R_i = \frac{p(A_i)p(B_i)}{p(A_i \wedge B_i)} \qquad i = 1, 2 \tag{12}$$

$$r_i = \frac{Corr(A_i, B_i)}{[p(A_i) - 1][p(B_i) - 1] + Corr(A_i, B_i)} \qquad i = 1, 2 \tag{13}$$

Proof: Let $(\mathcal{S}, p)$ be any probability space and $Corr(A, B) = p(A \wedge B) - p(A)p(B) > 0$ be a correlation in it. If we assume that there exists a common-cause C in $(\mathcal{S}, p)$ of the given correlation then, using the theorem of total probability

$$p(X) = p(X|Y)p(Y) + p(X|Y^{\perp})(1 - p(Y)) \qquad X, Y \in \mathcal{S} \tag{14}$$

we can write

$$\begin{aligned}
p(A) &= p(A|C)p(C) + p(A|C^{\perp})(1 - p(C)) && (15)\\
p(B) &= p(B|C)p(C) + p(B|C^{\perp})(1 - p(C)) && (16)\\
p(A \wedge B) &= p(A \wedge B|C)p(C) + p(A \wedge B|C^{\perp})(1 - p(C)) && (17)\\
&= p(A|C)p(B|C)p(C) + p(A|C^{\perp})p(B|C^{\perp})(1 - p(C)) && (18)\\
p(A|C) &> p(A|C^{\perp}) && (19)\\
p(B|C) &> p(B|C^{\perp}) && (20)
\end{aligned}$$

((18) follows from (17) because of the screening off equations (2)-(3), while (19)-(20) is just (4)-(5)).
Consider the system of 3 equations (15)-(17) with $t = p(A|C)$ and $s = p(B|C)$ as parameters. One can then express $p(C), p(A|C^{\perp})$ and $p(B|C^{\perp})$ from equations (15)-(17) as follows.

$$p(C) = \frac{Corr(A, B)}{[p(A) - t][p(B) - s] + Corr(A, B)} \tag{21}$$

$$p(A|C^{\perp}) = \frac{p(A) - t}{p(C^{\perp})} + t = \frac{p(A \wedge B) - p(A)s}{p(B) - s} \tag{22}$$

$$p(B|C^{\perp}) = \frac{p(B) - s}{p(C^{\perp})} + s = \frac{p(A \wedge B) - p(B)t}{p(A) - t} \tag{23}$$

One can verify by elementary algebraic calculations that for the conditions (15)-(20) to hold the two parameters t, s must be within the bounds

$$1 \geq \quad t = p(A|C) \quad \geq \frac{p(A \wedge B)}{p(B)} = p(A|B) \tag{24}$$

$$1 \geq \quad s = p(B|C) \quad \geq \frac{p(A \wedge B)}{p(A)} = p(B|A) \tag{25}$$

and, conversely, if t, s are within these bounds then the numbers $p(C), p(A|C^{\perp})$ and $p(B|C^{\perp})$ defined by the equations (21)-(23) are smaller than one and the conditions (15)-(20) hold. Consider now the function $(t,s) \mapsto p(C)(t,s)$ defined by

$$(t,s) \mapsto p(C)(t,s) = \frac{Corr(A,B)}{[p(A)-t][p(B)-s] + Corr(A,B)} \tag{26}$$

Elementary algebraic reasoning shows that if $Corr(A,B) > 0$ then the function $(t,s) \mapsto p(C)(t,s)$ is continuous and monotone decreasing in both t and s in the intervals specified by (24)-(25); therefore $p(C)$ takes on its minimum (respectively maximum) value at $\big(t = 1, s = 1\big)$ (respectively at $\big(t = p(A|B), s = p(B|A)\big)$) and these minimum and maximum values are

$$\begin{aligned} min\ p(C)(t,s) &= \frac{Corr(A,B)}{[p(A)-1][p(B)-1] + Corr(A,B)} \\ &= \frac{Corr(A,B)}{p(A^{\perp} \wedge B^{\perp})} \\ max\ p(C)(t,s) &= \frac{p(A)p(B)}{p(A \wedge B)} \end{aligned}$$

So the numbers R_i, r_i $(i = 1,2)$ in (12)-(13) are just the maximum and minimum values of the possible probabilities of the common-causes C_i of the correlations $Corr(A_i, B_i)$; furthermore, the continuity of $(t,s) \mapsto p(C)(t,s)$ implies that any number in the interval $[r_i, R_i]$ can in principle be the probability of a common-cause C_i of the correlation $Corr(A_i, B_i)$ $(i = 1,2)$. It follows then that a necessary condition for the two correlations $Corr(A_1, B_1)$ and $Corr(A_2, B_2)$ to have a common common-cause is that the intersection of the two intervals $[r_i, R_i]$ $(i = 1,2)$ is nonempty. The inequality (11) is just an expression of this requirement. Thus Proposition 4 is proven. As a corollary we have

Proposition 5 *If*

$$max\{r_1, r_2\} > min\{R_1, R_2\} \tag{27}$$

(with R_i and r_i defined by (12)-(13)) then the correlations (9)-(10) cannot have a common common-cause in $(\mathcal{S}, p)$ or in whatever extension $(\mathcal{S}', p')$ of $(\mathcal{S}, p)$.

In view of this corollary, to prove Proposition 3 it suffices to display a probability space with two correlations $Corr(A_1, B_1)$ and $Corr(A_2, B_2)$ in it such that $r_2 > R_1$, where r_2 and R_1 are defined by (13) and (12). We claim that there exist two such correlations in the probability space $([0,1], p)$, where p is the Lebesgue measure on the unit interval. Indeed, let $Corr(A_2, B_2)$ be any correlation in $([0,1], p)$, then $r_2 > 0$ is a fixed real number, and it is elementary to find a correlation $Corr(A_1, B_1)$ in $([0,1], p)$ such that $R_1 = \frac{p(A_1)p(B_1)}{p(A_1 \wedge B_1)} < r_2$. Hence Proposition 3 is proved.

In fact, we have proved somewhat more than just Proposition 3, since the following statement is true:

Proposition 6 *Given any probability space $(\mathcal{S}, \nu)$ and a correlation $Corr(A_1, B_1)$ in it one can find an extension $(\mathcal{S}', \nu')$ of $(\mathcal{S}, \nu)$ such that there is a correlation $Corr(A_2, B_2)$ in the extension $(\mathcal{S}', \nu')$ and the two correlations $Corr(A_1, B_1)$ and $Corr(A_2, B_2)$ cannot have a common common-cause.*

Proposition 6 follows since one can form the standard product probability space $(\mathcal{S} \times [0,1], \nu \times p)$ which is an extension of both $(\mathcal{S}, \nu)$ and $([0,1], p)$, and for any correlation $Corr(A_2, B_2)$ in $(\mathcal{S}, \nu)$ with some r_2 we can find a correlation $Corr(A_1, B_1)$ in $([0,1], p)$ with $R_1 < r_2$, consequently these two correlations cannot have a common common-cause by Proposition 5.

3 Refinements

One can strengthen Proposition 3 slightly by weakening its assumptions; this can be done by weakening Reichenbach's definition of common-cause in the following way. While C and $C^\perp$ feature symmetrically in the equations (2)-(3) in Reichenbach's definition, this symmetry is broken by conditions (4)-(5): it is these conditions that specify C, rather than $C^\perp$, as the common-cause. Distinguishing C from $C^\perp$ may very well be justified on some intuitive grounds; however, from the perspective of the explanatory power interpreted as validity of Proposition 1, the asymmetry between C and $C^\perp$ is not needed: as eq. (6) shows, Proposition 1 remains valid if *both* inequalities in (4)-(5) are reversed. Hence, one can, without loosing the explanatory significance of the notion of common-cause, weaken Reichenbach's definition of common-cause of a (positive) correlation by requiring only that, in addition to the screening off conditions (2)-(3), the quantities $[p(A|C)-p(A|C^\perp)]$ and $[p(B|C)-p(B|C^\perp)]$ have the *same* sign. In view of eq. (6), this weakened definition amounts to the demand that C is a common-cause of a *positive* correlation if (2) and (3) hold for C and $C^\perp$; in other words, if both C and $C^\perp$ screen off the correlation.

We claim that this weakening of Reichenbach's notion of common-cause does not change the validity of Proposition 3: it remains true that different correlations do not, in general, have a common common-cause even if the common-cause is taken in the weakened sense just described; however, in this case the necessary conditions for two correlations to have a common common-cause become slightly weaker: One can follow exactly the same line of argument as in the proof of Proposition 3 with the only modification that the possible range of the common cause C_i of the $i-th$ correlation $(i=1,2)$ is now represented not by the single interval $I_i=[r_i,R_i]$ but by two intervals: I_i and $I'_i=[r'_i,R'_i]$ where

$$R'_i = \frac{p(A_i^\perp)p(B_i^\perp)}{p(A_i^\perp \wedge B_i^\perp)} \tag{28}$$

$$r'_i = \frac{Corr(A_i^\perp, B_i^\perp)}{p(A_i \wedge B_i)} \tag{29}$$

Consequently, the necessary condition for $Corr(A_1,B_1)$ and $Corr(A_2,B_2)$ to have a common common-cause in the weakened sense is that either I_1 or I'_1 intersect with either I_2 or I'_2; to put it concisely: the necessary condition is that

$$(I_1 \cup I'_1) \cap (I_2 \cup I'_2) \neq \emptyset \tag{30}$$

Elementary algebraic manipulations show that one can explicitly formulate this latter condition (30) in terms of the probabilities of the four events involved as follows

$$\frac{p(A_1)p(B_1)}{p(A_1 \wedge B_1)} + \frac{p(A_2^\perp)p(B_2^\perp)}{p(A_2^\perp \wedge B_2^\perp)} \geq 1 \quad \text{and} \quad \frac{p(A_1^\perp)p(B_1^\perp)}{p(A_1^\perp \wedge B_1^\perp)} + \frac{p(A_2)p(B_2)}{p(A_2 \wedge B_2)} \geq 1 \tag{31}$$

or

$$\frac{p(A_1)p(B_1)}{p(A_1 \wedge B_1)} + \frac{p(A_2)p(B_2)}{p(A_2 \wedge B_2)} \geq 1 \quad \text{and} \quad \frac{p(A_1^\perp)p(B_1^\perp)}{p(A_1^\perp \wedge B_1^\perp)} + \frac{p(A_2^\perp)p(B_2^\perp)}{p(A_2^\perp \wedge B_2^\perp)} \geq 1 \tag{32}$$

Let us summarize all this in the form of a proposition:

Proposition 7 *The necessary condition for two correlations (9)-(10) to exist a $C \in S$ such that both C and $C^\perp$ screen-off both correlations is that conditions (31)-(32) hold.*

The above reasoning also shows that if instead of two correlations one is given a finite set $Corr(A_i,B_i)>0$ $(i=1,2,\dots N)$, then a necessary condition for the existence of a single C which is a common-cause in the weak sense of *all* the correlations $Corr(A_i, B_i) > 0$ is that

$$\cap_{i=1}^{N}\left(I_i \cup I'_i\right) \neq \emptyset \tag{33}$$

with I_i and I'_i defined by (12)-(13) and (28)-(29) by letting the index i run through $1, 2, \ldots N$. In the general case of $N > 2$ one also can give the explicit algebraic condition equivalent to (33) in terms of the probabilities of the events involved, this condition will be less and less transparent with the increase of N, however.

It should also be noted that condition (11) (respectively (30)) are not only necessary but also sufficient conditions for a common common-cause of *two* correlations to exist in the sense of Reichenbach (respectively in the weakened sense of common cause). This can be seen by the following reasoning. Reichenbach's conditions do not determine completely the type of the common-cause of a correlation: as the proof of Proposition 2 in [2] shows (and this can also be seen from the proof of Proposition 4) two numbers of the five in (8) (for instance $r_{A|C} = p(A|C)$ and $r_{B|C} = p(B|C)$ as in the proof of Proposition 4), are left unspecified by the Reichenbach's conditions. Therefore, if, given two correlations $Corr(A_1, B_1) > 0$ and $Corr(A_2, B_2) > 0$, one can choose C such that $p(C) \in (I_1 \cup I'_1) \cap (I_2 \cup I'_2)$, then one can choose freely the numbers $p(A_1|C)$ and $p(A_2|C)$, say, to fix two common-cause types, one for each of $Corr(A_1, B_1)$ and $Corr(A_2, B_2)$, and one can then apply Proposition 2 to conclude that there exists a C which is a common common-cause of the two correlations. If we have more than two correlations, then the situation changes and this argument is no longer valid because in the case of more than two correlations there may exist relations between the different correlations that entail additional constraints on the the admissible types of the common-causes of the correlations, constraints that may not be satisfiable. We wish to emphasize that no conditions are known that in the case of more than two correlations are necessary *and* sufficient for the existence of a common common-cause of the correlations.

The definition of common-cause given in section 2 follows closely Reichenbach's; in particular, the definition specifies a common cause for a *positive* correlation. It is clear, however, that *negative* correlations might just as much be in need of an explanation by a common-cause as could be positive ones. The only modification needed in Reichenbach's definition of common-cause to cover the case of negative correlations is reversing *only one* of the inequalities in (4) and (5). The resulting definition of common-cause yields a notion for which Proposition 3 remains valid because Proposition 7 remains valid with obvious modifications: If $Corr(A_1, B_1) < 0$, then $Corr(A_1^{\perp}, B_1) > 0$ and $Corr(A_1, B_1^{\perp}) > 0$, and both C and $C^{\perp}$ screen off the correlation between A_1 and B_1 if and only if they screen off the correlation between $A_1^{\perp}$ and B_1, which is the case if and only if C and $C^{\perp}$ screen off the correlation between A_1 and $B_1^{\perp}$; consequently, given two correlations, a negative $Corr(A_1, B_1) < 0$ and a positive one $Corr(A_2, B_2) > 0$, the necessary condition for C and $C^{\perp}$ to exist which both screen off the correlation is that (31)-(32) hold with the substitution $A_1 \to A_1^{\perp}$, equivalently, with the substitution $B_1 \to B_1^{\perp}$.

4 Closing comments

Proposition 3 shows that Proposition 2 cannot be strengthened in the following sense: While it is true that, given a finite set of correlations in a common-cause incomplete probability space the probability space can always be enlarged so that the larger one contains a common-cause of each correlation in the given set, these common-causes differ from correlation to correlation, and, in general, there exists no enlargement that contains a common common-cause of even *two* of the correlations in the finite set. One conclusion one can draw from this is that the notions of common-cause and of common common-cause are radically different; that is to say, if two correlations have a common

common-cause, then the random events involved stand in a probabilistic relation content of which is not exhausted by the individual relations of the common-causes to the correlations explained by them. Formulated differently: The assumption that two correlations have a common common-cause is much stronger than the assumption that each of the two correlations has its own common-cause, and, while Proposition 2 shows that one *cannot* conclude exclusively on the basis of knowing the probabilities of the events that common-causes of correlations do not exist, knowing the probabilities of the events involved one *can* exclude common-common causes. Being aware of this, one should be extremely cautious when requiring that an explanation of a set of correlations should be in terms of a common common-cause: such a requirement should always be carefully argued. It should be kept in mind in particular that Reichenbach's Common-Cause Principle, the metaphysical claim that if there is a correlation between events that cannot be due to a direct causal connection between the correlated events then there must exist a common-cause of the correlation, has nothing to do with multiple correlations and their (generally nonexistent) common common-causes. Hence from the fact that common common-causes of a given set of correlations do not exist one cannot conclude that Reichenbach's Common-Cause Principle is violated. In particular the fact that the notorious EPR correlations do not in general possess a common common-cause does not in and by itself entail that Reichenbach's Common-Cause Principle does not hold ([7], Chapter 11 in [6], [2], [3], [4] and [9]); not even if *certain* EPR type correlations can have a common common-cause, as Placek shows [5].

If one has the intuition, mentioned briefly in the introduction, that two correlations in a classical probability space should always have a common common-cause in a sufficiently rich random event structure, then another attitude one might take in view of Proposition 3 is that Reichenbach's notion of common-cause is inappropriate. The universal adequacy of Reichanbach's notion of common-cause has already been called into question, especially by Van Fraassen [10] and by Cartwright [1] – but on different grounds: their critique of Reichenbach's notion is not related to the issue of distinction between common-cause and common common-cause; rather, they see a problem with applicability of Reichenbach's notion to the case of correlations that are due to a conserved quantity. This is not the place to dwell into an analysis of Van Fraassen's and Cartwright's critique of Reichenbach's notion. We just wish to point out that it would be interesting to see how/whether the distinction between common-cause and common common-cause can be formulated in terms of the notion of common-cause proposed by Cartwright in [1], and, if this can be done, whether results similar to the ones presented here can be established.

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