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Einstein's Triumph over the Spacetime Coordinate System: A Paper presented in Honor of Roberto Torretti

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1. Introduction

Each student of Einstein must eventually make his or her their peace with Einstein's pronouncements on relativity and spacetime coordinate systems. Einstein saw the development of relativity as the ultimately successful struggle to overcome certain spacetime coordinate systems and thereby to implement a generalized principle of relativity. This signal achievement of relativity is embodied in its general covariance. We now hold spacetime coordinate systems merely to be convenient devices for smoothly labeling events. The selection of a coordinate system amounts to little more than a conventional choice of

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[1] Dr. Torretti has inspired my generation: in scholarship, by setting the standard in his researches in history and philosophy of space and time; and in humanity with his generosity and kindness. I take this opportunity to thank him personally for the stimulating model of scholarship in his *Relativity and Geometry* and related writings and for his encouragement, patience and instruction when I first worked in history and philosophy of space and time, especially during a year we shared at the Center for Philosophy of Science, University of Pittsburgh, in 1983-1984. He helped make it one the most exciting years intellectually of my life.

numbers, much like the selection of definition. How can one proclaim victory over a definition? If we are offended by a definition, the more appropriate attitude is just to decide quietly not to use it.

Dr. Torretti's celebrated *Relativity and Geometry* and related writings represent a landmark of scholarship. They provide our most detailed account of how Einstein's work in relativity theory changed physical geometry. It is presented in a comprehensive historical context with the uncompromised insistence that every geometric conception must be explicated to the highest standards of mathematical rigor. So when Dr. Torretti makes his peace with the problem of Einstein and spacetime coordinates in Section 5.5 "General Covariance and the Einstein-Grossmann theory," this latter insistence ensures that the peace will be uncomfortable—for Einstein. He takes Einstein's formulation of the postulate of general covariance and rephrases in language that mimics Einstein's 1905 statement of the principle of relativity of special relativity. Calling it the "principle of general relativity," Dr. Torretti explains why the similarity of the two relativity principles is only superficial. Unlike the case of the special principle, the general principle does not assert a physical equivalence of states of motion. Dr. Torretti's analysis is careful, thorough and leaves no room to quibble. So we are left with a puzzle. How could Einstein be so confused about the fundamentals of his own theory?

My goal in this paper is small. I do not want to dispute Dr. Torretti's careful analysis. Rather I offer an extended footnote to it. I want to try to explain what Einstein intended in his remarks about coordinate systems. There is, I believe, a natural reading for Einstein's claims that do make perfect sense. They require us to adopt a physical interpretation of relativity theory that is now no longer popular, so the natural reading will no longer have intrinsic interest. It will, however, allow us to make sense of Einstein's claims and his program.

2. "The Vanquishing of the Inertial System"

A Letter to Besso

When we face claims that are unintelligible in the writing of an Einstein, we are often tempted to dismiss them as remarks made in haste in the frenzied first moments of great discovery. Might they not be retracted or qualified in some essential way as time brings sober distance from those heady moments? While time mellowed Einstein, we can be sure this was not the case with his proclamations over coordinate systems. He brought the general theory of relativity to a generally covariant formulation in November 1915. Nearly 40 years later, after his theory had been much celebrated and its foundations subject to minute scrutiny, Einstein wrote to his lifelong friend and confidant, Michele Besso.
His letter of August 10, 1954, lays out a brief account of the essence of the general theory of relativity, explicitly intended to be free of entanglement with the history of the theory. (Speziali, 1972, p.525) [2]

> Your characterization of the general theory of relat.[ivity] characterizes the genetic side quite well. It is also valuable afterwards, however, to analyze the whole matter logically-formally. For as long as one cannot determine the physical content of the theory on account of temporarily insurmountable mathematical difficulties, logical simplicity is the only criterion of the value of the theory, even if it is naturally an insufficient one.
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> The special th.[eory] of r.[elativity] is really nothing other than an adaptation of the idea of the inertial system to the empirically confirmed conviction of the constancy of the velocity of light with respect to each inertial system. It does not vanquish the epistemologically untenable concept of the inertial system. (The

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[2] I thank Karola Stotz for help in this translation.

untenability of this concept was brought to light especially clearly by Mach and was, however, already recognized with lesser clarity by Huygens and Leibniz.)

The core of this objection against Newton's fundamentals is best explained through the analogy with the "center point of the world" of Aristotelian physics: there is a center point of the world, towards which heavy bodies strive. This explains, f[or] e[xample], the spherical shape of the earth. The ugliness in it is that this center point of the world acts on all others, but that all these others (i.e. bodies) do not act back on the center point of the earth. (One-sided causal nexus.)

It is just like this with inertial systems. They determine the inertial relations of things everywhere, without being influenced by them. (Really one ought better to speak of the aggregate of all inertial systems; however this is inessential.) The essence of the gen.[eral] th.[eory] of rel.[ativity] (G. R.) lies in the vanquishing [Ueberwindung] of inertial systems. (This was still not so clear at the time of the setting up of G. R., but was subsequently recognized principally through Levi-Civita.) In the setting up of the theory I had chosen the symmetric tensor g_{ik} as the starting concept. It provided the possibility of defining the "displacement field" Γ^l_{ik}…

Einstein briefly explained the notion of the displacement field and its independence from the metric g_{ik}. He continued:

But how is it that the displacement field really led to the vanquishing of inertial systems? If one has vectors with the same components at two arbitrarily distant points P and Q in an inertial system, then this is an objective (invariant) relation: they are equal and parallel. On this rests the circumstance that one obtains tensors again through differentiation of tensors with respect to the coordinates in an inertial system and that e.g. the wave equation represents an objective expression in inertial systems. The displacement field now allows such

> tensor formation by differentiation in relation to an arbitrary coordinate system. Therefore it is the invariant substitute of inertial systems and thereby--as it appears--the foundation of every relativistic field theory.

Einstein then continued to explain how the metric and displacement field are used to formulate general relativity and his unified field theory.

Its Unusual Treatment of Coordinate Systems

Einstein finds the essence of the general theory to lie in the vanquishing of inertial systems, that is, inertial coordinate systems. Part of his account is that these systems have the objectionable feature of acting without being acted upon. That aspect has been subject to much discussion and analysis. It is usually explicated by the notion of "absolute object," geometric objects that act but are not acted upon. In special relativity, the pertinent absolute object is the Minkowski metric.[3] Here I pass over the problem of explicating the absoluteness Einstein raises; I am interested in just one other aspect. Einstein's notion of the absolute inertial [coordinate] *system* has been transmogrified into an absolute geometric *object*, the Minkowski metric.

It is so tempting to say that this transformation is what Einstein really intended. But then we must be amazed at his tenacity in avoiding the assertion. His remarks to Besso mention the metric field and the displacement field, both geometric objects, but condemns the inertial *system* for its absolute character—and this forty years after his achievement of general covariance.[4]

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[3] For discussion see Norton (1993, Section 8).

[4] Similar remarks on inertial systems span Einstein's life. They appear, for example, as early as Einstein (1913, pp. 1260-61) and as late as a letter to George Jaffé of January 19, 1954 (Einstein Archive, document with duplicate archive control number 13 405).

2. Einstein's use of Coordinate Systems

Their Physical Content

There is a simple way to understand Einstein's remarks.[5] He did not regard coordinate systems as we now do, as essentially arbitrary systems of numerical labels of events. In his theorizing, they initially carried significant physical content. The journey to the completion of general relativity required the systematic elimination of this content.

That coordinate systems can be used to represent significant physical content is not the modern view and it is tempting to think that no other view is possible. But that narrowmindedness is quite incorrect. Our physical theories use mathematical structures to represent aspects of interest of the physical world. We routinely use a manifold that is topologically R^4 to represent the set of physical events in special relativity. Nothing prevents us using the structurally richer number manifold of quadruples of reals as this manifold. If we do use a number manifold in this way, then we are assigning quadruples of reals to events in spacetime. That is just what a coordinate system does.

A number manifold has considerably more structure than we use in standard theories of spacetime. It has a preferred origin (0,0,0,0), for example. How are we to interpret that? Does this preferred origin correspond to a real physical center point of the world? Whether it does or not cannot be decided purely by the mathematics of the theory. The mathematics can only affirm that (0,0,0,0) is indeed different from all other points in R^4, but not that the differences amount to nothing physically. This last judgment must be made by the physical interpretation we supply for the mathematical structures. The modern view is to discount it as physically insignificant. Einstein's default was the opposite. The various

[5] I have developed the approach to Einstein's use of coordinate systems sketched below in greater detail in Norton (1989, 1992).

features of coordinate systems represent physical features of the world. Most crudely, the origin (0,0,0,0) is a physical center point. In Einstein's program, we must find a way of depriving coordinate systems of this default physical content.

…and How it is Systematically Denied

Einstein used a single technique that was not his own invention. He used a strategy codified by Felix Klein in the nineteenth century.[6] Each geometric theory would be associated with a class of admissible coordinate systems and a group of transformations that would carry us between them. The cardinal rule was that physical significance can be assigned just to those features that were invariants of this group. In special relativity, that group is the Poincaré group. The origin (0,0,0,0) is not an invariant; under translations within the group, the origin is not mapped back to itself. Thus it has no physical significance. But the light cone structure –the complete catalog of the pairs of events that are lightlike separated[7]—is invariant and thus has physical significance.

3. The Development of Relativity Theory

The Default Interpretation of Spacetime Coordinate Systems…

Einstein's natural starting point is to assign physical significance to the natural features of a coordinate system. Using the familiar (t, x, y, z) as the spacetime coordinates, we can list some of them:

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[6] For a more detailed account of the connection to nineteenth century geometry, see Norton (1999).

[7] In coordinate terms, the pair satisfies the condition $\Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2 = 0$, where (t, x, y, z) are the usual spacetime coordinates, Δ represents the coordinate differentials and the speed of light is set to unity.

(a) The origin (0,0,0,0) corresponds to a central point; the distinction between the x, y and z coordinates makes space anisotropic.

(b) The curves picked out by constant values of x, y and z are a state of rest.

(c) In a Lorentz or Galilean covariant theory, the set of all curves picked out by (b) for all coordinate systems are the inertial states of motion.

(d) Coordinate differences have metrical significance; they represent the possible results of clock and rod measurements by observers in the state of rest picked out by (b).

…and Their Loss of Physical Significance

The development of relativity theory brings the systematic elimination of these default physical interpretations. As our starting point, we might imagine a one-coordinate system theory. It would have all the physical structures of the list above (a)-(e). The first step had already been taken in the nineteenth century. The spatial sections of the spacetime are covered by coordinates x, y, z. The Euclidean character of space entails that we can use many coordinate systems related by translations, rotations and reflections. None of the structures of (a) are invariants of these transformations. They lose physical significance.

The Relativity of Motion

The theory would retain an absolute state of rest (b), however. That is eliminated by the transition to a Newtonian spacetime, with the characteristic group the Galilean group, or to special relativity with the characteristic group the Poincaré group. The states of rest (b) are no longer invariant.

The next step marks the starting point of Einstein's 1907 quest for his general theory of relativity. [8] Einstein sought to expand the covariance of his theory further so as to deprive the inertial states of motion (c) of physical significance. This, he believed, was achieved with his postulation of the principle of equivalence which now allowed him to extend the Poincaré group with transformations that represented uniform acceleration, although in only limited circumstances. Einstein immediately interpreted the expansion as representing an extension of the principle of relativity to acceleration. In this account, we see why: the inertial motions of (c) are no longer invariants of the admissible transformations.

Metrical Significance

Presumably this much was all Einstein expected in 1907. In 1912, Einstein realized that the development of his theory required him to take another step in depriving coordinates of physical significance. He saw an analogy between the problem of gravitation and relativity and the theory of curved surfaces of Gauss. The latter has led to a new mathematics in which one could use arbitrary coordinate systems and in which coordinate differences cease to have the direct metrical significance of (d). [9]

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[8] There has been very considerable investigation in recent decades of Einstein's passage to the general theory of relativity. They span from early work including Torretti (1983), Norton (1984) and Stachel (1980) to Renn (in preparation).

[9] Here I will report Dr. Torretti's repeated lament that the group structure—or lack of it—of Einstein's expanded coordinate systems brought many unintended problems apparently ignored by Einstein. For example (Torretti, 1983, p. 153) observes that the ranges of two coordinate charts may not overlap, so that the point transformation by induced the corresponding coordinate transformation may have degenerate properties. Einstein largely maintained a physicist's silence on these mathematical niceties.

Independent Existence

With this development, Einstein's quest for depriving coordinate systems of their default physical significance has taken an unanticipated turn. It proved to be a trifle in comparison to the final hurdle that Einstein needed to overcome in arriving at a generally covariant formulation of his general theory of relativity. Having failed to find what he thought were admissible generally covariant gravitational field equations in 1912 and 1913, Einstein eventually found a way to discount the failure. He developed arguments that purported to show that general covariance would be physically uninteresting, were it to be achieved. The best known and most important of these was the "hole argument."

The error of Einstein's argument ation is now well known. He had generated two intertransformable metric fields $g_{ik}(x^m)$ and $g'_{ik}(x^m)$ in the same coordinate system, x^m. He had assumed that the two fields represented two distinct physical possibilities. That proved to be the elusive error that took several years to find. Einstein presumed that it made sense to say that the two fields were in the same coordinate system. That tacitly accorded an existence to the coordinate system independent of the metric field defined on it. Figuratively, it meant that it makes sense to say that we can remove the first field from the coordinate system, leave a bare coordinate system behind and then deposit the second fieldin the very same coordinate system.

One of the final stages of Einstein's development of a generally covariant theory was to recognize that coordinate systems have no such independent existence. He described his error to Besso in a letter of January 3, 1916:[10]

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[10] Schulmann et al. (1998) Papers, Vol. 8A, Doc. 178; Einstein's emphasis. I have argued elsewhere that Einstein's according independent reality to coordinate systems may have had catastrophic effects at an earlier stage of his quest for general covariance. See "What Was Einstein's Fatal Prejudice?" in Renn et al. (in preparation).

There is no physical content in two different solutions G(x) [$g_{ik}(x^m)$] and G'(x) [$g'_{ik}(x^m)$] existing with respect to the same coordinate system K. To imagine two solutions simultaneously in the same manifold has no meaning and the system K has no physical reality.

4. Conclusion

These considerations, however, have little force with modern readers. We now proceed from a quite different starting point. We do not accord default physical significance to coordinate systems. If we wish to endow a spacetime with inertial structures, absolute or otherwise, we start where Einstein ended. We start by endowing a manifold with an affine connection (displacement field) whose natural straights are the inertial motions. In all this, coordinate systems are little more than convenient labels for spacetime events.

For Einstein, however, matters looked quite different. His default was to load physical content into the coordinate systems. The conceptual development through special to general relativity is characterized by depriving coordinate systems of their default physical significance in progressively greater measure. He had initially intended to end up just depriving coordinate systems of absolute inertial motions. Once Einstein had started the process, it could not be stopped. The natural development of the theory ended up forcing much more. The coordinate systems lost their metrical significance and, after much suffering, he finally recognized the need to dispense with a notion of independent existence he had tacitly accorded them.

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