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Semigroups and Symmetry
an Investigation of Prigogine's Theories

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Chapter 1

Introduction

Interviewers[1]: *We love your metaphors.*

Al Gore: Around 25 years ago, the Nobel Prize in chemistry was awarded to a Belgian named Ilya Prigogine [interviewers look confused]. This is not an arcane fact. He discovered what was in essence a new law of thermodynamics, and here's how it operates [drawing another diagram]. He studied what are called open systems, in which there is energy flowing in and energy flowing out, and the energy swirls in a recognizable pattern while inside the system. What he found was, that when the flow of energy is increased, and increased again, at some point it crosses a threshold beyond two things happen sequentially. No1, the pattern of the system breaks down.

Interviewers: *And becomes unpredictable?*

Al Gore: No, let me finish. The second consequence is that the energy spontaneously reorganizes

itself at a higher level of complexity, and something new emerges, with a different persistent pattern. In some ways this was the birth of complexity theory. Now, if you look at political systems around the world, you can see this phenomenon taking place.

This is not an attempt to explain why Gore lost the presidential elections, but the quotation serves well to illustrate the broad spin-off that Prigogine's theories have had from philosophy to physics, from New Age to politics. It has been claimed that

[1] www.redherring.com/mag/issue84/resources/mag-gore3-84.html

his theories will constitute a paradigm shift unheard of since the impact of Newton.[2] The acclaimed new era of Prigoginianism still arouses controversies between adepts and critics. In order to settle some of these disputes a critical assessment is necessary, which at the same time gives an overview of the work of Prigogine in philosophy and physics.

Prigogine has devoted his whole scientific work to a revalidation (or rediscovery) of time. How can physics treat time as a mere parameter, in contrast to our everyday experience of time in which the distinction between past, present and future is so important. Furthermore, we see all sorts of asymmetrical phenomena around us taking place. Physics (and reality) as a whole is just not time symmetric according to Prigogine, and what is needed therefore is a revolution in thinking about time.

One of the main themes in the foundations of statistical physics centers around the directionality of time captured beautifully in Eddington's phrase time's arrow. The modern version of this problem is to reconcile the observed macroscopic asymmetry of processes with the acclaimed fundamental microscopic timesymmetric laws. This reductionist problem has an old history in physics. The reduction of thermodynamics to statistical mechanics has frequently been used as an example of a successful theory reduction (Nagel 1961). However precisely with the directionality of time there are major problems to overcome.

The main problem is due to the status of the second law of thermodynamics which states that entropy should increase (for isolated systems) in time, which is clearly a time asymmetrical law. At the same statistical mechanics predicts that the Gibbs fine-grained entropy remains constant. How can we reconcile these conflicting predictions?

Since Boltzmann's work on deriving a mechanical interpretation of the second law in his celebrated H-theorem the interpretation of the second law has been the center of controversies. Different strategies have been developed since to address this problem. First of all we can interpret the second law as having only a statistical validity as expressed by Maxwell's demon; or we can claim that the validity of the second law is dependent on a special sort of initial conditions which apply in our universe as alluded sometimes by Boltzmann; or we can claim that the second law gives a definition of the direction of time as maintained by Reichenbach; or we can claim an objective fundamental status for the second law as pursued by Prigogine. Note that there are logically of course at least three different possibilities: we can either reject the second law as being unimportant to the direction of

[2] Alvin Toffler speaks in *Order out of Chaos* ([62, p.xi-xxxi]) about 'the leap from Newtonianism to Prigoginianism' and the 'Prigoginian paradigm'.

time, after all what is precisely its meaning and validity? We can simply advocate a form of anti-reductionism, why should macroscopic processes be reducible to microscopic ones in the first place? Or we can revise statistical mechanics such as to allow an increasing entropy in time. Two well-known possibilities of the latter approach are coarse-graining and interventionism. Both of these approaches are rejected by Prigogine as these approaches constitute examples of extrinsic irreversibility, due to respectively epistemological constraints or outside disturbances. In contrast Prigogine's approach is in line with Boltzmann's original attempt to give a mechanical derivation of the second law, in trying to derive what he calls intrinsic irreversibility i.e. irreversibility due to the dynamics of a system alone. Kaon decay is an example of such an intrinsically irreversible system.

Beside the physical problem of considering the asymmetry of processes in time, time is of course also an interesting philosophical topic. In the philosophy of time there are different debates. One of the oldest has to do with topological questions as 'does time have a beginning?' (Kant, Augustinus). A second debate centers around the relation between time asymmetry, causality and determinism (Earman, Price). A third debate is concerned with the experience of time. Every person has a fundamental experience of time, namely that it has a definite direction towards what we conventionally call the future. That is a definite time asymmetry as well. Where does the experienced asymmetry come from?

A first possible approach consist in an analysis of so-called tensed facts (indexical propositions) within the philosophy of language. This debate is due to McTaggerts's distinction between time and tense. It is basically concerned with the question whether propositions as 'it is now 4 o'clock' are true (Prior, Mellor). Realism regarding tensed facts is called "becoming". A more elaborate approach towards time has been developed by philosophers as Heidegger, Bergson and Whitehead.

What makes Prigogine starting-point so fascinating is that it is precisely a combination of a physical and a philosophical approach, since he claims that his interpretation of the second law gives a justification for our intuitions about time. Once we acknowledge the fundamental status of what Prigogine calls irreversible processes we can unify our understanding of time overcoming the barriers between a mere chronological interpretation of time (in physics) and our subjective psychological experience of time.

On second thought, Prigogine's work does does not consist of one single well-described theory. In fact, several theories developed in different mathematical formalisms ought to be distinguished that have often been conflated in the literature. The basic aim of my thesis is to give a critical assessment of Prigogine's theories.

Up till now it remains unclear what Prigogine has precisely accomplished so far. A chronological review of his theories is also missing. As this would require at least a book length treatment, I have limited my research and have chosen to focus in this paper on the relationship between semigroups and symmetry in Prigogine's theories. Let me try to explain this briefly.

In statistical physics there have been developed numerous approaches in deriving irreversibility. However all these approaches have the same deficit according to Prigogine, namely that they overtly neglect or suppress information about the system. Prigogine claims that his theories about irreversibility can overcome this problem by introducing a transformation between the ordinary deterministic representation, where time evolution is described by a unitary group, towards a new representation, which is described by a semigroup evolution. The claimed properties of the operator which effects the transformation are that it is symmetrybreaking without a loss of information and thus a form of intrinsic irreversibility.

A semigroup is loosely speaking a group without an inverse element. If we consider therefore processes in time, having the semigroup property expresses, that these processes all have the same time orientation. An example of this is a Markov chain, which is an example of a stochastic process without a memory (or to introduce the difficulty at once, a process without a foresight). Once we break symmetry, we select the future oriented semigroup and hence derive intrinsic irreversibility. Whether Prigogine has succeeded in developing a viable alternative for coarse-graining or interventionism in a new theory about intrinsic irreversibility will be the main question of this paper.

There are several reasons for choosing semigroups and symmetry as my main object of research. My main reason however is that the relation between semigroups and symmetry constitutes the core of Prigogine's work and is an easy way to understand both Prigogine's philosophical and scientific claims. Moreover this angle of reflection on the kaleidoscopic Prigoginian work stands at the cross-roads of both philosophical and physical arguments that I will hope to disentangle. Especially Prigogine's philosophical starting point, which lies in the ideas developed by Heidegger and Bergson has my interest.

A myriad of other points of interest are raised by Prigogine, such as the meaning and use of probability, the impact his theories have on determinism, the ontology he advocates (he argues that the concept of trajectory has to be abandoned in physics). But others have already followed these paths; Batterman has investigated the probability concept, Bricmont has criticized Prigogine's use of the concept of determinism, and Bishop has scrutinized the notion of trajectory in Prigogine's thinking, to mention only a few. Although the work of Prigogine extends over 60 years of time, I have restricted myself especially to the 1970-85 period as in this 'creative period' lots of new ideas were generated that have attracted a lot of (critical) attention. Later developments are often grounded in these early ideas.

1.1 Prigogine: a short scientific biography

With 42 doctorates honoris causa and hundreds of publications, the person Prigogine himself attracts attention. Not only the wide spectrum of his work as a scientist but also his work for UNESCO and the major role he has played in the international lecture and journalistic circuit, make him a fascinating character. Prigogine was born in Moscow a few moths before the revolution in 1917.[3] The family left Russia in 1921 because of the difficult relationship it had with the new regime. Until 1929 they lived as immigrants in Germany, until they left for Belgium. Prigogine acquired Belgian nationality in 1949.

After completing his fourth year in chemistry at the Free University of Brussels, Prigogine decided to study thermodynamics there, focusing on the special significance of time. He received his Ph.D. in 1941. Prigogine started working in Brussels with a great number of different physicists which led to the birth of the so-called Brussels School in the late forties. The Brussels School is not a clearly defined movement, traceable to a certain date. The name is commonly used to refer to all the work that has been done by, or centering around Prigogine.

An important coworker in the early period was Théophile De Donder, whose merit it was according to Prigogine that he stressed the importance of entropy production while relating it to the pace of a chemical reaction expressed by a new function called 'affinity'. Hendrik Kramers was also an enthusiastic supporter. The study of irreversible phenomena, using the results of Onsager, led to the so called theorem of minimum entropy production in 1946, applicable to non-equilibrium states. Prigogine applied the theorem to problems in theoretical biology, relating the stability of steady states to the stability of biological populations. The linear thermodynamics of irreversible processes had already led to numerous applications, as shown by authors such as Meixner, de Groot and Mazur.

In the years from 1946 till 1967 Prigogine worked on the question what happens far from equilibrium in the non-linear range. In this period Glansdorff played a major

[3] For more information on the life of Prigogine and the birth of the Brussel's school see the web site of the Nobel Foundation (www.nobel.se/chemistry/laureates/1977/prigogine-autobio.html.)

role. This work led in 1967 to the notion of dissipative structure and was awarded the Nobel Prize for chemistry in 1977. In 1967 Prigogine was also appointed director of the Texas Institute of Technology in Austin. Since then, his work is sometimes referred to as that of the Brussels-Austin group (BAG).

The press release of the Nobel foundation writes “the great contribution of Prigogine to thermodynamic theory is his successful extension of it to systems which are far from thermodynamic equilibrium. This is extremely interesting as large differences compared to conditions close to equilibrium had to be expected. Prigogine has demonstrated that a new form of ordered structures can exist under such conditions, and he has given them the name ‘dissipative structures’ to stress that they can only exist in conjunction with their environment (..) these structures have led to many fundamental discoveries and applications in diverse fields of human endeavor, not only in chemistry”.

In his Nobel Prize inauguration speech in 1977 Prigogine mentions two important steps in his career that have lead to his present concerns. First of all there is the birth of non-equilibrium statistical physics at the end of the forties, after the pioneering work of Yvon, Kirkwood, Born and Green and Bogolyobov. The failure to apply the Born and Green formalism in order to extend Boltzmann’s method to dense systems led Prigogine to a first question: is it possible to develop an ‘exact’ dynamical theory of irreversible phenomena, without supplementary approximations, in which irreversible phenomena play an active constructive role?

In 1955 Léon van Hove worked with the Brussels group on the deduction of the ‘master equation’ for anharmonic systems, and on phase transitions, which has lead to the branch of statistical mechanics that deals with so-called ‘exact’ results. The impetus of Van Hove lead Prigogine in collaboration with Balescu, Brout, Hénin and Résibois to a formalism of non-equilibrium statistical mechanics from a purely dynamical point of view, without any probabilistic assumptions. This formalism using a ‘dynamics of correlations’ is summed up in the 1962 book *Non-equilibrium Statistical Mechanics* ([57]).

The second step that is important in this period is the relation between thermodynamics and dynamics. It questioned the very nature of dynamical systems and the limits of Hamiltonian description. The eventual aim was to develop a new interpretation of irreversibility. If irreversibility does not result from supplementary approximations, it can only be formulated in a theory of transformations which expresses in explicit terms what the usual formulation of dynamics does hide. In this perspective the kinetic equation of Boltzmann corresponds to a formulation of dynamics in a new representation. In conclusion, dynamics and thermodynamics become two complementary descriptions of nature, bound by a theory of star-unitary transformations. This approach (called causal dynamics or sometimes subdynamics) was according to Prigogine more than ever the result of a joint effort: Rosenfeld, Wentzel, de Haan, George, Grecos, Henin, Mayné and Theodosopulu were the main collaborators.

These contributions are judged, as far as I can see, to be valuable contributions in the study of non-equilibrium thermodynamics (or statistical mechanics NESM as Prigogine calls it) and as such can be found in some basic books on the study of thermodynamics and statistical physics. However, with the publication of *From Being to Becoming* ([61]) in 1980, the reception of Prigogine's ideas seems to change. *From Being to Becoming* not only serves as a popularization (and a summary) of the ideas that had led to the Nobelprize, but also encompasses a good part of the agenda of Prigogine's future research.

In the late seventies results of Misra and Courbage changed the perspective of the BS. Working in an extended Koopman formalism a 'new complementarity' was derived between probabilistic and deterministic descriptions. Especially in the period between 1977 and 1983 an enormous amount of articles has been published, breathing a sometimes revolutionary spirit. In a sometimes very associative (or as I will demonstrate superficial) manner links are made in these works with e.g. Gödel theorems, the philosophy of Bergson, the measurement problem of quantum mechanics (QM) etc.

This revolutionary spirit was enhanced by the publication of a 'popular science' book in 1984 called *Order out of Chaos* ([62]) subtitled *Men's new dialogue with nature* co-authored by Isabelle Stengers, where philosophically speaking the sky is the limit. Not only the claim that physics has forgotten about time, but also subjects as Snow's clash of two cultures, determinism and free will are discussed, and it is suggested that Prigoginianism is the answer to all these questions. Another popularization followed in 1988 *Between Time and Eternity* and in 1997 *The End of Certainty*. Criticism and scorn of the scientific community on the one hand, fame by the general public on the other hand, were the result of these books. The Sokal-Bricmont affair, in which also Prigogine figures, may illustrate the scepticism of the scientific community.[4]

In the years after 1986 again a change in perspective seems to occur based this time on the rigged Hilbert space formalism developed by Bohm and Gadella. Important co-workers now become Petrosky, Antoniou and Tasaki.

Unnecessary to say, the work of the Brussels School has attracted a lot of attention. From the (late) eighties till the mid nineties various authors investigate the

[4] www.physics.nyu.edu/faculty/sokal

claims of the Brussels School. Some of them criticize the physical content of the theories such as Obcemea and Brändas (1983 [47]), Batterman (1991 [11]). Others direct their criticism on the philosophical claims of Prigogine, such as Borzeskowski and Wahsner (1984 [20]), Sklar (1986 [75]) and Price (1997 [55]). Some authors criticize both physical and philosophical arguments such as Verstraeten (1991 [81]), Karakostas (1996 [37]) and Bishop (2001 [13]).

1.2 Outline

I have chosen the following methodology and outline of this paper. In the second chapter *Time's Arrow* I will analyze Eddington's phrase in more detail in order to pinpoint Prigogine's specific approach to the problem of time's arrow. In order to do so, I will review two main contributors to the foundations of statistical mechanics, Boltzmann and Eddington, and put Prigogine's thinking in perspective. It will be shown that the modern puzzle of time's arrow presupposes a reductionist assumption. Using a framework of Uffink, Prigogine's use of the term irreversibility will be assessed. Next I will sketch the scientific and the philosophical perspective on Prigogine's quest for revalidating time. The physical perspective consists in arguing for a new interpretation of irreversibility: intrinsic irreversibility. The primal aim of the philosophical perspective is to criticize the *Leugnung der Zeit* inherent in contemporary science and is inspired by continental philosophers as Bergson and Heidegger. I will argue that there is a tension between these two perspectives. This tension will be resolved by treating the philosophical and physical theories in two separate chapters.

The third chapter *The Kac Ring Model* serves mainly an educational purpose by discussing the so called Kac ring model. It will serve to define and clarify the concept of semigroups in relation to the reversibility paradox. The second purpose is to introduce the heuristic which I think lies at the heart of Prigogine's theories about intrinsic irreversibility. The heuristic consists of two steps: (1) a symmetry breaking step leading from a unitary evolution to two possible semigroup evolutions (2) a selection step which selects one of the semigroups. The SBSS heuristic makes it possible to distinguish between criticism on Prigogine in two classes. Criticism on the validity of SBSS itself as a possible theory of irreversibility (chapter five); criticism on Prigogine's use of it (chapter four).

In the fourth chapter *Semigroups and Irreversibility* I will investigate Prigogine's physical theories in more detail, bearing the philosphical framework of the first chapters in mind. My focus in this chapter will be on the relation between irreversibility, symmetry and semigroups. In order to be able to sketch the development of Prigogine's theories concerning this subject matter, I will work in this chapter in a chronological order, though concentrating especially on the period between 1969-1985.

I will argue that there is not a single theory of irreversibility but that several different approaches have to be distinguished properly, that up till now have not been disentangled precisely in the literature. In a chronological order the approaches are: causal dynamics; new complementarity; entropy as a selector of initial conditions; an alternative to QM; rigged Hilbert spaces; embedding. The first three of these approaches are accordingly treated in a separate section divided in (i) the mathematical formalism (ii) the interpretation of the formalism (iii) the analysis within the framework SBSS. I will argue first of all, that the development of Prigigine's thinking can indeed by assessed within the SBSS framework. Secondly I will show that some critics of Prigogine have conflated these different approaches. Finally I will argue that Prigogine so far has not been able to provide a satisfying answer to his core motivation of grounding intrinsic irreversibility.

In chapter five *Puzzling Prigogine* I will discuss the validity of the mechanism SBSS itself by reviewing criticisms of Sklar, Karakostas, Price and Bricmont in view of the problem of time's arrow. I will argue that many controversies are due to a disagreement about what is actually puzzling about time's arrow. Prigogine's proposal of crediting irreversible processes with a fundamental status is connected to the philosophical question that was raised in chapter two, namely, whether Prigogine adheres to an anti-reductionist position. I will distinguish between two possible routes. The monistic route consists in perceiving reality as asymmetrical and therefore criticizes the limited theoretical practice of physics. It is not directly related to reductionism. The second route has a dualistic ontology consisting either of reversible and irreversible processes or of unstable and stable systems. The second ontology is more in agreement with the aim of deriving intrinsic irreversibility for unstable systems. In what ways the dualistic route implies anti-reductionism will be discussed. Conflicting statements of Prigogine make it impossible to argue in favor of any of these routes.

A final note on methodology. Problematic for a research on Prigogine's theories are the following. The first problem is related to the fact that Prigogine's theories are often published in threefold; (i) a highly mathematical result, published in a mathematical article (ii) a physical elaboration on the mathematical result published in a physical paper (iii) a few years later, the popularization of the intuitions behind the idea. However, an extensive treatment in which the three aspects are intertwined is missing. Secondly, it is not always clear whether a specific idea or quotation can be attributed to Prigogine, or to one of his coauthors. If I am not sure whether a specific thought or theory can be attributed to Prigogine himself, I will refer rather with BS (Brussels School) or when discussing the later years with BAG (Brussels-Austin Group).

Chapter 2

Time's Arrow

> In any attempt to bridge the domains of experience belonging to the spiritual and physical sides of our nature time occupies the key position
>
> –Eddington 1968

Thinking about time is one of the most time-consuming activities known to me. The question what time is, has fascinated human beings since the dawn of modern thought. Aristotle associated time with motion, but added: 'there must also be a soul which counts'. Kant's antinomies about time, which led to the conclusion that time is unreal, are well-known today. As Augustine remarked, we all know what is meant when we use the word time, but if we are asked to define it we are perplexed. Luckily since ancient times some progress seems to have been made. Zeno's paradox can be resolved using infinitesimal calculus and with the advance of relativity theory the idea that space and time should be conceived as separate and absolute has been challenged. This has led to today's most prominent view on time, that it is one of the four dimensions of an unchanging so-called 'block-universe' (James). The modern model-theoretical way of perceiving theories does not demand a single definition of time (cf. in geometry a point is a primitive notion as well), but the unease our perplexity still arouses is not easy to overcome.

An important distinction was made in 1908 by McTaggert between two ways of saying when things happen. One way is to say whether they are past, present or future. Events ordered this way by their temporal relations to the present are called the A-series. The other way of ordering, called the B-series, is by how much later etc. they are, not than the present but with respect to each other. McTaggert's contribution consisted of showing, how many important questions about time are really questions about these two series of respectively tense and time.

How can we distinguish between time and the other dimensions of spacetime? McTaggert's answer would be that time is the dimension of change. If we now define change as a variation of time in properties of a certain thing we are faced with a circularity. McTaggert's solution to this problem was that what makes temporal variation change is that it entails changing facts about how things are now. We can visualize this as the flowing of the A series along the B-series. Hence the 'flow of time' accounts for the difference between the time- and space dimensions of the manifold.

The question now becomes pressing whether beliefs as, for example, 'it is now 8 o'clock', represent reality. In other words, is there something in reality which makes my A-series beliefs true (and if not, howcome it is one of the things I am most certain of). We all have certain beliefs that 'time goes on', that we are all growing older. Formulated in more contemporary notions, is the above proposition, a fact or is it merely projected into reality? The viewpoint that these convictions are indeed facts in reality (and hence that grammatical tenses have counterparts in reality (Uffink) is sometimes referred to with the concept of 'becoming'. This standpoint adheres to realism with respect to tensed facts. I will call the anti-realistic view which denies these convictions the status of facts, but as being mere projections psychological time.

The notions 'flow of time' and 'becoming' are captured under Eddington's phrase: 'the arrow of time'. It is my main aim in this chapter to analyze this phrase in more detail in order to pinpoint Prigogine's specific approach to time's arrow.

The outline of this chapter is as follows. First I will review two main contributors to the foundations of statistical mechanics, Boltzmann and Eddington, and put Prigogine's thinking in perspective. Secondly it will be shown that the modern puzzle of time's arrow presupposes a reductionist assumption. Using a framework of Uffink, Prigogine's semantics of the term irreversibility will be assessed. Next I will sketch the scientific and the philosophical perspective on Prigogine's quest for revalidating time. The physical perspective consists in arguing for a new interpretation of irreversibility: intrinsic irreversibility. The primal aim of the philosophical perspective is to criticize the *Leugnung der Zeit* inherent in contemporary science and is inspired by continental philosophers as Bergson and Heidegger. Finally, I will argue that there is a tension between these two perspectives. This tension will be resolved by treating the philosophical and physical theories in two separate chapters.

2.1 Boltzmann

In order to understand the motivations that have inspired Prigogine, a little background has to be given about the historical debates centered around Boltzmann in deriving a dynamical foundation of the second law of thermodynamics, in the form of his H-theorem. In reaction to Maxwell's derivation in 1867 *On the Dynamical Theory of Gases* of the distribution of velocities of a gas in thermal equilibrium, Boltzmann's approach was to define a quantity H with respect to which he argues (i) that H takes it minimum possible value when the gas has the Maxwell distribution, and (ii) that when the gas does not already have the Maxwell distribution, the effect of collisions is to ensure that H always decreases. Boltzmann accordingly identifies -H with the entropy as introduced by Clausius, so as to interpret the H-theorem as an expression of the increase of entropy over time, whatever the initial distribution. In other words the H-theorem is his long-sought mechanical analogue of the second law.

Yet the theorem was paradoxical because it described irreversible processes, but based them on reversible mechanisms, and in 1876 Loschmidt launched the **Umkehreinwand** to criticize it. In the argument, Loschmidt imagined the reversal of the velocities of all individual particles in a system with decreasing H and concluded that in the reversed system H would increase. Where does Boltzmann's derivation go wrong? In his 1877 answer to Loschmidt, Boltzmann answers that there are initial conditions under which H increases as well as initial conditions under which H decreases, but that the number of the latter is overwhelming. Accordingly, in order to derive the second law statistical considerations are needed over and above mechanics. Huw Price in *Time's Arrow and Archimedes' Point* mentions Maxwell's Demon, the imaginary creature that segregates fast and slow molecules of a gas, making it hot in one region and cold in another, thereby illustrating something which might happen by accident. This lead to the conclusion that "if the second law is to be grounded on a statistical treatment of the behaviour of the microscopic constituents of matter, it cannot be an exceptionless principle"(Price [55, p.28]).

It was Burbury in 1894 who identifies the so-called Stosszahlansatz, also called by Boltzmann the 'hypothesis of molecular chaos' as the ground of the reversibility problem. Indeed, how did Boltzmann manage to derive an asymmetrical conclusion of the increase in time of H from the underlying time-symmetric mechanics in the first place? In fact it is Maxwell's assumption that the frequencies of velocities of colliding particles are independent. This is an asymmetric assumption, because we expect the velocities of particles to become correlated as a result of their collision, and at the same time do not expect the incoming components of a collision to be correlated if they have never encountered into one another in the past. Furthermore, the assumption is not only an initial condition but is valid during the whole time period under consideration. As Price notes, the criticism did not initially focus on the Stosszahlansatz (but against the exceptionless character of the second law), and by the time it did, Boltzmann's statistical ideas had already taken a different track.

These statistical ideas are developed in *Further Studies on Thermal Equilibrium between Gas Molecules* in 1877. The generalized distribution function no longer describes the microscopic state of the gas but characterizes the macroscopic state of the gas: it has a probability itself instead of denoting probabilities of the velocities of the particles. Boltzmann distinguishes clearly between the microstate of a gas described by a point $x = (q_1, q_2, ..., q_N, p_1, p_2, ..., p_N)$, in $\Gamma = R^{6N}$ called the phasespace, and the macrostate of the gas characterized by a distribution $f(r, v)$ over the single particle space μ with coordinates (r, v) $(\mu = R^6)$. This μ space is subdivided in a large number of tiny cells. Every macroscopic state in μ can therefore be realized by different microscopic configurations in Γ. This opens the possibility for the interpretation of the entropy of a macroscopic state as a measure of its probability in terms of these possible microscopic realizations. If all possible microstates are assumed to be equally likely, the gas will spend a lot of time in macrostates that can be realized in very many ways, and little time in macrostates that can be realized in very few ways.

Why does entropy increase in this view? Simply because from a given starting point there are very many more microstates to choose from that correspond to higher (or equal) entropy states, than microstates that correspond to lower entropy macrostates. Herewith equilibrium is defined as the macrostate with a maximum of possible microscopic realizations. As Boltzmann already noted, there is no asymmetry in this statistical derivation. The two unwelcome consequences, according to Price, of this statistical approach are (1) to the extent that it explains why entropy normally increases toward (what we call) the future, it also predicts that entropy should increase toward (what we call) the past (2) it makes it rather mysterious why entropy should be so low now.[1]

In 1894 Zermelo pointed to another problem in the H-theorem called the **Wieder-**

[1]Price gives Boltzmann an overwhelmingly positive position. In Uffink [79] a more critical review of Boltzmann ideas can be found which point to technical dilemmas, such as (1) that in Boltzmann's statistical derivation, there is no interconnection between the particles, and (2) the procedure of subdividing the phasespace into cells is arbitrary and leads to different results. But also interpretational dilemmas such as (3) different conflations of the term 'probability' by Boltzmann and more profound the fact (4) that for Boltzmann entropy functions as an objective property of the microstate.

kehreinwand. Using Poincaré's recurrence axiom Zermelo shows that a physical system is bound eventually to come arbitrarily close to any possible state, even one of very low entropy. In 1896 Zermelo argued that this result is incompatible with the attempt to account for the asymmetry of thermodynamics in mechanical terms.[2] Boltzmann's response is basically, that every system strives to the most probable state, equilibrium, and that the time to return to the initial state is incredibly long. Fluctuations are possible but very improbable. These discussions still continue today.[3] Discussing the Kac ring model in chapter two I will return briefly to these paradoxes. Now I will address the question how Prigogine stands in this debate.[4]

To honor Boltzmann 100 years after the derivation of the H-theorem a special edition was issued in the Acta Physica ([59] 1973). Prigogine's contribution entitled *The statistical interpretation of non-equilibrium entropy* is very instructive for the way Prigogine sees his work in line and in contrast to Boltzmann's. Prigogine summarizes these differences in the following scheme:

$$\begin{aligned}\text{Boltzmann: Dynamics} &\rightarrow \text{Stochastic theory} \rightarrow \text{Entropy}\\ \text{Prigogine: Dissipative systems} &\rightarrow \text{Lyapounov function} \rightarrow \text{Entropy}\end{aligned} \tag{2.1}$$

Prigogine states that it is one of the most interesting aspects of his own approach that it eliminates any appeal to stochastic processes. He hopes that his theories bring us nearer to the original (pre-1877) goal formulated by Boltzmann, to give the 'mechanical derivation' of the second law. Prigogine adheres to Boltzmann's answer to Zermelo's paradox (the recurrence time increases beyond imagination with the

[2] According to Mackey "Zermelo was right in his assertion that the entropy of a system whose dynamics are governed by Hamiltons equations, or any set of ordinary differential equations for that matter, cannot change. He was wrong, however, to base his arguments on the results of the Poincaré theorem. The fallacy in the argument is to be found in his implicit assumption that densities (on which the behaviour of entropy depends) will behave like points and also be recurrent.(..) just because points are recurrent densities need not be."(Mackey [42, p.45-6])

[3] As Uffink demonstrates the debate between Zermelo and Boltzmann rests partly on Babylonical misunderstandings about terminology, and therefore different authors give Boltzmann different appraisal. Uffink argues that Boltzmann's position is not transparent after the debate, Price argues instead that Boltzmann already foresaw Zermelo's criticism by pointing to his statistical formulation.

[4] For the sake of completeness, I mention that in 1911 Paul and Tatyana Ehrenfest made an attempt to clarify Boltzmann's work. Uffink mentions two points of contribution. First in distinguishing between the evolution of the most probable state and the most probable evolution, the H-theorem can be saved as we see it as a proposition about a concentration curve i.e. the evolution of the most probable state. This is called 'the statistical H-theorem'. Secondly, using conditional probabilities, they show that even for a time symmetrical curve, a decrease can be, given that H is high at a certain moment (conditionality), more probable than an increase.

number of degrees of freedom of the system). However, the answer to Loschmidt's paradox Prigogine argues, is not so straightforward. Spin echo experiments[5] have shown that over limited periods anti-Boltzmannian behaviour (corresponding to an increase of H_B) is observed. Prigogine grants Kac to have solved the logical problem involved, but Kac did not show how the reconciliation of time reversibility and irreversible behaviour is possible in classical or quantum mechanics. Prigogine's sees his own theories as an attempt to answer Loschmidt's paradox in obtaining a mechanical derivation of the second law.

2.2 Eddington

In his 1927 Gifford lectures [29] Eddington gives his views about the second law of thermodynamics in two famous chapters titled *The running down of the universe* and a chapter titled *Becoming*.

> "I shall use the phrase 'time's arrow' to express this one-way property of time which has no analogue in space. It is a singularly interesting property from a philosophical standpoint. We must note that (i) It is vividly recognised by consciousness (ii) It is equally insisted on by our reasoning faculty, which tells us that a reversal of the arrow would render the external world nonsensical (iii) It makes no appearance in physical science except in the study of organisation of a number of individuals. Here the arrow indicates the direction of progressive increase of the random element."([29, p.69])

What does Eddington mean with this increase of the random element? He gives the example of shuffling of a deck of cards to illustrate this meaning and says "when-

[5]Systems of nuclear spins in crystals can be constructed which display a fascinating combination of two properties (1) they show a kind of evolution toward equilibrium (2) they can be demonstrated not to have dissipated their original information about their non-equilibrium situation into the outside world by means of interaction with that world. The nuclei are prepared with spins parallel. As time goes on, the system of nuclei evolve to systems with the spin axes 'randomly' distributed. But a clever trick which serves as a 'Loschmidt demon' and reverses the precessional motion of the spins (sort of) leads them to evolve back in time to their original state of parallel spin axes. This can be done even if the nuclei interact with another by means of spin-spin coupling. The reproducibility of the initial state of the system indicates plainly that the original information has not been lost by dissipation in the outer world. Yet the initial evolution of the system of spins shows an at least 'apparent' continual rerandomization which makes the evolution suitable for description the usual statistical mechanical way"([75, p.203-4] Sklar with reference to Hahn).

ever anything happens that cannot be undone, it is always reducible to the introduction of a random element analogous to that introduced by shuffling"(p.64). Eddington makes a distinction between primary laws, which forbid certain things happening because they are impossible and secondary laws which forbid certain things happening because they are improbable. In a famous quotation Eddington writes that "The law that entropy always increases (..) holds, I think, the supreme position among the laws of Nature (..) If your theory is to be found to be against the second law of thermodynamics I can give you no hope; there is nothing for it, than to collapse in deepest humiliation"(p.74). The second law is therefore according to Eddington the fundamental law in Nature and it states the increase of the random element. What is the relation between the second law and time's arrow?

In a region of equilibrium "we loose time's arrow (..) when the random element has reached its limit and become steady, the arrow does not know which way to point. It would not be true to say that such a region is timeless (..) Time is still there and retains its ordinary properties, but it has lost its arrow: like space it extends but it does not 'go on"'(p.79) i.e. "there can be no statistical criterion for a direction of time when there is thermodynamic equilibrium, i.e. when entropy is steady and ceases to indicate time's arrow"(p.80 in a footnote). Hence it seems, that for Eddington, the arrow is an expression for a property of time in the world, namely its direction. This direction is the approach to equilibrium demanded by the second law expressed as the 'running down of the universe'.

But how then, does time's arrow relate to our ordinary sense of time, in which we distinguish between yesterday, now and tomorrow (the A series). "Our present problem is to find the linkage between entropy which provides time's arrow in the symbolic world and the experience of growing or becoming which is the interpretation of time's arrow in the familiar world"(p.88). To answer this question Eddington returns to the problem of the shuffling of the deck of cards.

> "The crux of the matter is that, although a change described as sorting is the exact opposite to a change described as shuffling we cannot imagine a cause of sorting to be the exact opposite of a cause of shuffling. Thus a reversal of the time-direction which turns shuffling into sorting does not make the appropriate transformation of their causes (..) Whereas the cause of shuffling can be inorganic, the cause of sorting is 'the prerogative of mind' (..) We cannot believe that it is merely an orientation with respect to the time-direction which differentiates us from inorganic nature (..) to restore coherency we must postulate that by this change of direction something else has been reversed (..) 'becoming' has been

turned into 'unbecoming"'(p.93-4).

Eddington thus makes a distinction between 'becoming unshuffled' and 'unbecoming shuffled' and identifies sorting with the former, and a reversal of time with the latter. He concludes that evidently, "'becoming' gives a texture to the world which it is illegitimate to reverse". Why is reversing illegitimate?

Eddington first remarks an interesting feature about the relation between becoming in the familiar world and time's arrow in the symbolic world, namely, that the subjective and the objective have been reversed. Becoming, firmly grounded in our consciousness, is an objective fact while entropy is a more or less subjective notion to be compared with beauty. In a Kantian fashion, Eddington tries to account for this objective status of becoming. "It is so wielded into our conscience that a moving on of time is a condition of consciousness. We have direct insight into "becoming" (..) If I grasp the notion of existence because I myself exist, I grasp the notion of becoming because I myself become. It is the innermost Ego of all which *is* and *becomes*"(p.97). Eddington speaks about a divorce in physics between time and time's arrow. Physical time by no means corresponds to anything indicated by the time of consciousness, as in physics time is arrowless. It is consciousness that declares and detects time's arrow(p.100).

But there is not only a philosophical reason for this objectivity, Eddington hints at a physical reason as well. "Our consciousness somehow manages to keep in close touch with the material world, and we must suppose that its record of the flight of time is the reading of some kind of clock in the material of the brain (..) a better analogy would be an entropy clock (..) it seems to me, therefore, that consciousness with its insistence on time's arrow and its rather erratic ideas of time measurement may be guided by entropy-clocks in some portion of the brain"(p.101).

It is clear that Eddington does not give a unique meaning to the phrase time's arrow. First it seemed a property of the universe as a whole (the symbolic world) chained to the fundamental character of the second law. At other places the arrow is a property not of the symbolic world (which is called a world of shadows) but of our consciousness (the familiar world) and related to becoming and the flow of time. It is therefore not easy to make Eddington's use of time's arrow very precise.[6] It refers to a whole complex of concepts ranging from an inherent characteristic of time, the objectivity of the second law, to ideas as becoming and the flow of time, and finally the meaning of time.

[6]In that sense Prigogine and Eddington are alike.

For a good understanding of Prigogine it is, I think important not to underestimate the influence Eddington's original ideas may have had on Prigogine. First of all, there is a similarity of basic ideas: (i) the claimed fundamental status for the second law (ii) the emphasis on the objectivity of becoming. But in other remarks of Prigogine, Eddington resounds just as well. Prigogine alludes to Eddington's entropic clock when he writes: "a watch has no time in our sense. Everyday it goes back into its own past. It is we who have time, and this expresses the fact that, like all chemical systems, we belong to the category of highly unstable dynamical systems for which an object such as T can be defined"(Prigogine [35, p.244]). Secondly there is a remarkable similarity in terminology. As we will see subsequently, concepts as creation, organization, dynamical are employed by Prigogine as well. Most important question for our purpose is of course what the meaning is of Prigogine's use of Eddington's phrase time's arrow. This question will be pursued in the next section.

2.3 What actually is the Problem of Time's Arrow?

We have seen how Eddington's use of the 'arrow of time' was difficult to render precise as it denoted a whole complex of concepts. It is interesting to see how Eddington's phrase has been perceived since.

It is illustrative to consider briefly a discussion that was held in Nature (1956-7) centered around a contribution of Karl Popper [87]. Popper's main point was that the widely believed idea that all reversible mechanical processes involve an increase of entropy, and that 'classical' (that is, non-statistical) mechanics can describe physical processes only in so far as they are reversible in time, is a myth. The counterexample Popper gave is known today as Popper's pond. Suppose a film is taken of a large surface of water initially at rest into which a stone is dropped. The reversed film will show contracting circular waves of increasing amplitude. Immediately behind the highest wave crest, a circular region of undisturbed water will close in towards the centre. This cannot, argues Popper, be regarded as a possible physical process since it would require a number of coherent generators. Popper concludes: "although the arrow of time is not implied by the fundamental equations, it nevertheless characterizes most solutions"(Popper 1956).

Grünbaum and Hill reacted to Popper's remark. They make a distinction between two sorts of systems: open and closed, in the following sense. "In open systems there always exists a class of allowed elementary processes the inverses of which are unacceptable on physical grounds by requiring a *deus ex machina* for their production". In this sense Grünbaum and Hill propose a generalization of the remark made by

Popper. They write "the non-entropic irreversibility of open systems not only contributes to time's arrow on the cosmic scale, but can also be seen to play a part in the anisotropy of the time in our ordinary environment".

Important for our purpose is that whereas Eddington spoke about the arrow as entropy increase, Grünbaum speaks about non-entropic irreversibility. Whereas Eddington spoke about the arrow as a property of time, Popper speaks about processes in time. Where for Eddington time's arrow was directly related to questions about consciousness and experience of time, no longer recourse is made to the experience of time. If we compare Popper and Grünbaum, with Eddington's original use, we may conclude that the phrase time's arrow has changed in meaning from the broad complex of concepts towards the more specific problem of accounting in physics for asymmetrical processes in time.

Two modern authors on the arrow of time describe the problem as follows. Savitt defines in *Time's Arrows Today* ([73] 1995), "an 'arrow' of time as a physical process or phenomenon that has (or, at least, seems to have) a definite direction in time whose time reverse does not (or at least, does not seem to) occur. Savitt quotes the seven 'arrows' as distinguished by Penrose in 1979.[7] Savitt rather uses the category label *problems with the direction of time* than the metaphor of the arrow.

Also Huw Price has mixed feelings about the phrase 'arrow of time', because he judges it as ambiguous. Price distinguishes in *Time's Arrow: an Archmidean Point* ([55] 1996) between the asymmetry *of* time and the asymmetry of things *in* time. He uses a spatial analogy to clarify this point.[8] There is then the taxonomy problem[9] that aims at characterizing the different temporal arrows -asymmetries of things in time- and explain how they relate to one another. The second problem called the genealogy problem by Price is to explain why there is *any* significant asymmetry of things in time, given that (almost all[10]) the fundamental laws of physics appear to be symmetric with respect to time.

The modern example (playing cards is perhaps a bit old-fashioned), traceable to Popper, frequently used to demonstrate, in Price's words, the genealogy problem of time's arrow, is to record on videotape for example the event of dropping a glass

[7](i) The decay of the neutral K meson (ii) The process of Quantum measurement (iii) Second law of thermodynamics (iv) Radiation (v) Direction of psychological time (vi) The expansion of the universe (vii) White holes.

[8]Price considers an elongated table covered with cutlery. He makes the distinction between the asymmetry of the elongated table from end to end, and the asymmetry of the contents on the table along the same axis.

[9]Which I will not try to answer.

[10]The decay of the neutral K meson is an exception.

of milk on the kitchen floor. Playing the tape backwards displays a process which is hilarious and not frequently observed in real life. However playing a tape of a scattering incident backwards is not hilarious at all. Actually it is probably impossible to tell in which direction the tape has to be properly played. How to reconcile the macroscopic asymmetry of observed daily life processes, with the symmetrical microscopic fundamental laws (such as the laws of Newton) is the modern puzzle of time's arrow. The conceptual step involved in the above sketched development is that we no longer discuss metaphysical questions concerning becoming and (the meaning of) time itself, but discuss instead the physical problem regarding the (a)symmetry of processes in time.

For the problem of the direction of time to be a problem we need to presuppose a certain kind of reductionism with respect to laws of the macroscopic domain to the microscopic domain. At the heart of the motivation for this reductionist claim, lies the idea that microscopic laws are more fundamental. I will define this reductionist assumption accordingly as

- **RA** Reductionist Assumption: macroscopic laws are reducible to microscopic laws

But are microscopic laws more fundamental? This question will prove important for understanding Prigogine's position. Moreover, in order to hold a reductionist position, we have to argue that we can meaningfully distinguish between the micro- and macroscopic domain. Is this always possible? And if so, does Prigogine advocate the distinction between the microscopic and the macroscopic domain?

With this brief historical reconstruction we see that the concept of time's arrow has gradually shifted from Eddington's original broad category label denoting concepts as the 'approach to equilibrium' and of 'becoming', since the contributions of Popper and Grünbaum, towards the more specific problem of reconciling the asymmetry of processes at the macroscopic domain with the time symmetrical laws at the microscopic domain. We have identified RA as the assumption needed for the 'arrow of time' to constitute a problem.

What does Prigogine mean with the arrow of time? Does he subscribe to the modern use? My answer would be: not necessarily so. I think that Prigogine understands the problem of time's arrow as inherently related to the second law and as an expression of the approach to equilibrium. To give a typical example in which Prigogine mentions time's arrow: "Time's arrow, the entropy, according to Eddington's famous expression, is now explicitly displayed [in causal dynamics]"([59, p.445]) or consider "distance from equilibrium and therefore the arrow of time"([84]). In that respect Prigogine is closer to Eddington's intention than to Popper and Grünbaum. Prigogine rather uses the words 'time paradox' when he speaks about the later more specific problem of reducing asymmetrical macroscopic phenomena to symmetrical microscopic equations. It is unfortunately not an easy matter to define precisely what Prigogine means when he speaks about the arrow of time since the second law is of course one of the major time asymmetrical laws describing asymmetrical processes in time and in that sense the quotations above do not provide a definitive answer, but serve as mere circumstantial evidence. There is of course another possibility to scrutinize Prigogine's meaning of the use time's arrow which is by questioning whether Prigogine adheres to RA. This question will be taken up in chapter five.

For the moment I note that the major conceptual step involved in thinking about time's arrow is that the modern concept of time's arrow no longer considers time itself, but only processes in time. We have to be cautious not simply to assume that for Prigogine this distinction is important as well. This distinction will be scrutinised in investigating the motivations of Prigogine.

2.4 Irreversibility Defined

In the previous section we have already encountered the concept of (ir)reversibility a number of times, although it is not an unproblematical notion. In order to deal with the arrow of time clearcut definitions have to be used. I will use the instrumentarium as developed by Uffink in *Bluff your way in the second law of thermodynamics*, in which Uffink investigates what the second law of thermodynamics actually says and what kind of relationship it has with the arrow of time. The basis of the second law Uffink notes, consists in a claim that certain processes are impossible. But what does the word (im)possible exactly mean? Uffink distinguishes between three different uses of the word 'possible':

- 'Allowed by some given theory'. That is, the criterium for calling a process possible is whether one can specify a model of the theory in which it occurs. It is not not relevant whether the reversed processes occurs in the natural world. It is sufficient that the theory allows them in some possible world.

- 'Available in the actual world'. Advocated by Planck and many other 19^{th} century physicists, for whom the claim that a process is impossible is a statement that transcends theoretical boundaries.

- A third sense of 'possible' is 'available to us'. This reading makes the notion dependent on the human condition.

Uffink distinguishes three themes that are important in relation to the arrow of time:

A) We have already encountered the first in the introduction to this chapter. It is the idea of the flow of time, also called becoming. Becoming constitutes "a different ontological status to the events in the past, present and future. Present events are the only ones who are 'real' or 'actual'. The past is gone and forever fixed (..) The flow of time is then regarded as a special ontological transition: the creation or actualization of events. This process is often called *becoming*" (Uffink see also Poidevin [54]).

B) The second theme that Uffink mentions is the idea of symmetry under time reversal. We call a theory or law time-symmetric if the class of processes that it allows is time-symmetric. The mathematical form of the laws themselves (and a given choice of *R*) determines whether the theory is time symmetric or not. Note also that the term 'time-reversal' is not meant literally. The theory is called time-symmetric if the class *W* of possible worlds is closed under time reversal, i.e. if the following holds:

$$\begin{aligned} P &= \{s_t \in \Gamma : t_i \leq t \leq t_f\} \\ P^* &= \{(Rs)_t : -t_f \leq t \leq -t_i\} \\ \text{If } P &\in W \text{ then } P^* \in W \end{aligned} \tag{2.2}$$

where R is the transformation that turns a state in its 'time reversal' state, and hence P^* denotes the time reversal of a process.

C) Another theme concerns *irreversibility* which is most commonly predicated of processes instead of theories. However in the philosophy of science literature the concept is intimately connected with time-asymmetry of theories. More precisely, one calls a process *P* allowed by a given theory irreversible if the reversed process *P'* is excluded by this theory. Conversely every time-asymmetric theory does admit irreversible processes in this sense. Therefore discussions about irreversibility and time-asymmetry coincide for the most part. However, in thermodynamics the term 'irreversibility' is commonly employed with other meanings.

These other meanings are as follows.

- (1) There is the concept of irrecoverability, which differs from time-asymmetry in at least three respects: (i) for a recovery the only thing that counts is the

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retrieval of the initial state (ii) a complete recovery also involves also the environment (iii) the notion of possible used reflects the second meaning 'possible in our actual world'. Thus (ir)recoverability does not imply, and is not implied by time-(a)symmmetry.

- (2) The concept 'quasi-static' which is neutral with respect to time-reversal: a process is quasi-static (irreversible) if it exemplifies non-negligible deviations from equilibrium.
- (3) Last, part of the physical literature employs the term in order to denote (in)determinism in the evolutions allowed by the theory. A process is then called reversible because the evolution is an invertible mapping. An advocate of this definition is Mackey.

Because of these ambiguities Uffink suggests to abandon the term irreversibility for sake of clarity, and use asymmetry in the above sense instead. Difficult in assessing Prigogine is that Prigogine does not distinguish properly between these three themes (as I will argue when discussing Prigogine's motivations) while at the same time using expressions from all categories. I will first turn my attention to Prigogine's use of irreversibility.

2.4.1 Prigogine's Conflating Definitions

I will try to state exactly what Prigogine's definitions of (ir)reversible are using the framework introduced earlier. In the following quote Prigogine states his definition of a reversible law:

> "Die Eigenschaft der Reversibilität lässt sich ganz einfach ausdrücken: das dynamische Gesetz ist so beschaffen, dass eine (imaginäre oder auf dem Computer durchführbare) Umkehroperation $v \to -v$ der Geschwindigkeiten jedes Punktes des Systems einer Operation der Umkehrung der Richtung des Zeitflusses $t \to -t$ äquivalent ist."[11]

Presumably, this means that a law is called 'reversible' when reversing both the directions of time and velocities leaves the particular law invariant (although one may doubt whether 'equivalence' is the same as 'invariance'). A typical irreversible process from the point of view of thermodynamics according to Prigogine is described

[11] Prigogine/Stengers/Pahaut 1991 quoted from Sandbothe [71, p.13])

by the Fourier equation of heat conduction.

$$\frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial x^2} \tag{2.3}$$

which is obviously not time invariant. When we substitute *-t* for *t* we obtain the anti-Fourier equation, with *K* changed into *-K*. Irreversible processes are thus processes allowed by non time invariant laws. Time invariance can be compared in Uffink's scheme with B: the mathematical form of the laws themselves determines whether the theory is time-symmetric or not. The quotation also makes clear that Prigogine uses possible in the sense 'allowed by some given theory' as he speaks about an imaginary velocity inversion.

However, at other points Prigogine employs other meanings of the word irreversible process. For example Prigogine states:

> "Let us remind the very definition of an irreversible process as introduced in thermodynamics. According to Planck's definition it is a process which once performed leaves the world in an altered state: By no experimental device whatever the ingenuity of the experimenter, should it be possible to restore the initial state."([59, p.411])

> "If it were possible to use velocity inversion or similar experiments to destroy the entropy produced [before reversal] there would exist according to Planck's definition *no* irreversible process and *no* second law of thermodynamics. Fortunately this is not so with our new statistical interpretation of entropy, we may even construct thermodynamical cycles involving periods of time where the system retraces its past."([60, p.8])

Implicit in these remarks is that Prigogine subscribes to Plancks definition of irreversbility (i.e. irrecoverability of the initial state). Here Prigogine shifts his use of 'possible' towards the second sense as distinguished by Uffink 'available in the actual world'.

Prigogine often uses a more liberal third meaning of an irreversible process: a process in which the entropy increases.

> "Now if we consider the velocity inversion experiment the positive entropy produced (..) would be compensated by a negative entropy production, in contradiction with the very definition of an irreversible process."([59, p.411])

A reversible process is likewise a process in which there is no entropy increase. The third definition is less restrictive, since we can think of a process in which entropy increases, but which is recoverable. What is meant by 'entropy production' and 'the system retraces its past' will be discussed in chapter four.

In his more philosophical papers Prigogine advocates and argues for a new meaning for the concept of time as will be put forth in the next section where the motivations of Prigogine's work will be discussed. Prigogine opposes the use of time as a parameter in physics and argues for the constructive role of irreversible processes. Even more so, time itself is argued to be irreversible. Let me remark here, that time invariance reduces time to a mere parameter just as well.

For the moment I conclude that a discrepancy between Prigogine's philosophical aims and physical work seems to emerge. Moreover, Prigogine's use of the term irreversibility is in danger of being inconsistent. The discrepancy between Prigogine's philosophical and scientific perspective will be analyzed in more detail when discussing his motivations.

2.5 Motivations of Prigogine

> "For us believing physicists the distinction between past, present and future is only an illusion, however persistent"(Einstein)

On a wall in Ilya Prigogine's office of Texas at Austin is the Albert Einstein quote shown above, blown up to poster size. To Prigogine, time is the forgotten dimension, and he has directed his whole scientific research efforts towards a revalidation of time. In this section I will try to sketch the most important Leitmotive in the thinking of Prigogine. First I will discuss the revalidation of time from a scientific perspective concentrating on especially the early (pre 1970) work of Prigogine in non equilibrium statistical mechanics (NESM), which is an extension of thermodynamics to open systems. Generalized thermodynamics claims a universal validity for the second law, and argues for a non-destructive (and hence creative; order out of chaos), interpretation. The fact-like status of the second law is point of departure. In later years (see the scientific biography in section 1.1) Prigogine's programme changes. The emphasis changes towards developing a new interpretation of irreversibility. Prigogine no longer departs from the validity of the second law, but in line with Boltzmann, tries to give a justification for irreversibility at all levels. Prigogine will be seen to advocate a non-statistical (not due to coarse-graining; intrinsic), universally valid (irreversibility at all levels) interpretation of irreversibility.

This new theory is claimed to constitute the change *from being to becoming*.

Secondly I will discuss the revalidation of time from a philosophical perspective thereby focusing especially on the most prominent inspirator Prigogine mentions: Bergson. The philosophical perspective addresses the question what the role of the human experience of time (phenomenological or psychological or lived time) is within reality. Prigogine is often interpreted as trying to unify our understanding of time expressed by the slogan *new dialogue with nature*.

2.5.1 The Scientific Perspective

At the end of the nineteenth century the second law was feared as having quite detrimental effects. To mention the least, it would lead to the so called 'heat death'. Eddington expresses this when he writes about the running down of the universe. At the same time, as we have seen previously, Maxwell's demon illustrated that the second law is not an exceptionless principle i.e. is only approximately valid, necessitating a statistical interpretation as for example worked out by Boltzmann in 1877. Prigogine opposes both tendencies.

Rather than viewing the second law as a negative factor, Prigogine believes that it is actually a source of organization and order. These ideas have been developed in the early work of Prigogine on non-equilibrium statistical mechanics. Whereas as Bishop writes, "one of the fundamental laws of conventional equilibrium thermodynamics, it is typically treated as valid only at or near thermodynamic equilibrium. Prigogine and his colleagues believe that some appropriate generalization of the second law should be applicable to nonequilibrium systems as well" ([13, p.3]). In this section I will first describe Prigogine's older work which is rooted in biology and chemistry. Thereafter I will discuss his more recent ideas about intrinsic irreversibility, which are developed in opposition to statistical interpretations of irreversibility.

2.5.1.1 The Constructive Role of Irreversibility

In the years from 1946 till 1967 Prigogine develops his theory of non-equilibrium thermodynamics dealing with so-called dissipative structures in two books *Thermodynamics of Irreversible Processes* ([56]) and *Non-equilibrium Statistical Mechanics* ([57]). I will discuss here only the main points.

According to Prigogine the entropy production dS of an open system may be ascribed to two separate terms: the term dS_e, which refers to the heat exchange with the external medium, and the term dS_i which refers to the irreversible processes taking place within the system:

$$dS = d_eS + d_iS, \text{with } d_iS \geq 0 \tag{2.4}$$

which for systems in a steady non-equilibrium state reduces to

$$dS = d_eS + d_iS = 0,\ d_eS = -d_iS \leq 0 \tag{2.5}$$

Prigogine imposes the so-called **local equilibrium assumption** so that in any given region, entropy is represented by a thermodynamic function of the same parameters as at Gibbs equilibrium. Once this assumption is adopted the entropy production dS_i per unit of time takes the more explicit form:

$$P = \frac{d_iS}{dt} = \int_V \sum_\alpha X_\alpha J_\alpha dV \tag{2.6}$$

The J_α represent the rates of the various processes (heat flow, diffusion) and the X_α are the corresponding generalized forces (gradients of temperature and concentration). At thermodynamic equilibrium $J_\alpha = 0$ and $X_\alpha = 0$ for all irreversible processes simultaneously. It is therefore quite natural to assume, at least for conditions near equilibrium, linear homogeneous relations between the flows and the forces. Such a scheme automatically includes such empirical laws as Fourier's law, which says that the flow of heat is proportional to the gradient of temperature.

Linear thermodynamics of irreversible processes is dominated by two important results. The first is expressed by the Onsager reciprocity relations. The second result states that for the linear nonequilibrium region around the Gibbs equilibrium, the entropy production P of the steady state is minimum for a given generalized force X_α. This result is the well-known **minimum entropy production** theorem of Prigogine.

$$dP \leq 0 \tag{2.7}$$

The theorem of minimum entropy production expresses a kind of 'inertial' property of nonequilibrium systems. When given boundary conditions prevent the system from reaching thermodynamic equilibrium (i.e. zero entropy production) the system settles down in the state of 'least dissipation'. For years attempts were made to extend these results to the nonlinear range. It therefore came as a surprise that in systems far from equilibrium the behaviour could be quite different.

A famous example is the Rayleigh-Bénard flow (or instability). It occurs when a liquid is heated from below. As heating intensifies, the mixture suddenly begins to "self-organize", "taking on a striking spatial structure sometimes resembling miniature stained-glass cathedral windows"([84] Prigogine). The phenomenon intrigued scientists because these patterns resembled living cells, in that within each cell, ordered molecular motion occurs. Prigogine reasoned that if such a phenomenon was possible in fluid dynamics, it would be possible in chemistry and biology as well. This self-organization, as Prigogine calls it, of matter represented to him a critical link between animate and inanimate matter. It could even provide a clue to the spontaneous eruption of life beginnings.

In order to understand far from equilibrium structures, Prigogine developed in cooperation with Glansdorff techniques to deal with these. The use of stability theory in the form of Lyapounov functions led to the following result

$$\frac{1}{2}\frac{\partial}{\partial t}\delta^2 S = \sum_\alpha \delta J_\alpha \delta X_\alpha \tag{2.8}$$

The right hand side is called the 'excess entropy production' and may become unstable at a critical distance far from equilibrium. Prigogine has done a lot of theoretical research on chemical reactions in order to investigate these instabilities. These techniques are called **generalized thermodynamics**.

Until the late sixties experiments did not substantiate these views. The Zhabotinsky Reactions however confirmed Prigogine's theory. Just as he had predicted, the reactions, which require a continuous outside source of energy, occur at states far from equilibrium; and like animate matter itself, they are self-organizing. The concentrations of the various chemicals oscillate with clocklike precision, changing the solution from red to blue at regular intervals. The effect is what Prigogine calls *order out of chaos*, and the structures are called dissipative structures.

The interpretation and the importance of generalized thermodynamics is formulated by Prigogine in 1971 as follows:

> "The french philosopher Henri Bergson (1907) called the second law of thermodynamics the most 'metaphysical' of all laws of nature. Whether a compliment or a criticism, this applies also to the 'generalized thermodynamics' we develop in this monograph. Classical thermodynamics is essentially a theory of 'destruction of structure'. One may even consider the entropy production as a measure of the 'rate' of this destruction. But in some way such a theory has to be completed by a theory of 'creation of structure' lacking in classical thermodynamics. We have seen that in addition to the 'excess entropy production' which seems to be

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the basic quantity whose behaviour characterizes the occurence of new structures and their stability (..) Therefore we may hope that the far from equilibrium approach we develop in this book may act as an element of unification bringing closer problems belonging to a wide range of disciplines" (Glansdorff and Prigogine [30, p.xxi])

Two observations are in order. The first observation is the opposition Prigogine sees between the destructive character expressed by the second law and our common perception that we do not see destruction around us at all. How can we reconcile the second law of thermodynamic describing the increase of disorder with the emergence of all kinds of new structures around us? Prigogine argues that thermodynamics has to be completed by a theory of creation. Developing such a theory of creation is his main scientific goal. Crucial in such a theory is the role of irreversibility which will be discussed subsequently. Prigogine sees irreversible processes as (1) having a constructive role in biological development (cf. *From Being to Becoming* [61]) (2) being as real as reversible ones (3) being deeply rooted in dynamics. All these convictions are captured in the slogan *from being to becoming*.

It is therefore important to realize that the notions of creative and of becoming are used by Prigogine not in a very strict sense. 'Becoming' is I think most properly understood as a broad category label, and not in the specific meaning Uffink attributes to it as the (realistic) position that grammatical tenses have counterparts in reality. 'Creative' is best understood as the opposite to 'destructive', and hence not in the specific meaning of a continuous creation or actualization of the present. In short, the notions of becoming and creative are often used without refering to the human condition. I will discuss the latter point when I turn to the philosophical perspective of Prigogine.

My second observation is about *order out of chaos*. Both Schulman and Bricmont criticize the so called empirical evidence of the Zhabotinsky reactions for Prigogine's interpretation.[12] Prigogine, according to these authors, mispresents what is at stake. The confusion arises if we reason "a structure is initiated by a source of heat, which is usually a source of disorder (..) but what is needed, of course, is a temperature difference between the two plates. So, if one heats up from below, one must have cooling from above, which acts as a refrigerator, so it requires some 'ordered' source of energy"([21, p.33] Bricmont). There is strictly speaking, no 'order out of chaos' as Prigogine favors and no contradiction whatsoever with the second

[12] Attacks on Prigogine's theory of dissipative structures, can be found at www.santafe.edu/s̃halizi/notebooks/prigogine.html

law of thermodynamics.

This criticism is not completely fair for two reasons. Most important is of course that Prigogine has his generalized version of the second law in mind extended to open systems, and not restricted to closed systems. The second reason is that the criticism depends on the definitions of order and chaos that are being used. Prigogine sees the Zhabotinsky reactions and his own developed theoretical model for studying autocatalytic reactions, called the 'Brusselator', as empirical evidence for his scientific enterprise in claiming a new status for the second law of thermodynamics. It is important to realize that Prigogine's later scientific work is based on certain convictions, that are according to Prigogine, deeply rooted in empirical evidence. Prigogine's scientific perspective in that sense remains primarily that of a chemist and less that of a physicist.

2.5.1.2 Intrinsic Irreversibility

Beside arguing for an extension of the domain of application of the second law, in his pre-1970 work, Prigogine also claims a different status for it. In most interpretations of the second law, it is claimed that it is not an objective law, but has only statistical validity, cf. Maxwell's demon. Probabilistic interpretations are developed, using models applying coarse-graining i.e. using models that provide less specific information than the exact points in phase space.[13] Probability and herewith irreversibility is then deliberately introduced in the dynamics and is often justified by an appeal to epistemological reasons as for example measurement limitations.

> "In all fundamental theories (be it classical dynamics, QM or relativity theory) entropy is conserved as a result of the unitary (or measure-preserving) character of the evolution, in flagrant contradiction with the formulation of the second law of thermodynamics. As a result, the second law has usually been regarded as an approximation or even as being subjective in character. By contrast, in the approach to the problem of irreversibility developed by us, the law of entropy increase and, therefore, the existence of an 'arrow of time' is taken to be a fundamental fact. The task of a satisfactory theory of irreversibility is thus conceived as the study of the fundamental change in the conceptual structure of dynamics, which the law of entropy increase implies." (Misra and Prigogine 1983 [46, p.421])

[13] Boltzmann's 1877 idea to count the number of microscopic states that give rise to a certain macroscopic state is in fact a form of coarse graining.

Prigogine opposes a statistical interpretation of the second law. It is an objective law, and entropy increase is a fundamental fact. But whereas in NESM the second law was used in the framework of thermodynamics, Prigogine now attempts to change the conceptual structure of dynamics in accordance with the fundamentality of the second law. This requires a new interpretation of irreversibility, which is called by Prigogine 'intrinsic irreversibility'. In chapter four I will discuss the different approaches that have been developed towards this goal.

An important distinction is made between two types of irreversibility: *extrinsic* and *intrinsic*. Extrinsic irreversibility is possible in two ways. First, due to the interaction of a physical system with its environment, where in the absence of an environment, the system evolution would be reversible. In Bishop's words, the environment is both necessary and sufficient to produce irreversibility. Decoherence (For All Practical Purposes), in quantum mechanics uses this route. Secondly, the above described forms of coarse graining also constitute a form of extrinsic irreversibility, justified by appealing to epistemological reasons.

By contrast, intrinsic irreversibility refers to irreversible behaviour due to the dynamics of a physical system. Environmental effects may be necessary, but are not sufficient conditions. Irreversibility becomes an objective property of certain systems. These systems are called 'intrinsically random' (and from 1983 onwards intrinsically irreversible) and Prigogine tries to find necessary and sufficient conditions for a system to be intrinsically random.

Prigogine's new interpretation is hence in line with the pre-1877 work of Boltzmann aiming at a mechanical derivation of the second law expressed by the word intrinsic. But it is more than a mere attempt to give a justification. Irreversibility is not something just for the macroscopical level, irreversibility attains at all levels. For Prigogine irreversibility can be characterized as: universal (irreversibility at all levels), intrinsic, and creative (constructive). It is questionable whether the concept of irreversibility can bear such a heavy load.

This change in scientific perspective from NESM towards intrinsic irreversibility has been misunderstood by some authors. Prigogine's aim to develop a microscopic theory of intrinsic irreversibility is described by Sandbothe as "unnötiger Ballast"(p.106). Sandbothe points to what might be perceived as a tension between the philosophical and the scientific perspective on the revalidation of time. Common sense imposes the idea of an asymmetrical concept of time, in which past and future are distinct. At the same time we have an important asymmetrical law such as the second law of thermodynamics. Why should we still bother to justify this law? Are not our philosophical arguments about time enough to justify the fundamentality and validity of the second law?

A similar idea is expressed by Verstraeten. Verstraeten [81] says that "lawlike approaches, which are based on the second law, to explain, asymmetric time evolution and the 'arrow of time' are redundant as this second law also gets its physical meaning by its fact-like origin. Hence Prigogine's attempt - in searching for a microscopic origin - to prove that the second law of thermodynamics is as fundamental as the classical laws of mechanics is chaos out of order"(Verstraeten p.653). Actually, I think that Verstraeten and Sandbothe misunderstand the change in perspective from NESM towards intrinsic irreversibility.[14] The shift is, as we have seen, not merely a shift from 'assuming' towards 'justifying' the second law, but encompasses an extension from the mere macroscopic validity of irreversible behaviour towards the new idea that irreversibility attains to all levels, not just the macroscopic one.

I do agree that there is a tension between the scientific and the philosophical perspective on revalidating time, though for other reasons. My main reason is that these perspectives address different questions about time that should be separated instead of intertwined. This argument will be developed in the next section.

2.5.2 The Philosophical Perspective

As we have seen, for Eddington the role of consciousness was important for a proper understanding of time's arrow. The question then becomes pressing what the relation is between physical time and our human experience of time. What is Prigogine's conception of time like? How should time be revalidated? These questions are the focal point of the philosophical perspective on revalidating time. First I will discuss the historical analysis Prigogine gives of the *Leugnung der Zeit*. Then I will turn my attention to the main inspirator of Prigogine, namely Bergson, and scrutinize the *new dialogue with nature*.

2.5.2.1 Zeitvergessenheit

Prigogine claims in *Order out of Chaos* that there have been successively three steps, that have led to the modern physical standpoint regarding time (the block universe view), in increasing order of abstraction and idealization. The first step is the reduction of the different forms of change to just one, i.e. movement.

[14]Verstraeten seems to overlook this change in perspective totally when for example discussing the Baker's transformation. Verstraeten totally overlooks the importance of the results of Misra in constructing Λ ([81]).

> "Die Dynamik kennt nur eine Art von Veränderung, nur einen 'Prozess', und das ist die Bewegung. Die qualitative Vielfalt der Veränderungen in der Natur wird auf die relative Ortsveränderung materieller Körper zurückgeführt (..) Schon Aristoteles unterschied verschiedene Modalitäten des Wechsels, vor allem den Ortswechsel (lokale Bewegung) und den Eigenschäftswechsel (Veränderung). Sowohl die klassische als auch die relativistische oder Quantenphysik haben sich auf die Zeit, betrachtet als Bewegung, konzentriert. Es schien, als ob die Zeit als qualitativer Wechsel ausserhalb ihres Horizontes läge"(Prigogine and Stengers[15])

Whereas Aristotle still distinguished between four different causes (materialis, efficiens, formalis, finalis) related to change, change has become restricted to *loco motio*.

The second idealization step consists in freeing the interpretation of *loco motio* from its Aristotelian properties (Seinsweise des Körpers) towards a quantification in a mathematical sense. This is illustrated by the difference between Kepler and Newton. Whereas Kepler interpreted tangential forces as *causa finalis* of the motion of planets, Newton was only interested in the *causa efficiens* of the radial forces. The consequence of the second tendency is that the 'why' question has been totally eliminated on behalf of the 'how' question.

The third step, due to Newton, consists in the extension of the domain of applicability of the calculations of movement and can be seen as an universalisation tendency.

> "Die formulierung der Newtonschen Bewegungsgesetze vereinigte zwei konvergierende Entwicklungen: in der Physik die Keplerschen Gesetze der Bewegung der Planeten und Galileis Gesetze des Falls von Körpern, in der Mathematik die Formulierung der Differential-oder 'Infinitesimal'-rechnung."(ibid p.63)

Whereas the realms of the spheres and of the earth used to be seen as categorically different, they are now both seen as instances of the same underlying mechanical principles. Modern differential calculus has turned time into a line without direction, in which future and past have been reduced into mere coordinates. Leibniz in 1704 for the first time mentions time as a parameter (and later Lagrange and Laplace), leading eventually to Kant's antinomies, in which time is argued to be unreal.

[15] *Dialog mit der Natur* (Neue Wege naturwissenschaftliches Denkens München 1981 p.26)

These three successive steps of idealization and abstraction are called the **Leugnung des Pfeils der Zeit**[16]: in physics time has been reduced to a parameter. Prigogine vehemently opposes this notion of time as only a parameter, as a label of events. “Die klassische Wissenschaft (..) leugnete (..) das Werden und die natürliche Vielfalt, die Aristoteles als Attribute der sublunaren, minderen Welt auffasste.”(Prigogine and Stengers 1981 p.281). We thus seem again to arrive at the necessity of a movement from being to becoming.

One may ask why the reduction of time to a mere parameter is objectionable. The elucidation he gives is the consequences the reduction has for the concept of a trajectory, namely that it leads to the notion of reversible, deterministic trajectories in physics. Prigogine and Stengers mention: “die grundlegenden Merkmale der Trajektorien *T* sind Gesetzmässigkeit, Determiniertheit und Reversibilität (..) Es ist somit in jedem Augenblick alles gegeben”(ibid). This contradicts our experience in two important ways: (i) there *is* a distinction between past and future (ii) the future is still open and undetermined (only the past is determined). The former has to do with the importance of irreversibility at all levels, the latter with Prigogine’s belief in chaos and bifurcations. This is the main reason for dismissing trajectories as the corner stones of physics, leading to a rhetoric in which they should be eliminated form physics. Boltzmann and Bergson are credited for having acknowledged ‘die Zeitvergessenheit der klassischen Physik ins wissenschaftliche Bewusstsein’.

Prigogine not only provides us a historical perspective consisting of these three idealization and abstraction steps in the history of classical physics, but he also gives an argument to explain why the three steps have taken place. The argument is, that whenever Zeitvergessenheit is there, also the fact that we have forgetten is immediately forgotten. In *Entre le temps et l’éternité* Prigogine and Stengers identify as main reason for the Zeitvergessenheit, the fundamental status for the human being of its time-horizon.[17]

> “Diese Unterscheidung zwischen Vorher und Nachher ist (..) in solchem Masse Bestandteil unserer Erfahrung, dass wir diese gar nicht beschreiben können, ohne diesen Unterschied vorauszusetzen. Deshalb konnten die

[16]Prigogine and Stengers *Das Paradox der Zeit* (Zeit, Chaos und Quanten München 1993 p.9)

[17]The term Zeitvergessenheit was proposed by Michael Sandbothe in his study *Zeitlichkeit und Selbstorganisation: Ilya Prigogines Theorie Irreversibeler Prozesse und Martin Heideggers Zeitlichkeitsanalyse: ein Gegenkozept zur Aktuellen Konvergentzthese* (1994 [71]). Sandbothe contrasts Prigogine’s theory with Heidegger’s *Sein und Zeit*. His study is interesting because it aims explicitly on clarifying Prigogine’s concept of time, regardless of the validity of his scientific theories, and tries to interpret him in the continental tradition of philosophy.

> Physiker die in der Dynamik enthaltene Negation dieser unterschiedung wohl auch erst aus ihren Gleichungen herauslesen, als sie durch das Problem der irreversibele Prozesse dazu genötigt wurden."(Prigogine and Stengers 1988 p.33)

The above argument to explain why physics has forgotten about time may strike some physicists as out of place. Actually it is basically the same argument as Heidegger has put forward in *Sein und Zeit*. In the prologue of *Sein und Zeit* a twofold Seinsvergessenheit is lamented: we have not only forgotten what Sein is, but on top of that we have also forgotten this act of forgetting. It is necessary to restate and re-understand the importance and the sense of the question about Sein. The aim of this question is to demonstrate that the interpretation of time as a possible horizon of any experience of time is necessary. An important starting point for Heidegger lies in the principle of ontological difference between Sein and Dasein. To understand the sense of Sein, we have to start by analyzing Dasein. This semantical analysis has no other point of departure than this Da of Dasein. Heidegger's analysis starts henceforth with a criticism on what he calls objectifying, as it is responsible for Seinsvergessenheit. In objectifying the variety and richness of relations in which any subject is necessarily involved vanishes, leading to concepts as subject and object. But these are mere constructs, that prohibit answering real philosophical questions such as 'Why there is something and not nothing?'. This criticism on objectifying is the starting point of the phenomenological approach developed first in the works of Husserl (making it the structure of our consciousness) and later radicalized by Heidegger (making it the structure of reality).

One can develop a stronger argument in favor of Prigogine. One could argue that physics is an enterprise that by its very nature and purpose, is limited and abstractive. It considers only those aspects of reality which can be connected to, or expressed in terms of general laws and repeatable events. This aim of physics leads to the discarding of all aspects of reality which are accidental, non-repeatable, and specifically tied to a unique 'here-now'. In this point of view, the fact that physics finds no evidence for tensedness is not an argument that time is unreal, but rather that physics persistently ignores this aspect of reality. Prigogine can accordingly be interpreted as formulating a direct attack on the reductionist practice of the physical, perhaps even on the whole scientific enterprise. Following this line of thought (due to Uffink) the narrative of Zeitvergessenheit would have a strong anti-reductionist flavor.

2.5.2.2 Bergson

> "Since my adolescence, I have read many philosophical texts, and I still remember the spell *L'evolution créatrice* cast on me. I felt that some essential message was embedded, still to be made explicit, in Bergson's remark: 'The more deeply we study the nature of time, the better we understand that duration means invention, creation of forms, continuous elaboration of the absolutely new.'"(Prigogine [85])

We have seen that Bergson is credited by Prigogine as providing a turning point in thinking about time. The quotation makes clear that Prigogine has been inspired, to say the least, by Bergson's ideas about time and creation. The question then seems to be justified, to what extent Prigogine is influenced by Bergson. In other words, is Prigogine a Bergsonian?

Bergson is one of the great French philosophers of the nineteenth century, but his current influence seems to have been marginalized. Bergson proposed a pragmatic inherently dualistic philosophy involving a distinction between intellect and intuition leading to a radical distinction between space and time. The intellect is characterized by the power to separate in space and fixate in time, and is therefore unable to understand life. According to Bergson science has to be completed by a metaphysics of duration. In order to understand what the latter means we have to turn our attention to the other mental power, intuition. Whereas Kant wrote 'Anschauungen ohne Begriffe sind leer', intuition is a philosophical method of immediate experience i.e. a method of experiencing reality without a priori employing concepts or theories.[18] This raises the question of course, what we will find if we experience reality immediately.

In reality there are no distinct concrete things, reality consists of an endless stream of becoming. This view of reality has been called by Griffin pantemporalism. Bergson makes a distinction between mathematical time, which is a form of space, and lived time or also called *durée*, which is one of the most difficult concepts in Bergsons philosophy to understand. In a classical study on Bergson, Capek distinguished between four aspects of duration: continuity, succession, heterogeneity, and survival of the past. "Continuity, interpreted as the survival of the past, constitutes in Bergson's opinion a new, heterogeneous present, and vice versa. Classical physics implies an elimination of duration, or 'real time', from the physical world" (Capek[19]). Bergson's idea can be illustrated as follows. As an example consider

[18]Paraphrazed and translated from Bor ([19])
[19]quoted from Bor [19, p.269].

a moving object. Bergson claims that movement cannot be subdivided into positions at which the moving object can be found. Even an infinite number of points does not constitute a continuous movement. Whereas physics claims to have solved Zeno's turtoise paradox by infinitesimal calculus, Bergson questions the validity of the mathematical definition of continuity.

I do not want to analyze Bergson's philosophy in more detail, nor give an evaluation. What I want to conjecture however, is that the main source for Prigogine's ideas about Zeitvergessenheit and the inspiration in claiming a revalidation of time both stem from Bergson. Another observation to make is that Bergson uses the word 'becoming' to describe the pantemporalism of reality. As I noted earlier Prigogine uses 'becoming' not in the narrow sense of realism with respect to tensed facts, but as a broader category label. It may well be that he has Bergson's image of pantemporalism in mind when he speaks about the movement from being to becoming, but this remains of course a mere speculation.

There is also a second way to perceive the influence Bergson has had on Prigogine. Prigogine's scientific work might be seen as an attempt to make Bergson's essential message more precise. I would like to quote an example to illustrate how Prigogine perceives his scientific work in relation to Bergson:

> "We may speak of the 'duration' of the present. It is interesting that philosophers (especially Bergson and Whitehead) have emphasized the need to attribute to the present some kind of impressible duration. The view of the second law as a dynamical principle precisely lead to this conclusion"([64, p.246]).
>
> However Bergson confused the issue by stating that 'lived time' had to remain outside the reach of physics. In this sense progress can now be made by comparing astronomical time not to lived time, which is a concept difficult to make precise, but to internal time, which is related to dissipation and is therefore at the basis of the very possibility of life (..) It is this rediscovery of time, not the old time, according to which the watch is eternally going to its own past, but an internal time, which corresponds to activity and finally creative processes as envisaged by Whitehead (..) which makes our period as exciting and full of promise for the future as the great period between Galileo and Newton"([64, p.249]).

Prigogine sees his scientific work as the quotation indicates, as an attempt to make Bergson's ideas about duration precise, by introducing a so-called internal time operator (this will be discussed in more detail in chapter four). This is necessary since Bergson's concept as we have seen is difficult to make precise. On the other hand Prigogine greatly differs from Bergson with respect to his dualism. Prigogine is clearly not a Bergsonian, in the sense that he wants to transform physics to unify the mathematical and psychological concepts of time, whereas Bergson is inherently dualist about the realm of the intellect (physics) and intuition. As Prigogine puts it, 'lived time' has to be incorporated within the real of physics.

In the same article Prigogine makes clear that he by no means wants to rehabilitate Bergson, who has been ruthlessly criticized by Russell (a true philosophical assassination according to Bor and unjustified!) and who bravely lost in his discussions about time with Einstein (*Mélanges* [12]). Prigogine says: "I have found it always astonishing that Bergson had such a difficult time understanding Einstein, and also that Einstein remained *nearly*[20] all his life, hostile to the problems at the core of Bergson's work. It seems to me that the progress in our understanding of time (..) permits us to avoid some aspects of this conflictive situation"(ibid p.247). In other words, Prigogine does not want to choose sides but to resolve the conflict. This attempt at resolvement has been described as *unifying our understanding of time*:

> "Daraus ergibt sich einerseits die Versuchung, der wie sogar bei Einstein begegnen, die Existenz der Zeit oder die Geschichte zu leugnen, und andererseits entstehen daraus die Einsprüche von Philosophen wie Bergson, Whitehead, Husserl oder Heidegger, die in dieser Verleugnung einen Offenbarungseid des wissenschäftlichen Vorgehens sehen. Seltsamerweise können wir heute die Möglichkeit einer Synthese ins Auge fassen, die diese beiden Aspekte der Zeit miteinander verbindet"[21]

To be frank, I am rather skeptical about this attempt at unification. It is an easy thing to claim a unification, it is more difficult to achieve the unification. Let me give my arguments for this skepticism. First of all, the mentioned philosophers as far as I see might agree in their refutation of the way physics treats time, but they greatly

[20]Though Prigogine is keen to quote a passage of Einstein, in which Einstein seems to show more doubts about 'time as illusion': "what is essential in this is the fact that sending a signal is, in the sense of thermodynamics, an irreversible process, a process which is connected with the growth of entropy (whereas according to our present knowledge, all elementary processes are reversible)"(Einstein). Prigogine: "it is very interesting to note here that Einstein could not avoid considering irreversibility as an essential ingredient of our world picture"([64] p.248) in which sympathy for Bergson resounds.

[21](Prigogine and Pahaut (1985 p.26); cited in Sandbothe ([70, p.195-6]).

differ about the meaning of time. So it is no simple matter to unite the views of all the philosophers mentioned.

There is an enormous amount of literature about the relations between Prigogine and Heidegger: Sandbothe [71]; Cramer; Eigen; Hohfeld; Jantsch; Kroes; McKinney [72]. Sandbothe interprets Heidegger as claiming a temporalization of time: "time itself is meaningless; time is temporal"(Heidegger). Heidegger's analysis of this temporalisation is a pluralization of the concept of time (authentic/inauthentic/vulgar conception) according to Sandbothe. Prigogine's theories constitute in Sandbothe's words also a temporalization of time (although a gegenständliche and not a reflexive) with the aim of developing a selfconsistent interpretation, which encompasses both the world as experience and us who describe it. This is according to Sandbothe precisely the meaning of Prigogine's so-called *new dialogue with nature* or also called the notion of a participatory universe. The notion of participation has to be seen as opposite to the ordinary spectator-object dichotomy and would be an expression of the phenomenological approach advocated by Prigogine.

Problematic in a lot of these philosophical studies is that they are based only on the popular books of Stengers and Prigogine, in which as we have seen in discussing Zeitvergessenheit allusions are made to Heidegger. These studies take the importance of the rudimentary human experience of time as their starting point and judge the Nobelprize winning work of Prigogine as scientific evidence for their position, without assessing the work of Prigogine. There is nothing wrong with that of course, but if we take a simple look at a scientific paper of Prigogine we see directly that Prigogine in no way wants to incorporate the phenomenological approach in his scientific work. Heidegger is never mentioned in any of the scientific papers. I am therefore rather skeptical whether Prigogine's new dialogue truly aims at incorporating Husserl's and Heidegger's phenomenological legacy. The notions of a new dialogue and participation are unfortunately too vaguely defined in Prigogine's vocabulary to warrant sharper conclusions.

Griffin [35] has written about the relation between Prigogine and Whitehead. I will give a very brief (and superficial) characterization here. Whitehead's process philosophy starts from the analysis that the natural sciences methodologically abstract from the full concreteness of the entities or processes they study. To jump from the mere fact that time is not present in physics to the conclusion that time is unreal in an ontological sense is fallacious. Physical methodology ignores half the evidence, since it only considers things as they are perceivable by their measurement apparatuses. In considers therefore only the extrinsic side and not their intrinsic nature. The extrinsic process is called transition, and the intrinsic process is called concrescence. All the features of time are however rooted in the intrinsic reality of events, in the process by which they become concrete. It is here that the event includes the past events into itself and it is this inclusion that makes time irreversible. "Although there is a space-time continuum, this is a structure of potentiality. The temporal process is not in fact continuous; it is not constituted by a series of instants"(Griffin p.41). Whitehead's philosophy tries to incorporate physical theories of relativity in his ideas about time.

What this superficial characterization makes clear is that Whitehead's analysis of time is enormously different from Heidegger's. Whereas Heidegger analyzes the human experience of time, Whitehead analyzes physical spacetime. Whereas for Heidegger time is an undefinable concept, for Whitehead time *is* irreversible. Hence I conclude that time is a greatly differing concept in Heidegger's and in Whiteheads philosophy and simply mentioning their names side by side is not enough to acieve unification.[22]

This brings my to my second argument in favor of a skeptical point of view on unification. Prigogine seems to assume that the scientific and the philosophical perspective on revalidating time are intertwined. Prigogine is often interpreted as having claimed that the second law provides a physical foundation for the flow of time aspect of our experience. There are however problems with reconciling the philosophical and the scientific perspective. We have seen that Uffink distinguished between three aspects of time. The first was becoming or the flow of time, the second was the concept of symmetry under time reversal, and the third has to do with irreversibility. I will argue now that Prigogine conflates these three aspects.

There is an important distinction to be made between speaking of time itself, and speaking about the (a)symmetry of processes in time. Whitehead concludes that time is irreversible. Irreversibility is a defining characteristic of time. When Prigogine speaks about irreversibility he speaks about irreversible processes. An irreversible process is a process in which entropy increases, that is, a process in which entropy increases in time. But than a primitive concept of time is already presupposed. The protagonist of unification has to give an argument, what is meant by an irreversible process in a non-circular way. The first problem is due to a conflation of time and processes *in* time. The second problem, as we have discussed in section 2.4.1, is due to conflating notions of possible and hence conflicting notions of irreversibility.

A third problem consists in the concept of becoming. Is it a category label, as opposed to the static concept of being, or is it an ontological claim about the pantemporalism of reality. The former is problematic by its impreciseness; the latter

[22] Although I do believe that Bergson's and Whitehead's views on time are rather close.

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is in danger of falling prey to the epistemic fallacy. Psychological observations about time (our experience of it, aging) by itself are not enough to warrant ontological conclusions regarding whether time itself exists. We might all be suffering from a persistent illusion.

A final problem exists in conflating theoretical and empirical considerations about time's arrow. Bishop says "both [I will distinguish more] approaches share in this quest [in explaining the thermodynamic arrow of time] a kind of vacillation between seeking an explanation in the dynamics of the observed model and taking the empirically observed direction of the thermodynamic arrow as a fundamental fact"(Bishop [**?**, p.11]) As we will later see, in order to derive irreversibility Prigogine uses arguments over and above the dynamics alone, appealing to such observational facts as that we see equilibrium in the future direction but not in the past, or a form of causality. To use the adverb intrinsic for these demonstrations seems to me questionable.

These confusions can be traced to Eddington's ambiguous use of the arrow metaphor. Is the arrow a definition, or a property of time; are we talking about time in physics or the human experience of time. This concludes my skeptical arguments. The burden of proof still lies with the unificationist. Up till then it is better to disentangle questions instead of intermingling them.

I conclude that Prigogine is not a Bergsonian in a literal sense. He is too much of a physicist to incorporate Bergson's dualism. Prigogine wants to transform physics in order to unify our understanding of time. At the same time the influence of Bergson must not be underestimated. Bergson is frequently mentioned in his scientific papers (in contrast to e.g Heidegger), and Prigogine sees his physical theories as rendering Bergson's ideas more precise. Prigogine clearly symphatizes with Bergson's ideas such as: the loss of time in physics; the difference between time and duration; the creative power of evolution (self-organization). Earlier I conjectured that Prigogine not necessarily follows the modern viewpoint on what is the problem of time's arrow, which was developed since Popper and Grünbaum. Now I conjecture that Prigogine not necessarily uses the word becoming in the sense of realism with respect to tensed facts, but as an expression of Bergson's ideas about pantemporalism, in which reality exists as an endless stream of becoming and time is constructive and creative.

2.6 Conclusions

- The author that best foreshadows Prigogine is Arthur Eddington. There is a striking similarity between several statements of Prigogine and Eddington.

As we have seen, the second law for Eddington should be accompanied with awe. Similarly Prigogine claims a fundamental status of the second law, both in scope and whilst interpreting it, as an objective law deeply embedded in nature. Even the terminology of 'order' and 'creation' and 'organization' can already be traced to the original work of Eddington.

- We have noticed that since Eddington's introduction of the phrase 'time's arrow' in order to describe the approach to equilibrium, today's time's arrow problematic is more focused. It is concerned with the reduction of asymmetrical macroscopic phenomena to symmetrical microscopic laws. Prigogine's use of 'time's arrow' is closer to Eddington's than to the modern point of view.

- We have also seen that for the modern problem of time's arrow to be a problem we need a reductionist assumption. It is not easy to answer the question whether Prigogine is an anti-reductionist i.e. whether Prigogine rejects RA. The main motivation that lies at the heart of all Prigogine's theories is that physics has reduced time to a mere parameter and hence needs a revalidation of time. This observation certainly has an anti-reductionistic flaw. This question will be of central importance in the last chapter.

- There is a change in scientific perspective from NESM towards intrinsic irreversibility, which has been misunderstood by some authors. The shift is as we have seen not merely a shift from 'assuming' towards 'justifying' the second law, but encompasses an extension from the mere macroscopic validity of irreversible behaviour towards the new idea that irreversibility attains to all levels, not just the macroscopic one. Prigogine's later work is in line with Boltzmann's original attempt to obtain a mechanical derivation of irreversibility. Intrinsic irreversibility is due to the dynamics alone, without taking recourse to a form of coarse graining or interventionism.

- Prigogine frequently mentions continental philosophers that have inspired him in developing his ideas about time, such as Heidegger, Husserl and Whitehead. I argued that Prigogine in no way truly incorporates any of the consequences of the phenomenological approach. The influence of Heidegger and Husserl is as far as I can see therefore only superficial. Prigogine symphatizes and flirts, but does not try to incorporate their philosophical legacy in his physical theories.

- Bergson's (and Whiteheads) influence was conjectured to be far greater and

Prigogine's work might be percieved as rendering Bergson's ideas about duration and becoming more precise. Prigogine's use of these terms is thus non standard (McTaggart's sense). Prigogine is however not a Bergsonian in a literal sense, since he opposes Bergson's dualism. Prigogine wants to transform physics in order to unify our understanding of time.

- The unification of time was argued to be wanting, since it seems to confuse several issues about time. There is a tension between the philosophical and the scientific perspective in Prigogine's work. They can better be perceived separately. The burden of proof lies with the unificationst to argue in what ways these perspectives can be intertwined in a non-circular way.

These conclusions bring me to a first general criticism of Prigogine's thinking. The concepts being used remain vague, have a non standard meaning or are inconsistently used. This was illustrated of concepts as time's arrow, irreversibility, order out of chaos, the new dialogue with nature, and from being to becoming. This makes it rather difficult to assess Prigogine's position.

Two important questions were raised in this chapter. The first question is whether Prigogine succeeds in his attempt of deriving intrinsic irreversibility in providing an alternative to coarse-graining or interventionism, which is the main topic of chapter four. The second question is what the precise meaning is of taking irreversibility to be a fundamental fact at all levels. This question will be pursued in chapter five in relation to the reductionist assumption. In the next chapter I will first discuss the Kac ring model as a prelude to the physical theories of Prigogine and introduce a scheme for assessment.

Chapter 3

The Kac ring model

It is a mere accident that we have no memory of the future

–Bertrand Russell 1921

As a prelude to the physical content of the next chapter I would like to consider some paradoxes that can be illustrated with the Kac ring model due to Mark Kac (1959 [36, p.99]) see also Thompson ([77]) Schulman ([74]) and Bricmont ([21]).[1]

The model is shown in figure 3. A series of *n* points $P_1, P_2, P_3, ...P_n$ are arranged on a circle. Some of these points do, some do not, have a marker. There are *m* such markers. At the *n* halfway points between any neighboring points P_i and P_{i+1} there are *n* balls, some black, some white. These balls are moving together in elementary steps; in one such elementary step, all balls move simultaneously one unit counterclockwise in the circle; those who sweep across a marker change color, the others do not. The problem is to predict the color of the balls after *t* steps.

The Kac ring model has all the properties of a mechanical system which are essential here. First it is (at least it seems) reversible; if the balls are stopped at any moment and rotated clockwise they will sweep across the markers in an inverted time sequence, and the pattern will be the same as previously when they return to their original positions. Second, it has a Poincaré cycle: after two complete revolutions of the system, every ball will be at its starting point with its starting color. In order to write down the equations of motion for the system, let us denote by *B(t)* the number of black balls at time *t* and by *W(t)* the number of white balls. We have then obviously

$$B(t) + W(t) = n \tag{3.1}$$

[1] This section contains paraphrases of the treatments given by Thompson and Schulman.

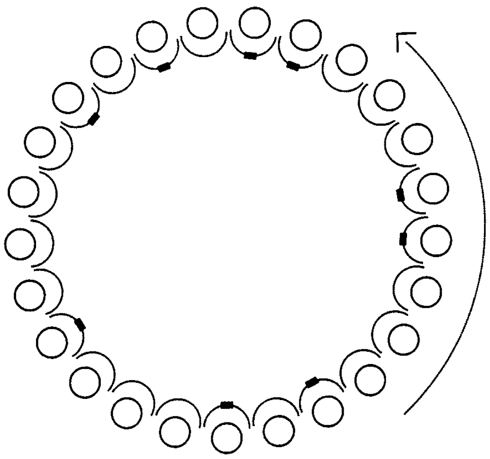

Figure 3.1: *Picture of the KAC ring model. Black or white balls move counterclockwise in single steps across points P_i; they change color if there is a marker at the point they cross. Ball color not shown.*

We further have to distinguish the black and white balls according to whether they have a marker ahead of them or not. We denote the number of ones that do by *b(t)* and *w(t)*. This definition entails a second 'conservation law'

$$b(t) + w(t) = m \tag{3.2}$$

The numbers *B* and *W* at the time $t + 1$ can be predicted from the definitions given as being

$$B(t+1) = B(t) - b(t) + w(t),\ W(t+1) = W(t) - w(t) + b(t) \tag{3.3}$$

We can solve the equations by making the natural assumption that the color of the ball and the property of having a marker ahead of it, are uncorrelated (the Stosszahlansatz) which can be expressed as

$$\frac{m}{n} \equiv \mu = \frac{b(t)}{B(t)} = \frac{w(t)}{W(t)} \tag{3.4}$$

Thereupon we have

$$B(t+1) - W(t+1) = (1 - 2\mu)\{B(t) - W(t)\} \tag{3.5}$$

and by iteration

$$B(t) - W(t) = (1 - 2\mu)^t \{B(0) - W(0)\} \tag{3.6}$$

This equation, also called the 'equilibration proof', is quite interesting because it seems in Thompson's words "to be both 'right' and 'wrong'". Since 1-2μ is a number between -1 and +1, the difference in the number of the two kinds of balls decreases exponentially towards zero i.e. we approach a situation in which there is the same number of balls of either kind, regardless of the number of white and black balls we start with and the number of markers. In other words, "the system goes gray exponentially rapidly, with oscillation if $\mu \geq 1/2$"(Schulman). However, this solution is wrong because it violates two earlier mentioned mechanical features of the model, namely of being reversible and of being periodic in time. In the original words of Kac, "the conclusion is clearly untenable because our model is completely reversible".

The intended analogy with Boltzmann's derivation of the H-theorem is obtained if we identify the color of each ball with the velocity of the particle, the two types of intervals can then be seen as different results of collisions; the velocity changes or remains constant. As the distribution of markers does not change over time, they can be seen as fixed scatterers. ([79] Uffink p.122)

3.1 Entropy in the Ring Model

What about the increase of entropy in this process? As a natural choice of a macroscopic[2] description, consider the parameter δ, defined as

$$\delta = \frac{B(t) - W(t)}{n} \tag{3.7}$$

It is a measure of greyness, corresponding to all balls being white if $\delta = 1$, $\delta = -1$ all black and $\delta = 0$ grey. Because of the discrete nature of the ring model, δ does not take a continuum of values, but is restricted to

$$\delta = \frac{2W - n}{n} \text{ for W = 0,..., n} \tag{3.8}$$

[2]The term 'macroscopic' is not unproblematical. It suggests the existence of collective or large-scale properties for a system. That there are such properties, that entropy could have a meaning even without an absolute definition of 'macroscopic', are central questions according to Schulman.

We can now define the entropy *S*, in light of our considerations in section 2.1 as[3]

$$S(\delta) \equiv k \log[\#\text{microstates consistent with } \delta] = k \log \binom{n}{W} \tag{3.9}$$

This expresses entropy as the number of (micro)states consistent with a certain macroscopic greyness i.e. the number of states of the balls consistent with having W of them white is the number of ways of choosing W objects out of *n*. The total number of microscopic available states to the balls of the system is 2^N. Using Stirlings's approximation, the definition of δ and equation 3.6 we see that the entropy $S(\delta)$ increases monotonically.

3.2 Statistical Physics

I will now turn my attention directly to the untenable conclusion we arrived at with the so-called 'equilibration proof'. Luckily we can also write down the exact equations of motion for the ring model. Hence we can examine our equilibration 'proof' that $\delta(t) \sim (1-2\mu)^t\delta(0)$. It is convenient to denote

$$\begin{aligned} \epsilon_i &= +1 \text{ if there is no marker at } P_i \\ \epsilon_i &= -1 \text{ if there is a marker at } P_i \end{aligned} \tag{3.10}$$

Further

$$\begin{aligned} \eta_i(t) &= +1 \text{ if the ball at the position preceding } P_i \text{ is black at time } t \\ \eta_i(t) &= -1 \text{ if the ball at the position preceding } P_i \text{ is white at time } t \end{aligned} \tag{3.11}$$

Taking the site-numbering to be counterclockwise, the time evolution of the system is given by

$$\eta_j(t) = \epsilon_{j-1}\epsilon_{j-2}\epsilon_{j-3}....\epsilon_{j-t}\eta_{j-t}(0) \tag{3.12}$$

which gives us the exact result

$$\delta(t) = \frac{1}{N}\sum_j \Big[\prod_{l=0}^{t-1}\epsilon_{p+l}\Big]\eta_p(0) \tag{3.13}$$

[3]The necessity for the logarithm is due to the extensive character that we demand of the entropy concept.

To proceed beyond this point, it is convenient to introduce an ensemble. We can think of different ensembles such as ensembles of variously defined objects, the number m of active sites, or of initial ball conditions.[4] Schulman (and also Thompson) choose an ensemble of ring models, all consisting of n balls, but differing in the distribution of the active sites.

As the site variables ϵ_i for this particular ensemble are random variables, we want to evaluate the following expectation value:

$$\Big\langle \delta(t) \Big\rangle = \sum_p \Big\langle \Big[\prod_{l=0}^{t-1} \epsilon_{p+l} \Big] \Big\rangle \eta_p(0) \tag{3.14}$$

If we assume that the marker distribution is random, then the ensemble average of any t consecutive factors ϵ_i is independent of p. Therefore the summation sign can be brought in front of the η's. We can now calculate the expectation value. The sum of the η's equals δ. Assuming that the we are dealing with uncorrelated variables we can calculate the ϵ average. For uncorrelated variables, the average of a product is the product of the averages, so that for example $\langle \epsilon_1 \epsilon_2 \epsilon_3 \rangle = \langle \epsilon_1 \rangle \langle \epsilon_2 \rangle \langle \epsilon_3 \rangle$. This gives

$$\begin{aligned} \langle \epsilon \rangle &= (+1)Pr(\epsilon = +1) + (-1)Pr(\epsilon = -1) \\ &= (+1) \times \frac{N-m}{N} + (-1) \times \frac{m}{N} \\ &= 1 - 2\mu \end{aligned} \tag{3.15}$$

Using an ensemble of rings with different number of active sites in which the ϵ's are independent random variables, we have reproduced the results of equation 3.6.[5] The equilibration 'proof' has been made into an honest proof. In Schulman's words:

[4]Gibbs' method consists of taking an ensemble over different initial states, but for the ring model this does not work, "because the dynamics are not rich enough"(Schulman p.69).

[5]Thompson [77] p.402 gives a more elaborate calculation of the $epsilon$ average. First he introduces the probability of having s markers among t points as:

$$\begin{aligned} p(s,t) &= \frac{t!}{s!(t-s)!\mu^s(1-\mu)^{t-s}} \; 0 \le t \le n \\ p(s,t) &= \frac{(2n-t!)}{s!(2n-t-s)!\mu^s(1-\mu)^{2n-t-s}} \; n \le t \le 2n \end{aligned} \tag{3.16}$$

We have to distinguish between the two situations. When t is between n and $2n$, a certain number of factors ϵ occurs twice and give 1. This plays an important role in resolving the recurrence paradox.

> "What we need for a real proof is thus independence of the variables $\{\epsilon\}$. Statistical mechanics then makes a virtue of necessity. Instead of claiming our equilibration assertion to be true for individual systems, it only makes the claim for ensembles of systems, entire collections of rings each with a different arrangement of active sites. For any particular ring there may be anomalous behaviour. But when arranged over the ensemble, the smoothed behavior prevails."([74, p.25] Schulman)

3.3 The Two Classical Paradoxes and Irreversibility

In chapter two we have already touched briefly on Zermelo's recurrence paradox and Loschmidt's reversibility paradox which were arguments against the validity of the Boltzmann equation. We can now use the ring model to illustrate and resolve these paradoxes.

3.3.1 The Recurrence Paradox

Recurrence for the Kac ring model is obvious. Start with all balls white and consider the state $2n$ times later. Every ball has been in every active site twice. Therefore it has switched color an even number of times and is now white again. No matter how you begin, you return to the original state. Isn't this in flagrant contradiction with the second law?

There seem to be then (at least) two ways of dealing with the paradox. The recurrence paradox can be resolved according to Schulman because of the following theoretical considerations. We have to realize that once $t \geq n$, some site variables ϵ_q will appear more than once in the product of the right hand side of equation 3.14 and hence give necessarily one. Strictly speaking the 'equilibration proof' has been turned into an honest proof only for $t < n$. Therefore mathematically the argument

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The ensemble average is now given as:

$$\langle \epsilon_1 \epsilon_2 \epsilon_3 ... \rangle = \sum_{s=0}^{t} (-)^s p(s,t) \tag{3.17}$$

which after substitution leads to:

$$\langle B(t) - W(t) \rangle = (1-2\mu)^{n-|n-t|} \{B(0) - W(0)\} \tag{3.18}$$

at the heart of the recurrence paradox breaks down and hence there arises no paradox in the first place.

Thompson's answer is based on practical considerations. If we think of *n* as being very large, say 'beyond the range of human experience' than the difference between $B - W$ dies exponentially away. Thompson's is in line with Boltzmann's original answer that in reality n is incredibly large[6], so there will occur no paradox in practice. Prigogine as we have seen in chapter two, agrees to Thompson's point of view.

3.3.2 The Reversibility Paradox

Let us consider next the reversibility paradox as quoted from Schulman. "Suppose you start the system in a state of less than maximum entropy; for example take 75 % of the balls to be white. Let it go for a moderate time *T*. It becomes grayer and has higher entropy. Now we run the inverse process. For the inverse dynamics the rules are slightly different: balls move *clockwise* and change color when they *enter* active sites, rather then when they leave. However, if you go back to our arguments on the time dependence of δ you will see that nowhere did we use those details. So δ should decrease, and entropy increase with either rule. Therefore after another *T* steps with the inverse process the system should be yet grayer, with yet higher entropy. But the inverse process brings us back to exactly the original state, 75 % white. Therefore the 'proof' of entropy increase cannot have been correct."([74, p.23])

We can write down exactly what happens when reversing. For the reverse process we have:

$$\eta_p(t) = \epsilon_p \eta_{p+1}(t-1) \tag{3.19}$$

So the following equation represents one step with the forward process and one step with the backward process:

$$\eta_p(t+1) = \epsilon_p \eta_{p+1}(t) = \epsilon_p \epsilon_p \eta_p(t-1) \tag{3.20}$$

The important fact is now that ϵ_p appears twice, and the square is always one. However, the 'proof' of equilibration assumed that the site variables were uncorrelated, as shown above. But when you get two particular ϵ's twice in a row this assumption breaks down, because "they *are* correlated, about as correlated as they can get"(Schulman p.26). The proof that δ increases for the reverse evolution does not

[6] Actually unimaginable huge. According to Kac, Boltzmann reacted to the Wiederkehreinwand with the words: 'wait for it then'!

hold, because it forbids the use of correlated site variables. Therefore, mathematically the argument at the heart of the reversibility paradox fails and in that sense Schulman claims that the paradox is resolved.

3.3.3 Have we explained Irreversibility?

It is fascinating to see how much disagreement there exists about the question whether the ring model involves (or illustrates) an explanation of irreversibility. Bricmont for example says:

> "It has many simplifying features (..) However, it has *all* the properties that have been invoked to show that mechanical systems cannot behave irreversibly, and therefore it is an exact counterexample that allows us to refute all those arguments (and to understand exactly what is wrong with them): it is isolated (the balls plus the scatterers), deterministic, reversible, has Poincaré cycles and is not ergodic.[7]"([21, p.42])

In Bricmont's opinion, the problem of time's arrow has already been solved by Boltzmann, as illustrated by the ring model. Also Thompson says the answer is yes:

> "This derivation is very instructive because it shows clearly the mathematical origin of irreversibility. Irreversibility arises because our answers are not strictly mechanical: to get the expected result, averages and limits must be taken in a way which is extraneous to an individual mechanical system. In the Kac ring model the average is taken over the markers, and the number *n* of members of the ring must be permitted to infinity. These processes are of such a nature that the original unbalance of black and white balls could never have arisen in the first place, except by external action outside of the rules of the model."([77, p.403])

Schulman's position is clearly different. The ring model does not at all show the mathematical origin of irreversibility.

[7] Ergodic theory is based on the idea to equate time average of an ensemble with the easier calculable phasespace average. There exist an hierarchy of mathematically defined properties, that can be predicated of systems, called the ergodic hierarchy, starting with ergodicity, via K-systems, to Bernouilli systems. The properties are either necessary or sufficient to predict the approach to equilibrium of the system involved. The ring model also nicely illustrates the role of ergodicity. As Bricmont notes,'although perfectly 'irreversible', [it] is not ergodic!' Indeed since it is periodic, no trajectory can 'visit' more than $2n$ microscopic configurations. But the 'phase space contains' contains 2^n configurations (two possibilities -black or white- at each site). So, only a very small fraction of the phase space is visited by a trajectory.

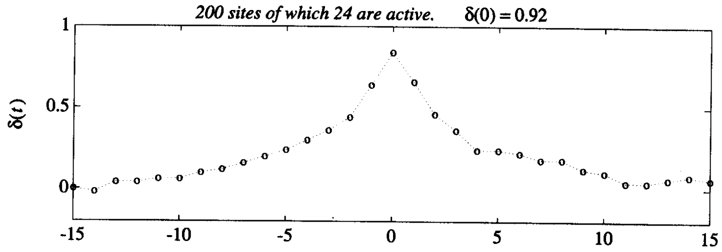

Figure 3.2: *Time symmetry. Moving in either direction away from the usual time-0 state (with over 90 % white), the entropy increases. The plots show a computer simulation in which both active sites and time-0 ball color were randomly selected, appropriately biased to give the indicated values ([74, p.28] Schulman.*

> "It is clear that all we have proved is, that if you take an unusual state and evolve from it in either time direction, its entropy increases."([74, p.27] cf.figure 3.3.3)

And therefore we have in no way derived the thermodynamic 'arrow of time'. How is it possible that there is so much controversy around the interpretation of the ring model?

One of the main disagreements is about how we have to understand the inverse operation of the reversibility paradox. Is it a purely mental operation, or an active real operation involving for example a twist of the ring? Bricmont seems to be in favor of the former interpretation since he writes that the ring model is isolated. Schulman is more in favor of the activist point of view. We will see in chapter four, that Prigogine (at least in his causal dynamics approach) also favors the activist point of view. He aims to eliminate Loschmidt's paradox by exploiting the necessity of an external influence in order to reverse. This external influence would involve a negative entropy flow from environment towards the system rendering the second law valid both before and after reversal.

But there is a more fundamental difference in point of view between Bricmont and Thompson on the one hand and Schulman on the other. As Schulman notes, "all we have proved is, that if you take an unusual state and evolve in either time direction, its entropy increases". Schulman's main observation is about the symmetry of the ring model between counter- and clockwise motion. Indeed, the direction of motion is inessential to obtain entropy increase. Schulman points basically to the more fundamental problem of the symmetry of statistical explanations. How can we possibly derive irreversibility from a symmetrical model? Thompson, for example, mentions that limits and averages have to be taken in order to derive irreversibility. But limits and averages are (time)symmetrical. Schulman's main point can be formulated as follows.

The reversibility paradox can be considered on two levels. At the surface level: given that we are evolving in either counter- or clockwise direction, we could reverse motion. Since our results do not depend on the direction of motion, we would have both an intropy increase and decrease as we return to our initial state. On a more fundamental level: if you take an unusual state and evolve from it in either direction of motion, its entropy increases.[8] The reversibility paradox conceived this way points to the symmetry of statistical explanations. This will be investigated in the next section.

3.4 Semigroups and Symmetry

In this section I will no longer follow Schulman, but develop further my own ideas regarding the symmetry of the Kac ring model and introduce and define the concept of a semigroup. Furthermore, I will scrutinize the claim that, as we have seen, the ring model is always assumed to be an example of a reversible model.

We have seen when discussing the reversibility paradox, that the dynamical laws of the clockwise motion are different from the counterclockwise motion. Indeed, using the same formulas as before, the same equations can be used, except that $t+1$ must be replaced with $t-1$. Strictly speaking the ring model would not be reversible judged by the form of the laws. However, most authors perceive this as a minor technical problem that can be overcome. As Bricmont notes "there is a small abuse here because changing the orientation amounts to changing the laws of motion (..) But I can attach another discrete 'velocity' parameter to the particles, having the same value for all of them, and indicating the orientation, clock- or counterclockwise, of their motion. Then, the motion is truly reversible, and the inversion operator *I* simply changes the velocity parameter"([21, p 38-9] cf. footnote 98). Bricmont proposal is thus to add a velocity parameter in order to have an invariant law of motion for both counter- and clockwise motion in the model.

[8]There might be a complication here. Given the symmetry of the model, it could be argued that there is no way of distinguishing between counter- and clockwise motion. They are enantiomorph processes. I will not delve deeper in this interesting substantivalist/ relativist discussion.

My own approach to the reversibility paradox is however different. First of all I note that in the reversed dynamics not only the law of motion has changed, but also the interpretation of the assumption of uncorrelatedness has changed. For the inverse dynamics the assumption is that there is no correlation between the color of the ball and the fact whether the point P_i *just passed* had a marker. Introducing an extra parameter conceals what is happening when reversing i.e. Bricmont's solution conceals the more fundamental reversibility problem. In order to express the latter, my strategy will be to show that the evolution equation of the Kac ring model, satisfies the properties of a semigroup. I will define a one-parameter semigroup first:

Definition A one-parameter semigroup on a phase space X is a family of bounded linear operators $T_t : X \to X$ parametrized by real $t \geq 0$ and satisfying the following linear relations:

- $T_0 = 1$
- If $0 \leq s, t < \infty$ then

$$T_s T_t = T_{s+t} \tag{3.21}$$

Claim: The evolution equation of the ring model satisfies the properties of a semigroup.

Proof: For the ring model we can define the phase space as the set of vectors $\eta = (\eta_1, \eta_2, ..\eta_n)$ consisting of 2^n points. Consider now our previous equation

$$\delta(t) = (1 - 2\mu)^t \delta(0) \tag{3.22}$$

It is of the form

$$f_t = T_t f_0 \tag{3.23}$$

in which I identify T_t with $(1 - 2\mu)^t$ and f_t with $\delta(t)$. Property (i) is evident. Property (ii) is easily seen to be fulfilled since $(1 - 2\mu)^{t_1}(1 - 2\mu)^{t_2} = (1 - 2\mu)^{t_1+t_2}$. As f is restricted to $-1 \leq f \leq 1$ (because δ is thus restricted), there exist in general no inverse element $T_t^{-1} = T_{-t}$, since in the inverse dynamics we divide and no longer multiply by T_t. $\square$

We see that the evolution equation for counterclockwise motion satisfies the properties of a semigroup. We can now inquire whether the same holds for clockwise motion. It is easily seen that there exists also a semigroup for clockwise motion but now restricted to negative values of t. Hence we get two semigroups, one for

$t > 0$ describing counterclockwise motion, and one for $t < 0$ describing clockwise motion. In line with Schulman's analysis we obtain for each direction of motion an evolution equation, described by a semigroup, showing entropy increase.

The question then of course arises whether we are then still justified in calling the ring model reversible? This depends of course on the definition of reversible that we advocate as discussed in section 2.4. If we perceive the model as an imaginary object, and velocity reversal as an imaginary operation, the model will be called reversible. If we judge strictly by the forms of the law, the model is irreversible (since velocity reversal changes the dynamics). If we follow Bricmont and introduce a velocity parameter, it would be judged reversible.

To establish an analogy with Prigogine's later ideas, I will advocate here Mackey's definition. Mackey chooses the following definition of irreversibility in *Time's Arrow: The Origins of Thermodynamic Behavior*

> "The difference between the definition of dynamical and semidynamical systems lies solely in the restriction of *t* and *t'* to values drawn from the positive real numbers, or the positive integers, for the semidynamical systems. Thus, in sharp contrast to dynamical systems, semidynamical systems are noninvertible and may not be run backwards in time in an unambiguous fashion (..) Situations like this are called irreversible in the physics literature, while mathematicians call them noninvertible."(1993 [42, p.3])

According to Mackey's definition of irreversibility, both motions of the ring model, since they are semidynamical systems are irreversible. A connection can then be made with Prigogine's later ideas about deriving intrinsic irreversibility.

3.5 Symmetry Breaking and Semigroup Selection

We have encountered above the steps of a heuristic that I will argue lies at the heart of Prigogine's theories aimed at intrinsic irreversibility. This heuristic I will call Symmetry Breaking and Semigroup Selection (SBSS):

- (1) First we have to give a symmetry breaking argument with two semigroups as result (in the ring model: the non-existence of an inverse element)

- (2) Secondly, we have to select one of the respective semigroups (argue against the physical relevance of the other semigroup)

The analogy with the work of Prigogine arises, if we identify counter- and clockwise motion with going in different time directions (past and future). A symmetry-breaking argument is accordingly given for certain types of systems (mostly dissipative systems or K-systems). These semigroups are interpreted by Prigogine respectively as describing the 'approach to equilibrium in the past and in the future'. Once we have two semigroups a selection argument is given why only one of these groups is realized. This semigroup then displays the intrinsic irreversibility of the system. The irreversibility is called intrinsical since it is a property of the dynamics alone of certain unstable systems.[9]

The advantage of using this heuristic in discussing irreversibility is that it can be used to distinguish between criticism directed at the viability of SBSS itself, and questions (assuming its validity) that apply within the framework of SBSS. The former evokes discussions about the philosophy of time, which I will take up in chapter five. In the latter, the strength of theories involving semigroups can be analyzed easily. What the BS basically needs to provide is thus two independent arguments: one in order to break symmetry, the second in order to choose one semigroup above the other. If these arguments can be reduced to one another, the theory is circular or begs the question.

SBSS will therefore be my main heuristic in assessing Prigogine's theories about intrinsic irreversibility in the next chapter.

[9] In the ergodic hierarchy, they are so-called K-systems. As the ring model is not ergodic, it is also not in Prigogine's sense 'intrinsically random' (this will be shown in the next chapter), and thus does not display 'intrinsic irreversibility' in Prigogine's sense. The analogy breaks down here.

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Chapter 4

Semigroups and Irreversibility

Time itself is meaningless, time is temporal

– Heidegger 1929[1]

In this chapter I will investigate Prigogine's physical theories in more detail, bearing the philosophical framework of the previous chapters in mind. My focus in this chapter will be on the relation between irreversibility, symmetry and semigroups. In order to sketch the development of Prigogine's theories concerning this subject matter, I will work in this chapter in a chronological order, though concentrating especially on the period between 1969-1985.

I will argue that there is not a single theory of irreversibility but that several different approaches have to be distinguished properly, that up till now have not been disentangled precisely in the literature. In a chronological order the approaches are: causal dynamics; new complementarity; entropy as a selector of initial conditions; an alternative to QM; rigged Hilbert spaces; embedding. The first three approaches is accordingly treated in a separate section divided in (1) the mathematical formalism (2) the interpretation of the formalism (3) the analysis within the framework SBSS.

I will argue first of all, that the development of Prigigine's thinking can indeed by assessed within the SBSS heuristic. Secondly I will show that some critics of Prigogine have conflated these different approaches. Finally I will argue that Prigogine so far has not been able to provide a satisfying answer to his core motivation of deriving irreversibility due to the intrinsic properties of the dynamics.

[1] The Concept of Time.

4.1 Causal Dynamics

The first approach that will be discussed is causal dynamics. This approach may be better known under the name subdynamics, although as I would like to demonstrate, subdynamics is one of the ingredients within the wider formalism of causal dynamics.[2] The formalism is put forward in two 1973 articles entitled *A Unified Formulation of Dynamics and Thermodynamics* (Prigogine, George, Henin and Rosenfeld Chemica Scripta 1973 [60]) and in *The Statistical Interpretation of Non-equilibrium Entropy* (Prigogine Acta Physicae Austriaca [59]) and is sometimes referred to as the PGH theory of irreversibility. In the latter article Prigogine introduces the notion of a "'causal" or "obviously causal" formulation of dynamics'([59, p.402]). A handicap in the study of causal dynamics is that the formalism is not developed in a single article, at a certain point in time, but in several articles in a period of time (roughly between 1966-1977) by several authors (the above + Grécos, Turner, Balescu) each using its own notation.

4.1.1 The Formalism of Causal Dynamics

For a good understanding of the formalism of causal dynamics the starting points of the approach have to be laid bare. The first consists in the use of the Koopman formalism of statistical mechanics. The second starting point lies in the incorporation of the approach developed by Nakajima and Zwanzig of using projection operator techniques in the derivation of the so-called generalized master equation. After these techniques have been introduced I will discuss what Prigogine calls the two main ingredients of causal dynamics: subdynamics and the transformation to a new physical representation. Finally, the dissipativity condition (and its importance) will be analyzed, and shown to be a necessary condition for symmetry breaking.

4.1.1.1 The Koopman Formalism

A theorem due to B.O.Koopman (1931 [40]) introduced the possibility of using Hilbert Space methods for studying classical (in particular, Hamiltonian) systems.

> **Koopman 1931** Let (Γ, μ, S^t) be an abstract dynamical system with μ invariant. For instance, Γ could be the energy surface Γ_E of a Hamiltonian system, μ the invariant measure with respect to S^t, guaranteed

[2]Karakostas, Obcemea and Brändas and Bishop call the formalism subdynamics. Balescu has developed an own self-contained formalism of subdynamics ([10] chapter 15-18).

by Liouville's theorem. S^t is a one parameter family of point transformations with group properties. Let L^2_μ be the Hilbert space of square integrable functions with respect to μ. Then S^t induces a *unitary* transformation $U_t : L^2_\mu \to L^2_\mu$ such that for any $\rho \in L^2_\mu$

$$(U_t\rho)(x) = \rho(S^{-t}(x))\text{for } x \in \Gamma \tag{4.1}$$

with U_t the Koopman operator.[3]

The generator H of the group U_t is called the Hamiltonian of the system

$$U_t = e^{-iHt} \tag{4.2}$$

One usually identifies physical observables (e.g energy) of the classical system with appropriate real-valued functions on the phase space. In close analogy with quantum mechanics we can associate linear operators on a separable Hilbert Space with physical observables. In the Hilbert space formulation of classical mechanics such observables are naturally represented by multiplication by corresponding functions. The expectation value of an observable, corresponding to an operator, can then be defined as $(\phi, F\phi)$.

The Liouville equation for the density matrix ϱ of an isolated system (L does not depend on t) can be converted into an operator equation on Hilbert space called the Liouville von Neumann equation (LvN).[4]

$$i\frac{\partial \varrho}{\partial t} = L\varrho \tag{4.3}$$

where L is the hermitian Liouville operator (or better a superoperator because it acts on the operator ϱ). The Liouvillian is in general an unbounded self-adjoint operator and for Hamiltonian systems (with H denoting the Hamiltonian) is given by

$$L = \frac{1}{\hbar}[H, \circ] \tag{4.4}$$

[3]Bishop notes, that there is a point of confusion regarding Koopman's formalism. The Koopman operator is defined on L^∞ spaces and not on L^2. 'The BS apply Koopman's formalism to L^2 without comment or justification'. But the justification does not seem to be problematic since the Koopman operator is unitary.

[4]The Von Neumann equation can be obtained either by quantizing the Liouville equation or by starting from a Schrödinger equation and applying statistics to a fictitious ensemble of wave functions. It may, therefore, be characterized as the general equation of motion in physics.

The physical interpretation of the Koopman formulation of the LvN equation is however the same as in classical mechanics, it describes the evolution of the distribution ρ as it moves through phase space from the perspective of a fixed observer.

Prigogine notes that the LvN equation has as its basic property $L-t$ invariance[5]: if we perform the operations

$$\begin{aligned} L &\to -L \\ t &\to -t \end{aligned} \tag{4.5}$$

the equation remains invariant. But macroscopic equations involving thermodynamic quantities such as the Fourier equation do not present such an invariance: their time symmetry is broken according to Prigogine. It will be the eventual aim of causal dynamics to break this $L-t$ invariance. The formal solution reads

$$\varrho(t) = e^{-iLt}\varrho(0) \tag{4.6}$$

A formal integral representation of ϱ is given in terms of a contour integral (actually a Laplace-transform representation) involving the resolvent of the evolution operator $exp(-iLt)$ as

$$\varrho(t) = \frac{1}{2\pi i}\int_{C^+} dz e^{-iz(t-t_0)}\frac{1}{L-z}\varrho(t_0) \tag{4.7}$$

The Laplace transform (also called a one-sided Fourier transformation) of a (sufficiently regular) function of time $f(t)$ is defined as:

$$\hat{f}(z) = \int_0^\infty dt e^{izt} f(t) \tag{4.8}$$

where z is a complex variable. The well-known inversion formula is

$$f(t) = \frac{1}{2\pi}\int_{C^+} dz e^{-izt} f(t) \tag{4.9}$$

where C^+ is a contour in the complex z plane, parallel to the real axis, and lying above all singularities of $\hat{f}(z)$ see figure 4.1.1.1.

[5]Karakostas claims that 'direct application of this symmetry shows that the usual expressions for nonequilibrium entropy in classical or in quantum mechanics are constants of motion.' I fail to see this connection. Rather, it seems that the vaildity of Liouville's theorem is essential here

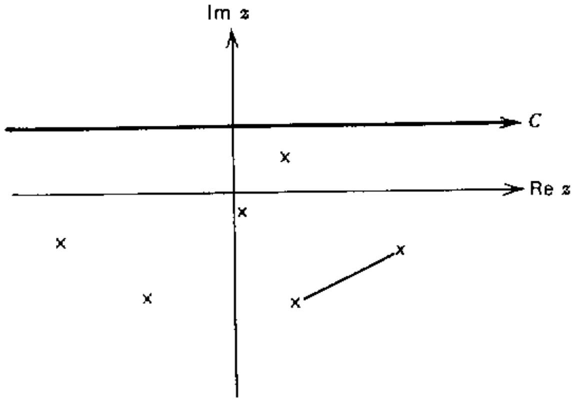

Figure 4.1: *Contour for the Laplace inversion formula*

4.1.1.2 The Nakajima-Zwanzig Approach

The time derivative of both $S(\varrho) = \int \varrho^2 dpdq$ and $S(\varrho) = -Tr\varrho \ln \varrho$ (fine-grained Gibbs entropy in the QM case) give zero due to the measure preserving character of the LvN equation. An important problem in statistical mechanics is therefore to obtain a satisfying derivation that would yield the desired approach to equilibrium.

The Nakajima-Zwanzig approach, developed independently by Nakajima (1958) and Zwanzig (1960) and developed in greater detail in *On the Identity of Three Generalized Master Equations* (Zwanzig 1963 [83]) is based on projection operator techniques. Projection operators are introduced which project onto the diagonal part of the density matrix, in order to decompose the full density matrix ϱ into the respective components ϱ_o and ϱ_c such that:

$$\begin{aligned} P + Q &= I \\ P &= P^2, Q = Q^2 \\ QP &= PQ = 0 \\ \varrho &= P\varrho + Q\varrho = \varrho_o + \varrho_c \end{aligned} \tag{4.10}$$

In the same way, the Laplace transform $\hat{f}(z)$ separates in diagonal and non-diagonal parts

$$\hat{f}(z) = P\hat{f}(z) + Q\hat{f}(z) = \hat{f}_o(z)\hat{f}_c(z) \tag{4.11}$$

Nakajima and Zwanzig use P and Q to separate the LvN equation in two parts

$$\begin{aligned} Pz\hat{f}(z) - P\hat{\varrho}(0) &= -iPL\hat{f}(z) \\ Qz\hat{f}(z) - Q\hat{\varrho}(0) &= -iQL\hat{f}(z) \end{aligned} \tag{4.12}$$

which gives

$$\begin{aligned} z\hat{f}_o(z) - \hat{\varrho}_o(0) &= -iPL\hat{f}_o(z) - iPL\hat{f}_c(z) \\ z\hat{f}_c(z) - \hat{\varrho}_c(0) &= -iQL\hat{f}_o(z) - iQL\hat{f}_c(z) \end{aligned} \tag{4.13}$$

They solve the second equation for $\hat{f}_c(z)$ and substitute the solution back in the first equation to obtain

$$z\hat{f}_o(z) - \hat{\varrho}_o(0) = -iPL\hat{f}_o(z) - iPL\frac{1}{z+iQL}\hat{\varrho}_c(0) - PL\frac{1}{z+iQL}QL\hat{f}_o(z)(S) \tag{4.14}$$

Inverting the Laplace transform we arrive at the integro-differential equation called the Generalized Master Equation (GME)[6]

$$i\partial_t\varrho_o(t) = PLP\varrho_o(t) + I(t) - i\int_0^t d\tau K(t-\tau)\varrho_o(\tau)\, t > 0 \tag{4.15}$$

where

$$\begin{aligned} I(t) &= PLPe^{-iQLQt}Q\varrho_c(0) \\ K(t) &= PLPe^{-iQLQt}QLP \end{aligned} \tag{4.16}$$

The restriction to $t > 0$ is directly related to the choice of the contour in the inverse Laplace transform. The generalized master equation is still exact and has two important properties. The first is the occurrence of a memory term $I(t)$ depending on the initial correlations $\varrho_c(0)$. It can be assumed that in most systems their effect should disappear as time goes on, and therefore $I(t)$ is often chosen as 0. The second

[6]Obcemea en Brändas note that “the starting point of [what they call] the theory of subdynamics is precisely the Nakajima-Zwanzig Generalized Master Equation related via inverse Laplace transform”(p.396). The generalized master equation derived by Prigogine and Résibois (1963), by Montroll (1961) and independently by Nakajima (1958) and Zwanzig (1960), were shown to be formally identical by Zwanzig (1964 [83]). Formally identical since NZ and PR use slightly different notations. PR use a second-quantized Hamiltonian, in the occupation number representation. Hence PR is obtained from NZ by replacing matrix subscripts (m, n) by their difference and their arithmetic mean.

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property is the non-local character of the collision term [last term] through which the time change of ϱ_0 at time *t* depends on the previous history of the system. This makes the generalized master equation non-Markovian (see appendix). The main problem is therefore to make the kernel $K(t-\tau)$ local in time, yielding an approach to equilibrium such as described for example by the Boltzmann master equation

$$\frac{\partial f(t)}{\partial t} = J(f(t)) \tag{4.17}$$

where f is a velocity distribution function and J is a collision operator whose exact form is unimportant for the moment.

Different approaches have been developed hereto by Van Hove and by Bogolyobov. The first uses the so-called Van Hove limit, which pictures the dynamics in the scaled time $t \to \lambda^2 t = \tau$. Problematic is then that the master equation holds not for all values of τ but only within some finite interval. Bogolyobov ensures the validity of a Boltzmann type of equation as follows: if there exists a clear cut separation in time scales, between the 'duration of the collision' t_{coll} and the 'relaxation time' t_{rel}, such that $t_{rel} \gg t_{coll}$ and if the memory term becomes negligible over times of the order of t_{coll} (short term correlations) the Boltzmann type of kinetic equation is recovered. However, Prigogine finds this derivation unsatisfying for two reasons. First although "the progress realized, in comparison with Boltzmann's original argument, is that the kinetic equation is now related to mechanics, the conditions under which this holds are quite restrictive ([60, p.10])". Secondly, the connection between the local second law of thermodynamics and the non-local master equation is difficult to establish.

The BS come therefore with a total different approach, described as 'the *construction* of a manifestly causal formulation of dynamics'. What this approach claims is that 'it is now possible to formulate NESM in a new form which combines causality and locality' in which 'causality is now incorporated into the differential equations (and not only as in the usual formulation in the integral representation of the solutions)'.[7] This approach requires two successive steps, the introduction of the concept of subdynamics and a transformation to the physical representation.

[7]Obcemea and Brändas claim that the aim of what they call subdynamics is to "derive the approach to equilibrium without recourse to any coarse-graining, time-smoothing or any loss of information (..) not by any limiting procedure or time-scaling, but by an "equivalence" relation. Irreversible dynamics is derived since the dynamical evolution is no longer described by a unitary group but by a semigroup evolution."([47, p.391]) It will be argued that Obcemea and Brändas conflate the causal dynamics approach with Prigogine's later ideas to obtain intrinsic irreversibility.

4.1.1.3 Subdynamics within Causal Dynamics

The article *Dynamical and Statistical Descriptions of N-Body Systems* (1969 [58]) is the first article where the concept of *independent-subdynamics*, as it is called there, is introduced. The later 1973 articles refer to this article when discussing subdynamics.

As we have seen in the Nakajima-Zwanzig approach, introducing the projection operators P and Q we can decompose the density matrix ϱ into the respective components ϱ_o and ϱ_c. The starting point of subdynamics is now to choose the projection operators such that $PH = H_0$ and $QH = \lambda V$, where we consider a system with a Hamiltonian H, which we decompose into an unperturbed part H_0 corresponding to the system's free motion, and a perturbation term V associated with the molecular collisions

$$H = H_0 + \lambda V \tag{4.18}$$

with λ a coupling constant associated with the strength of the interaction. For this reason $P\varrho$ is called the 'vacuum of correlations'.

Next Prigogine decomposes the resolvent of L into a sum of partial resolvents as[8]

$$\frac{1}{L-z} = [P + \mathcal{C}(z)]\frac{1}{PLP + \psi(z) - z}[P + \mathcal{D}(z)] + \frac{1}{QLQ - z}Q \tag{4.19}$$

where we have introduced the so-called collision operator,

$$\psi(z) = -PLQ\frac{1}{QLQ - z}QLP \tag{4.20}$$

the destruction operator

$$\mathcal{D}(z) = -PLQ\frac{1}{QLQ - z} \tag{4.21}$$

and the creation operator

$$\mathcal{C}(z) = -\frac{1}{QLQ - z}QLP \tag{4.22}$$

To proceed further, Prigogine separates what he calls the 'essential' singularity at $z = 0$ (denoted by a tilde) from the other singularities (denoted by a hat). Prigogine claims that we can identify, with the separation into two classes of contributions of

[8]For the tedious algebraic derivation I refer to Obcemea and Brändas [47, p.394-6].

the singularities, two operators of motion each satisfying the semigroup property (cf. equation 3.20) and accordingly separate the unitary operator U according to

$$\varrho(t) = \tilde{\varrho}(t) + \hat{\varrho}(t) = \tilde{\Sigma}(t)\varrho(0) + \hat{\Sigma}(t)\varrho(0) \equiv U(t)\varrho(0) \tag{4.23}$$

where $\tilde{\Sigma}$ is given by

$$\tilde{\mathbf{\Sigma}} \equiv \begin{pmatrix} e^{-i\theta t}(1+DC)^{-1} & e^{-i\theta t}(1+DC)^{-1}D \\ e^{-i\theta t}C(1+DC)^{-1} & e^{-i\theta t}C(1+DC)^{-1}D \end{pmatrix} \tag{4.24}$$

The (super)operators θ, C and D are closely related to the (super)operators $\psi, \mathcal{C}, \mathcal{D}$. The expression for $\tilde{\Sigma}$ is derived in the appendix.

Before I will continue the exposition of the formalism of causal dynamics, I will investigate the Σ operators, since there seems to be disagreement about their derivation and definition. In the original 1969 article of Prigogine, George and Henin (PGH) $\tilde{\Sigma}$ is derived in the asymptotic limit and is called therefore the asymptotic evolution operator.[9] It is claimed (1) that $\tilde{\Sigma}$ has the semi-group property and (2) that the contribution due to the $z = 0$ singularity is the only remaining for all times, but that $\tilde{\Sigma}$ can be defined for all times, together with some operator $\hat{\Sigma}$ which would include the effect of all other singularities. It remains unclear in this derivation whether $\hat{\Sigma}$ has the semigroup property as well.

Balescu and Wallenborn in *The time-evolution process in many-body systems* (1971 [9]) claim to have extended both semigroups Σ into full groups. The extension is due to the fact that they do not derive Σ using Laplace transforms but define Σ from the start. It is however unclear whether PGH ever subscribed to Balescu's analysis.

The best analysis of the properties of Σ I could find is given by Obcemea and Brändas which I will therefore briefly discuss. Their starting point is to explicitly perform the contour integration of the resolvent of L in equation 4.7:

> Bromwich Integral Theorem: one can evaluate the integral by closing the contour of integration so that the residue contribution due to the singularities of the reduced resolvent $(PLP + \psi_z - z)^{-1}$ is:
>
> $$\varrho(t) = 2\pi i \sum Res + \int cont. = \tilde{\Sigma}(t)\varrho(0) + \hat{\Sigma}(t)\varrho(0) \tag{4.25}$$
>
> where $\tilde{\Sigma}$ can be written as
>
> $$\tilde{\Sigma}(t) = (P+C)e^{-i\Theta t}A(P+D) \tag{4.26}$$

[9] In the appendix the '69 derivation is given.

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with

$$\Theta = \sum_n z_n \mid u_n\rangle\langle\overline{v_n} \mid$$
$$A = \sum_n \frac{1}{1-\mu_n'} \mid u_n\rangle\langle v_n \mid$$
$$D = \sum_n \mid \overline{u_n}\rangle\langle v_n \mid D(z_n)$$
$$C = \sum_n C(z_n) \mid u_n\rangle\langle\overline{v_n} \mid \tag{4.27}$$

where $\mid u_n(z)\rangle$ and $\langle v_n(z) \mid$ are defined in the spectral decomposition

$$PLP + \psi(z) = \sum_n \mu_n(z) \mid u_n(z)\rangle\langle v_n(z) \mid \tag{4.28}$$

Obcemea and Brändas now calculate the residue contribution directly using residue theory. First they close the contour of integration so that for $t > 0$

$$\frac{1}{2\pi i}\int dze^{-izt}\frac{1}{L-z}\varrho(0) = \Big\{\sum Res + \int cont.\Big\}\varrho(0) \tag{4.29}$$

Let

$$f(z) \equiv \psi(z) + PLP \tag{4.30}$$

then

$$Res(z - f(z))^{-1} = \lim_{z\to z_k}(z - z_k)(z - f(z)^{-1}) \tag{4.31}$$

They then make a Taylor expansion about the singular point $z - z_k$, calculate the residue contribution at the poles z_k and use the Bromwich theorem to obtain

$$\tilde{\Sigma}(t) = \sum_k (P + C(z_k))(1 - \mu_k'(z)_k)^{-1} \mid u_k(z_k)\rangle\langle v_k(z_k) \mid (P + D(z_k))e^{-iz_k t} \tag{4.32}$$

Obcemea and Brändas state that 'it is clear from this derivation that $\tilde{\Sigma}(t)$ which is the residue part, obeys a semigroup'. Obcemea and Brändas note that the singularity z_k in the second Riemann sheet has a negative imaginary part so that expression 4.32 shows an exponential decay iff the collision operator $\psi(+i0) \neq 0$. I will return to this observation when discussing the dissipativity condition.

This ends the analysis of the semigroup property according to Obcemea and Brändas. Note (1) that we did not perform an asymptotic limit to obtain $\tilde{\Sigma}$ and (2) that we did not derive the semigroup property for $\hat{\Sigma}$, whereas Karakostas and Prigogine 1973 claim that it does have semigroup properties. I consider it a drawback that a property that is so crucial in the formalism has been given such little attention.

The next step in the formalism is to introduce two operators, $\tilde{\Pi}$ and $\hat{\Pi}$ which are defined as the limit $t \to +0$ of $\tilde{\Sigma}(t)$ and $\hat{\Sigma}(t)$. Important properties of Π are

- $\Pi^2 = \Pi$ idempotent, but not hermitian $\Pi^\dagger \neq \Pi$
- $\Pi L = L\Pi$ commutes with the Liouville operator

Because of the first two properties Π can be seen as a projection operator, such that the projected evolution is stable under L. This is why the evolution constitutes a subdynamics since

$$\tilde{\varrho}_t = \tilde{\Sigma}(t)\varrho(0) = \tilde{\Pi}e^{-iLt}\varrho(0) = e^{-iLt}\tilde{\Pi}\varrho(0) = e^{-iLt}\tilde{\varrho}(0) \tag{4.33}$$

$\tilde{\Pi}$ is orthogonal to $\hat{\Pi}$ which allows the decomposition of the dynamics into two completely separated subdynamics associated with the mutual orthogonal subspaces $\tilde{\Pi}$ and $\hat{\Pi}$.

The relation between P and Π is clarified by Obcemea and Brändas as follows (NB: this observation cannot be found in any article of Prigogine on the matter):

$$\lim_{\lambda \to 0} \Pi = P \tag{4.34}$$

Using the explicit form of the operator Π we see that the correlations in $\tilde{\Pi}$ appear as linear functionals of the vacuum component

$$\tilde{\varrho}_c(t) = C\tilde{\varrho}_o(t) \tag{4.35}$$

Similarly in the $\hat{\Pi}$ subspace, the diagonal elements can be shown to be functionals of the correlations

$$\hat{\varrho}_o(t) = D\hat{\varrho}_c(t) \tag{4.36}$$

It becomes then natural to use in the decomposition of the complete density matrix as independent elements the 'priviliged' components $\tilde{\varrho}_o$ and $\hat{\varrho}_c$. It is now the purpose of transformation theory to transform to the so-called physical representation in which these priviliged components satisfy separate equations of motion.

4.1.1.4 Starunitary Transformation Theory

The second ingredient in causal dynamics is called by Prigogine starunitary transformation theory. A superoperator $Q(L)$ is called star-hermitian (denoted by *) iff invariant under the product

$$\text{(adjunction AND inversion } L \rightarrow -L) \tag{4.37}$$

The star-hermiticity of an operator can therefore be realized in (at least) two different ways: the operator may be either

$$\text{(hermitian) AND (even in } L) \tag{4.38}$$

or

$$\text{(antihermitian) AND (odd in } L) \tag{4.39}$$

As will be shown these two realizations of star-hermiticity precisely enable us to distinguish between both dissipative and non-dissipative systems.

Prigogine introduces two further superoperators χ_o and χ_c in the P and Q superspaces respectively[10]

$$\begin{aligned} \chi_o \chi_o^* &\equiv P\tilde{\Pi}P = (1 + DC)^{-1} \\ \chi_c \chi_c^* &\equiv Q\tilde{\Pi}Q = (1 + CD)^{-1} \end{aligned} \tag{4.40}$$

The various components χ_o, χ_c constitute particular elements of an operator called Λ. Since $\Pi = \Lambda P \Lambda^{-1}$ we obtain:

$$\mathbf{\Lambda} \equiv \begin{pmatrix} P\Lambda P & P\Lambda Q \\ Q\Lambda P & Q\Lambda Q \end{pmatrix} = \begin{pmatrix} \chi_o & -D\chi_c \\ C\chi_o & \chi_c \end{pmatrix} \tag{4.41}$$

The importance of the operator Λ is that it effects a transformation of the original density matrix satisfying the LvN equation to the so-called physical representation (denoted by the *p* superscript)

$$\varrho^p = \begin{pmatrix} \varrho_o^p(t) \\ \varrho_c^p(t) \end{pmatrix} \tag{4.42}$$

[10] The need for introducing a further operator χ is a direct consequence of the fact that the evolution operator θ which appears in the structure of Σ does not possess well-defined hermiticity properties, and hence may have imaginary eigenvalues. According to Karakostas most of the difficulties encountered in the BS approach in defining a nonequilibrium entropy beyond the 'Boltzmann approximation' may be traced to this fact. On the contrary the operator χ can be shown to satisfy definite symmetry properties with respect to star-conjugation operation.

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whose elements can be expressed as linear combinations of the initial priviliged components

$$\varrho_o^p = \chi_o^*(1 + DC)\tilde{\varrho}_o, \quad \varrho_c^p = \chi_c^*(1 + CD)\hat{\varrho}_c \tag{4.43}$$

through a star-unitary transformation

$$\varrho^p = \Lambda^{-1}\varrho \tag{4.44}$$

The gain in going to the physical representation resides in the fact that the original LvN equation is decoupled (or block-diagonalized) into

$$\begin{aligned} \frac{\partial \varrho_o^p}{\partial t} &= -i\Phi_o \varrho_o^p \\ \frac{\partial \varrho_c^p}{\partial t} &= -i\Phi_c \varrho_c^p \end{aligned} \tag{4.45}$$

with the new generator of motion

$$\Phi \equiv \Lambda^* L \Lambda \tag{4.46}$$

As both representations are considered equivalent under transformation Λ it is natural to require that all average values of observables should be kept invariant in going to either representation i.e.:

$$\langle O \rangle_t = TrO^\dagger \varrho = Tr(O^p)^\dagger \varrho^p \tag{4.47}$$

Using the Heisenberg evolution equation of observables, this amounts to requiring

$$\Lambda^{-1}(L) = \Lambda^\dagger(-L) \tag{4.48}$$

which defines a star-unitary transformation.[11] For a star-unitary transformation the inverse of the initial transformation is equal to its star-hermitian conjugate: $\Lambda^{-1} = \Lambda^*$.

The dynamical operator $i\Phi$ has the following important properties (i) starhermiticity (due to the star-hermiticity of Λ) (ii) the hermitian part is semi-positive definite i.e. $(i\Phi^e) \geq 0)$. The second property is responsible for irreversible behaviour as will be scrutinized later on.

[11]The Heisenberg evolution equation of observables $i\frac{\partial O(t)}{\partial t} = -LO(t)$ differs from the Liouville equation of states by a replacement of *L* by *-L*. We require thus for an observable *O* that $O^p = \Lambda^{-1}(-L)O$.

The essential new aspect according to the BS, is that the transformation to the causal representation breaks the $L-t$ invariance of the LvN equation. To see this remember that for the initial representation we have

$$\frac{\partial \varrho}{\partial t} = -iL\varrho \tag{4.49}$$

whereas in the p-representation

$$\frac{\partial \varrho^p}{\partial t} = -i\Phi\varrho^p \tag{4.50}$$

It is easily seen that both generators $(iL) = (iL)^*$ and $(i\Phi) = (i\Phi)^*$ are star-hermitian.

Let's return to the inverse Laplace transformation. According to Prigogine, "causality implies that the contour has to be traced in the upper half plane of the complex variable z (i.e. for Im $z > 0$). Equivalently the choice of such a contour corresponds to the solution of an initial value problem (..) with which we calculate the *retarded* solution of the Liouville equation"([60, p.7]). Hence the name causal dynamics. The $L-t$ invariance is reflected in the fact that we can likewise construct an advanced solution by tracing the contour in the lower half plane in the integral representation. This claim will also be scrutinized later on.

But whereas the $L-t$ invariance in the ordinary representation stems from the fact that (iL) is antihermitian and of course odd under L-inversion, the $(i\Phi)$ operator can be decomposed into an even and odd part with respect to L-inversion. As a consequence, the differential equations obeyed by the retarded solution ϱ_r^p and the advanced solution ϱ_a^p are different. This is in contrast with the traditional transformation theory such as the unitary transformation theory of quantum mechanics, in which no distinction is made between the advanced and the retarded solutions. To see this, note that because of $L-t$ invariance for the LvN equation time inversion is equal to L inversion, we have

$$\varrho^a = \Lambda(-L)\varrho^{a,p} \tag{4.51}$$

Herewith we see, where ϱ denotes a supervector again and $\Phi' = \Lambda^{*'}L'\Lambda' = -\Lambda^\dagger L\Lambda'$

$$\begin{aligned} \frac{\partial \varrho_r^p}{\partial t} &= -i\Phi\varrho^{r,p} = -i(\Phi^e + \Phi^o)\varrho^{r,p} \\ \frac{\partial \varrho_a^p}{\partial t} &= i\Phi'\varrho^{a,p} = -i(-\Phi^e + \Phi^o)\varrho^{a,p} \end{aligned} \tag{4.52}$$

Hence the symmetry of the Liouville equation is broken, if and only if the dynamical operator Φ contains an even part in L.[12] This is precisely so when the dissipativity condition (DC) is fulfilled.

> "The basic progress that has been achieved is that the evolution of the system *including causality*, may now be discussed at the level of the differential equations, without the introduction of non-local effects as they appeared originally in the master equation. It is in this very precise sense that we have obtained a new causal formulation of dynamics"(BS [60, p.16]).

4.1.1.5 Dissipativity Condition

Different authors seem to have struggled with DC. I have found three different definitions of DC in the work of Prigogine:

- DC_1: $\psi(+i0) \neq 0$ (ibid p.10). Obcemea and Brändas interpret this dissipativity condition as expressing that the collision operator is required to be non-hermitian.
- DC_2: $(i\Phi^e) \neq 0$ (ibid p.15). Karakostas does not question $(i\Phi^e) \neq 0$ and identifies $(i\Phi^e)$ simply as the irreversible part and $(i\Phi^o)$ as the reversible part of the evolution equation of causal dynamics ([37, p.391]).
- DC_3: $(i\Phi^e) > 0$ (*From Being to Becoming* [61, p.186]) Prigogine describes this version of the dissipativity condition as the condition which expresses the existence of a Lyapounov function Ω.

I will now argue that these three definitions are not equivalent.

From the definition of the collision operator equation 4.19 it follows that[13]

$$\psi(x+iy) = -PLQ\frac{QLQ-x}{(QLQ-x)^2+y^2}QLP - iPLQ\frac{y}{(QLQ-x)^2+y^2}QLP \tag{4.53}$$

The second term has a definite sign, namely < 0 for $y > 0$ and > 0 for $y < 0$. The second term vanishes in the limit $y \to 0$ if the spectrum of L is discrete, i.e. in a finite system. However if we perform the thermodynamic limit $N \to \infty$ and

[12] Note that the starunitariness of the Λ transformation is crucial to uphold the distinction.

[13] Kramer [41, p.316]; Prigogine [59, p.421-5].

$V \to \infty$ with $N/V = const$ before taking the limit $z \to +i0$, part of the spectrum of L will become continuous and the second term may not vanish. Then we obtain DC_1 (taking $x = 0$ for convenience)

$$\psi(+i0) \neq \psi(-i0) \tag{4.54}$$

which is called a dissipativity condition because the real part of ψ will lead to oscillatory solutions, whereas an imaginary contribution can lead to damping or growing solutions.

Secondly, we remark that the first term in equation 4.53 is odd in L and the second is even in L. We may decompose the collision operator therefore in an even and odd part with respect to L inversion. But it is not clear why this would imply that we also may decompose Φ likewise, since the relation between Φ and ψ still has to be established. DC_2 is not implied by DC_1. Sometimes it is simply claimed that "we expect on phenomenological grounds that Φ has an even part in L".

The biggest problem however lies in DC_3. In order to break symmetry we need DC_2. But from DC_2 we of course cannot infer DC_3. The support for DC_3 comes from the fact that $(i\Phi)$ is claimed to be semi-definite positive. But if we ask why this is so we find that this is a consistency test!, 'to make the distinction between retarded and advanced solutions meaningful'. Nor follows DC_3 from DC_1 as is claimed. 'To lowest order [DC_3] reduces to the requirement that the collision operator is semi-definite positive: $i\psi(0) \geq 0$. This can easily be checked' it is claimed because $i\psi(0)$ has even part in L, but this is of course different from DC_1. We see that DC_3 is not warranted by physical reasons such as DC_1 but is simply required. It is not surprising that in the next formalism that will be analyzed, the existence of a Lyapounov variable will be imposed from the start.

When is a system dissipative? In *Dissipative Properties of Quantum Systems* (1972 [34]) Grecos and Prigogine address this question. For quantum systems a necessary condition for dissipativity is the occurence of a continuous spectrum, i.e. in general the limit of a large system. For classical systems the situation is different, because the Liouville operator may have a continuous spectrum even for a system with a few degrees of freedom. Thus for classical systems dissipative phenemena may appear even for small systems. DC_1 is clearly a property of certain systems.

4.1.2 The Interpretation of Causal Dynamics

In the previous section we have stated and disentangled the difficult formalism of causal dynamics. In this section the interpretation that is given by the BS of the results of causal dynamics will be discussed. The focus will be at two important elements. The formalism of causal dynamics can be used to define a nonequilibrium entropy function that can be contrasted with the original Boltzmann H-quantity. This will be discussed in relation to the velocity inversion experiment. The second point of interest lies in the interpretation of subdynamics, where we will analyse the meaning of the Π orthogonal subspaces. The question is addressed in what sense this interpretation differs from coarse-graining.

4.1.2.1 Loschmidt's Paradox Eliminated

The specific problem that is investigated in the Chemica Scripta article is to develop a meaningful account of the velocity inversion experiment, which we have already encountered in our review of the Kac ring model. The BS want to give an unambiguous answer to the problem of velocity inversion, which will render the second law valid both before and after velocity inversion. The idea of the BS is that a statistical expression of entropy is needed, different from H_B, which depends explicitly on the correlations that are formed when reversing.

In order to derive this expression of entropy the BS first proves that in the physical representation the following quadratic functional of the density matrix

$$\Omega = Tr(\varrho^p)^\dagger \varrho^p \tag{4.55}$$

can only decrease in time.

Proof:

$$\begin{aligned}\frac{d\Omega}{dt} &= Tr(-i\Phi\varrho^p)^\dagger \varrho^p + Tr(\varrho^p)^\dagger(-i\Phi\varrho^p)\\ &= -Tr(\varrho^p)^\dagger(i\Phi + i\Phi')\varrho^p\\ &= -2Tr(\varrho^p)^\dagger(i\Phi^e)\varrho^p \leq 0\end{aligned} \tag{4.56}$$

In the second step the star-hermiticity of $(i\Phi)$ has been used. In the third step the odd parts of $(i\Phi)$ and $(i\Phi')$ cancel out so that only the even part enters in the evolution. Finally the semi-definite postiveness of $(i\Phi)$ leads directly to the desired result. □

Equilibrium will be reached when Ω takes its minimum value. We conclude that the proof is a direct consequence of DC_3.

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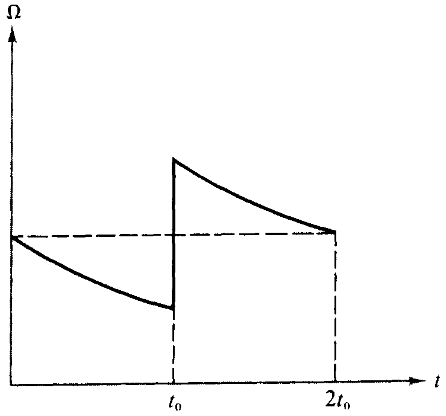

Figure 4.2: *Evolution of H with time when $\varrho_o(0) = I\varrho_o(0)$ and L is inverted at t_0 and $2t_0$ (isotropic initial velocity distribution and velocity inversion at t_0 and $2t_0$*

Here upon Prigogine introduces an entropy definition based on Ω, "which has a generality comparable to that of the phenomenological formulation of the second law"([60, p.19]), with the corresponding H-quantity as:

$$H - H_{equil} = \frac{1}{2}\log\frac{Tr(\varrho^p)^2}{Tr(\varrho^p)^2_{equil}} \tag{4.57}$$

and entropy (with *k* a constant, and ν the number of accessible states)

$$S - S_{equil} = -\frac{k}{2}\log[\nu Tr(\varrho^p)^2] \tag{4.58}$$

Prigogine concludes that the above model for entropy is automatically in agreement with linear non equilibrium thermodynamics. Secondly, the effect of correlations is included in such a way that the entropy is not an invariant of motion such as the Gibbs entropy (for details see ibid p.17-20).

The BS now apply their definition of S to the velocity inversion experiment, in order to test their theory. In agreement with the earlier work of Prigogine (chapter two) the entropy change is decomposed into an entropy production and an entropy flow. Introducing an inversion superoperator I, which supposedly anticommutes with L and is involutive ($I^2 = I$), the BS tries to demonstrate the validity of their nonequilibrium entropy definition as is illustrated in figure 4.1.2.1. The BS interprets this result as follows:

> "The entropy production will occur over the whole period of the cycle between 0 and $2t_0$. The entropy *flow* is associated here with the effect of the inversion operator I, and occurs in a discontinuous way at time t_0 (between t_{0-} and t_{0+}) and at time $2t_0$. The sign in 2.4 of the entropy received by the system is negative, i.e. the system will give to the external world more entropy than it receives"(ibid p.20).

The relation with our discussion of the reversibility paradox concerning the Kac ring model is clear. Prigogine is in favor of an active interpretation of reversal. 'The entropy flow is realized through the work done by external agents..'. There would be no antithermodynamical behaviour any more, since the entropy production of the system is always positive. It is compensated by the entropy flow at t_0 associated with the action of the inversion operator I. Even for velocity distributions that are not symmetric at the initial time $t = 0$, these observations hold. The change in H can likewise be shown to decrease, whereas H_B does not show an decrease for the velocity inversion experiment. Prigogine interprets these results as follows: "We believe that we have given an answer to the question of the 'mechanical interpretation of the second law', which has motivated Boltzmann's basic investigations"(ibid p.23). "We have in fact eliminated Loschmidt's paradox."(ibid p.24)

Two remarks are in place. It is questionable to claim that there is work done in reversing. The communis opinio is that this claim is untrue, since we can think of elastic scattering. Secondly, an active interpretation of reversal which results in an entropy flow seems to contradict the involutive property of I. Moreover, it is claimed that there is no discontinuity at $2t_0$. But then the effect of I would be dependent on the state it acts on. This asymmetrical assumption is obviously questionable.

But at the same time Prigogine gives a second and different explanation of the velocity inversion experiment, due to 'the molecular origin of the entropy decrease due to the inversion operator'. In the language of subdynamics, the inversion brings the systems out of $\tilde{\Pi}$ resulting in a component in the $\hat{\Pi}$ space. Therefore 'an *extension of the space of thermodynamic observables is necessary*'.

Both explanations seem to conflict in an important way. The first analysis presupposes a distinction between the system on the one hand and its environment on the other. The work done by an external disturbance is crucial. The second explanation is internal because it only resorts to subdynamics stressing the molecular origin of irreversibility. Important in the conflict is therefore the status of the $\hat{\Pi}$ subspace. This will be discussed in the next section.

4.1.2.2 Subdynamics: an Alternative for Coarse-graining?

In the 1969 article [58] where subdynamics is developed, Prigogine discusses the interpretation of what he calls the Π manifolds. The decomposition has a 'deep physical meaning' according to Prigogine. To see this he notes that the $\tilde{\Pi}$ manifold has 'most remarkable properties'. For instance all invariants such as the total energy lie in this manifold (a result by Turner), since

$$\tilde{\Pi}H = H \text{ and } \hat{\Pi}H = 0 \tag{4.59}$$

Prigogine therefore concludes that the energy is entirely 'localized' in $\tilde{\Pi}$. The information about the motion included in the complementary space $\tilde{\Pi}$ concerns only phase relations. The evolution in $\tilde{\Pi}$ leads to thermodynamic equilibrium, called the coherent part, and $\tilde{\Pi}$ describes the incoherent part, due to initial conditions which are destroyed by the dynamical evolution. In the end this component vanishes in agreement with taking the asymptotic limit.

Prigogine notes that when setting up an initial value problem (by imposing the set of observables at $t = 0$), we are in general not allowed to use the dynamical operator $\tilde{\Sigma}$ alone, but we have to use the complete operator U. Hence Prigogine states, we need a supplementary asymptotic element which would permit us to neglect the contribution of $\hat{\Pi}$. What does he have in mind? As an example Prigogine considers scattering:

> "We set up an wave packet at t_0. The evolution of this wave packet involves indeed the complete dynamical operator U. We are, however generally interested only in results which do not depend on the exact specification for the wave packet. To extract these results we reject the initial conditions to $t_0 \to -\infty$"([58, p.429]).

This argument deserves greater attention. Prigogine rejects on fore hand certain initial conditions because of reasons over and above the dynamics alone. These reasons are grounded in the fact 'that we are generally not interested in them'. This type of argument will be used more explicit in later approaches (as will be seen subsequently) when it will be developed into a selection argument of one semigroup instead of the other. Precisely at this crucial point (neglecting the $\hat{\Pi}$ subspace development) the subdynamics approach has received extensive criticism by Obcemea and Brändas (1983) and later by Karakostas (1996).

> "In [the BS] view, equilibrium is attained when the $\Pi\varrho$ component reaches the canonical distribution in its subspace, provided that 'the projection

> of ϱ into the complementary subspace gives a vanishing contribution'. To say however, that certain correlations (..) vanish is, at best, to make a statement about the average values of microscopic variables. The probabilities involved are the ordinary probabilities of statistical mechanics (as opposed to irreducible quantal probabilities) reflecting our ignorance or lack of interest in microscopic details. Thus the statement that the incoherent (..) component vanishes simply means that the corresponding information disappears from our description of the system. Therefore we regard the projector Π as being equivalent to a form of coarse-graining, because it provides a contracted description of the system, in that only the $\Pi\varrho$ portion of the information available in the full ensemble ϱ can be shown to exhibit irreversible behavior. A complete description would also require the evolution of the $\hat{\Pi}$-subspace."(Karakostas [37, p.384])

What can Prigogine argue against these accusations? Remarkably Prigogine admits (1969) that causal dynamics is a kind of coarse graining:

> "What the great founders of statistical mechanics had tried to achieve through the introduction of some *coarse-grained* distribution function in addition to a *fine-grained* distribution function is in fact automatically realized by the dynamics of the N-body system. The projection of ρ on the subspace $\tilde{\Sigma}$ is indeed a kind of coarse-grained distribution which carries all the information about the thermodynamical behaviour of the system"([59, p.420])

How is the misdirected coarse graining accusation of both Obcemea and Brändas and Karakostas understandable? What I want to conjecture is (for reasons that will become more clear after the second formalism has been assessed) that (1) different formalisms have to be distinguished (2) Obcemea and Brändas and Krakostas treat these formalisms as one and the same (3) in the later formalism Prigogine aims to derive intrinsic irreversibility (4) Obcemea and Brändas and Karakostas think that this claim also holds for causal dynamics (5) they criticize CD for coarse-graining. In support of this hypothesis is that (a) Obcemea and Brändas and Karakostas do not refer to the '69 article and probably have not read it (b) Prigogine himself confusingly seems to claim that both approaches are in support of each other, and hence (indirectly) that also subdynamics is an alternative for coarse graining. As we will see, in the fourth approach (late eighties), in which subdynamics is explicitly used, Prigogine claims to have obtained intrinsic irreversibility.

A second point to distill from the quote is the claim that there is no-information-loss. This is supported by the fact that Λ is an equivalence map and has an inverse. In the next approach this no-information loss-argument and the construction of a different non-unitary operator Λ comes to the foreground and will be the main argument in claiming an alternative for coarse-graining. The appeal to subdynamics will be abandoned.

Let us conclude so far that CD is effectively a form of coarse-graining. Before I will turn my attention to the next approach, I will sum up my observations so far regarding the causal dynamics approach within the SBSS heuristic.

4.1.3 Analysis within SBSS

In this section I want to summarize the causal dynamics approach within SBSS. The causal dynamics approach to irreversibility can be summarized concisely:

- (1) DC is a necessary condition for symmetry breaking. Symmetry breaking is exemplified, as the transformation (assuming the validity of causal dynamics) to the phsyical representation is different for the retarded and the advanced solutions of the Liouville equations

- (2) In dissipative systems a thermodynamic entropy can be associated with systems whose time evolution can be represented by a genuine semigroup. Causality requires us to select the retarded solution.

Regarding the first point, the following quotes illustrate the necessity of DC

> "For large isolated dynamical systems (and especially in the formulation of the laws of dynamics in some asymptotic limit[14] a central role in the symmetry breaking is played by the dissipativity condition.(..) Briefly speaking, this condition determines if the analytical continuation of the resolvent (..) defined in the upper half plane, has singularities in the lower half plane. It is only when the dissipativity condition is satisfied that differences appear between the retarded solution (which contains the effect of singularities in the lower half plane) and the advanced solution (corresponding to singularities in the upper halfplane)."([60, p.6])

[14] E.g. the limit of a large volume or the thermodynamic limit.

> "To lowest order the condition reduces to the requirement that the collision operator is semi-definite (..) and this is an absolute necessary requirement (..) in order to make the distinction between the retarded and the advanced solutions meaningful."(ibid p.16)

It was argued however that Prigogine does not properly distinguish between three different versions of DC. The result of this is that the proof of entropy increase is fallacious. It is important to note that in the next approach that will be discussed, Prigogine no longer attempts to derive an entropy decrease (based on DC_3) but simply postulates DC_3. In other words the existence of a Lyapounov variable is imposed.

Regarding the selection of the retarded solution, instead of the advanced solution, Prigogine has given different arguments in the course of years. I will discuss three of them. The first is a transcendental argument (made famous by Kant) due to Rosenfeld:

> Let us repeat a statement made forcefully by Rosenfeld: 'The inclusion of a specification of the conditions of observation into the account of the phenomena is .. an indispensable part of their objective description'. It is this account which introduces the distinction between retarded and advanced solutions. As we suppose that the observer may make this distinction it belongs to the type of primitive concepts which we have to introduce in our theoretical scheme at the start and which permits us to select the retarded form of causal dynamics. ([59, p.44])

The problem with this argument is that it rests upon a specific concept of causality related to the human condition which can be compared with for example the anthropic principle. The names 'causal dynamics' or 'the physical representation' may indicate that Prigogine sees the choice of retarded solutions self-evident and hence without need of explanation but this is not easy to maintain.

Prigogine's second argument seems to be that causality imposes us to solve an initial condition problem and hence to select the retarded solution, just as in electrodynamics. But the analogy with electrodynamics is fallacious.

In electrodynamics the wave equation (which can be derived from the Maxwell equations) is given by

$$\partial^{\nu}\partial_{\nu}A^{\mu}(r,t) = -4\pi j^{\mu}(r,t) \tag{4.60}$$

which is a time reversal invariant equation. The most general solution is given by

$$A^{\mu}(r,t) = c_1 A^{\mu}_{ret}(r,t) + c_2 A^{\mu}_{adv}(r,t) + A^{\mu}_{boundaries}(r,t),\ c_1 + c_2 = 1 \tag{4.61}$$

This expression can be simplified and written in either of the forms:

$$A^{\mu}(r,t) = A^{\mu}_{ret}(r,t) + A^{\mu}_{in}(r,t) \tag{4.62}$$

$$A^{\mu}(r,t) = A^{\mu}_{adv}(r,t) + A^{\mu}_{out}(r,t) \tag{4.63}$$

The choice between these solutions is conventional. This conventionality is reflected in the fact that when we take a Fourier transformation of A^{μ} the wave equation reduces to

$$(\omega^2 - k^2)A^{\mu}(k,\omega) = -4\pi j^{\mu}(k,\omega) \tag{4.64}$$

We are in general free to choose our contour as we like. Choosing the retarded solution is often justified by stating that due to causality advanced potentials are impossible. But in fact the form of the equations does not imply anything about causality. What is needed therefore is a special asymmetric boundary condition, such as the Sommerfeld condition $A_{in} = 0$, which states that there are no free fields in the past. The idea is that sources only produce retarded radiation. The Sommerfeld condition may be justified by a possible interpretation of causality, which states that sources have to pre-exist radiation, since they cause the radiation.

The problem with the analogy arises if we realize that in causal dynamics we are dealing with a Laplace transformation of the first order LvN equation. It is a well known property of Laplace transformations that the contour has to be traced above all singularities, and consequently there is no freedom left in choosing a different contour, and there is no possible reference to causality here. It is thus inappropriate to state that causality imposes us to trace a particular contour. Secondly, it is not possible to use an interpretation of causality using sources and radiation, as it is unclear in the causal dynamics approach what kind of physical processes ought to be identified with sources and effects. The analogy is hence fallacious. Resorting to causality is thus in need of a clarification. It is therefore not surprising that in later approaches the appeal to causality is marginalized.

Karakostas' main criticism of the BS is precisely that there is a lack of argument why they choose a contour in the upper half plane. Karakostas: "The inefficacy of this approach, (..) becomes evident by considering the fact that the time-reversal symmetry of the underlying dynamics allows us to construct equally well another transformation Λ_- off the limit $z \to -i0$, such that the new transformed states (..) will evolve now under a Markov process (..) for the negative direction of time"([37, p.392]). His criticism is first of all unfair since as we have seen the retarded-advanced picture is a crucial part of the causal dynamics approach. His criticism is also wrong since there is no freedom left in choosing a contour.

Sometimes a third argument due to George is given by the BS:

> "We may now give physical content to the relation between the direction of time and analytic continuation, following a rule first introduced by George (1973). The 'future' is described by processes which either conserve the type of correlation or create correlations. To such processes we associate the analytic continuation $-i\epsilon$ ('future') [and vice versa]"([65, p.468])

This argument is based upon the fact that we decompose the Hamiltonian into a an unperturbed part corresponding to the system's free motion, and a perturbation term corresponding to the interactions. The idea is then that if we start in an unperturbed state also called the vacuum of correlations, correlations between the particles are created due to collisions. Problematic in this explanation is however that interaction is a dynamical concept, whereas correlation is a probabilistic concept. I conclude that the problem inherent in all three explanantions is that they are essentially metaphorical.

4.2 New Complementarity

The concept of 'complementarity' is one of the guiding ideas Prigogine is fond of using. Already in 1969 he writes about the complementarity which he (supposedly) has demonstrated between the dynamical description of systems with few degrees of freedom and that of large systems. At other points Prigogine speaks about a complementarity between thermodynamics and mechanics. Prigogine even alludes to an interpretation that assigns the unitary operator 'mechanical time', and the evolution in the $\tilde{\Pi}$ space a 'thermodynamical time'.

In this section I will discuss another type of complementarity between probabilistic and deterministic descriptions, which I have christened the new complementarity (NC) approach. In this approach the intuitions about complementarity and a distinction in time are given a mathematical justification in the results derived by Misra in 1978. Misra extends the previously introduced Koopman formalism with a Lyapounov functional in order to impose the approach to equilibrium. This extension leads to the introduction of a microscopic entropy operator M which obeys certain commutation relations with L and an operator T, which is interpreted by Prigogine as the 'internal time' operator. This approach is sometimes referred to as the Misra-Prigogine-Courbage Theory of Irreversibility (Ordónez 1997 [48], Suchanecki and Weron 1989 [76]).

But there is another major ingredient in the NC approach, which has lead to some confusion in the literature. Again a nonunitary (however not starunitary) operator, confusingly called Λ, is introduced (following the results of Misra), which now constitutes a similarity transformation with a Markov process, it is claimed, without any 'loss of information'.

Whereas the causal dynamics approach was explicitly contrasted with Markov processes, in the new complementarity approach Markov processes become rudimentary as is illustrated by the following quotes.

> "As long as correlations are important, we certainly do not hope to describe dynamics purely in terms of stochastic process. Further on, the concept of transition probability used in Markov chains is in general not applicable in the frame of causal dynamics"(1973 [60, p.18]).
>
> "Indeed Markov processes provide the best possible models to represent irreversible evolution obeying the law of increasing entropy"(1979 [45, p.2])

On the interpretational level, this similarity transformation is interpreted as constituting the change from deterministic dynamics to probabilistic descriptions. One of the main arguments for this interpretation are the supposedly delocalizing properties of the transformation. This claim has attracted a lot of attention in the secondary literature and will be discussed. There is however another point of attraction in the NC approach, which consist in the revolutionary views that are sketched given by Prigogine based on analogies with quantum mechanics. These views certainly justify giving the new complementarity approach its name.

4.2.1 The Formalism of New Complementarity

4.2.1.1 Extending the Koopman formalism

In 1978 Misra (since then one of the members of the Brussels School) publishes a very influential article entitled *Nonequilibrium entropy, Lyapounov Variables, and Ergodic Properties of Classical Systems* ([44]).

In the Hilbert space formulation of classical mechanics physical observables (with appropriate real-valued functions on the phase space) are represented by the operators of multiplication by corresponding functions. There are of course a vast number of operators on L^2_μ that are not multiplication operators by such functions. However it is generally accepted that such operators can have no interpretation in terms of observable quantities of the physical system. Misra departs from this interpretation and considers the possibility that some of the physical observables of classical systems require operators other than multiplication by phase space functions for their mathematical representation. In fact using an important 'impossibility theorem' due to Poincaré (1889) more general operators are needed for the purpose of deriving an increase in entropy:

> **Theorem 1 (Poincaré):** There can exist no function on the phase space that is of definite sign and that increases monotonically to a maximum under Hamiltoninian evolution.
>
> *Proof*: This is a direct consequence of Poincaré's recurrence theorem, the proof of which is based on the Liouville theorem ([79, p.115-6]). A function of phase space will infinitely often assume its initial value. It cannot behave therefore in a monotonically increasing way (Prigogine). It holds even if the Hamiltonian is allowed to contain both even and odd polynomials of momenta so that the Hamiltonian evolution no longer possesses the time reversal symmetry (Misra).

Misra first proves the following proposition:

> **Proposition 1:** Let A be a (bounded) operator on L^2_μ. Then $\langle\psi, A\psi\rangle$ is a single-valued functional of the functions $\rho(\omega) \equiv \mid \psi(\omega) \mid^2$ (with ψ ranging over L^2_μ) if and only if A is an operator of multiplication by a function on the phase space.
>
> *Proof*: The if part of the proposition is trivial. To show the 'only if' part let us suppose that $\langle\psi, A\psi\rangle$ is single-valued on the functions $\rho(\omega) \equiv \mid \psi(\omega) \mid^2$. This is possible if and only if A commutes with all unitary operators U of the form
>
> $$(U\psi)(\omega) = x(\omega)\psi(\omega), \text{ with } \mid x(\omega) \mid = 1 \tag{4.65}$$
>
> Let N denote the set of all operators of multiplication be measurable and bounded functions on the phase space. Then it follows from standard results in the theory of von Neumann algebra that N is a maximal abelian von Neumann algebra; that is $N' = N$. Here N' denotes the set of all (bounded) operators that commute with every element in N. Moreover because N is a von Neumann algebra (with unity) every element of N can be expressed as a linear combination of unitary operators

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in N (of the above form). Thus A must commute also with all elements of N. Because $N' = N$, A belongs to N.

The consequence of proposition 1 in conjunction with Poincaré's result is, that the nonequilibrium entropy (which we are assuming is expressible in terms of a bilinear functional $\langle \psi, M\psi \rangle$ on L^2_μ) is not a single-valued functional of the Gibbsian ensembles.

Misra distilles another implication of this observation, namely, that if the nonequilibrium entropy is to be expressed in terms of the functional $\langle \psi, M\psi \rangle$ it is necessary that the operator M fails to commute with at least some of the multiplication operators by phase space functions. The upshot of this observation is that the set of observables of the classical system must, thus, contain noncommuting observables if nonequilibrium entropy is to be incorporated within its conceptual scheme.

> "The simultaneous determination of (or, at least, of certain of) the classical dynamical variables (i.e. phase space functions) and nonequilibrium entropy must thus be subject to uncertainty relations that flow from the just-mentioned noncommutatitivity. The notion of entropy must therefore be totally inapplicable in the conditions that allow the dynamical description of the system in terms of phase space trajectories with all classical observables completely determined. Similarly the conditions that permit an accurate determination of (nonequilibrium) entropy must exclude the possibility of accurate determination of trajectories in phase space. The dynamical descriptions in terms of phase space trajectories and thermodynamic description (in terms of nonequilibrium entropy) thus naturally emerge, in this approach as complementary descriptions analogous to the 'complementarity encountered in QM'" (Misra [44, p.1628])

Hence the title for this approach 'new complementarity'. Misra's strategy now consists in questioning whether there exist necessary or sufficient conditions for the existence of such an operator M. In order to do so, he gives first definitions of a Lyapounov variable and of a K-flow:

Definition: A bounded operator M on L^2_μ will be called a Lyapounov variable of the system if it satisfies the following conditions (i) M is nonnegative (ii) $\psi \in D_L$ ($\equiv$ the domain of L) implies $M\psi \in D_L$

(iii)$i[L, M] \subseteq -D$ (the commutator implies the existence of an operator D) with D self-adjoint and non-negative (iv) $\langle\psi, D\psi\rangle = 0$ iff $\psi(\omega) =$ a constant almost everywhere on Ω.

Definition: For the dynamical system (with Liouvillian L) to be a K-flow it is necessary that here exists a system of imprimitivity for the group e^{-iLt} (t real) in $H_0^{\perp}$. H_0 is the one-dimensional subspace spanned by constant-valued functions on the energy surface. The projection onto it will be denoted with P_0. Misra first states the following theorem (by Arnold en Avez 1968, Sinai 1964):

Theorem 2: For dynamical systems that are K-flows there necessarily exists a family $F_\lambda(-\infty < \lambda < \infty)$ of projection operators with the following properties:

- $F_\lambda \leq F_\mu$ if $\lambda < \mu$
- $\lim_{\lambda\to\infty} F_\lambda = P_0^{\perp}$
- $\lim_{\lambda\to-\infty} F_\lambda = 0$
- F_λ is (strongly) continuous in λ
- $e^{-iLt} F_\lambda e^{iLt} = F_{\lambda+t}$ for all real λ and t.

The consequence of the last property is that the self-adjoint operator

$$T = \int_{-\infty}^{\infty} \lambda dF_\lambda \tag{4.66}$$

which has F_λ as its spectral family of projections acts as a canonically coniugate variable to L. It satisfies the Weyl commutation relations

$$e^{iLt} e^{isT} = e^{ist} e^{isT} e^{iLt} \tag{4.67}$$

as well as the canonical commutation relations

$$[T, L] = iI \tag{4.68}$$

Misra then proves the following theorem:

Theorem 3: In order that a classical system admits the existence of a Lyapounov variable it is necessary that it satisfies the condition of mixing. Conversely, a sufficient condition for the existence of a Lyapounov variable is that the dynamical evolution satisfies the conditions of a K-flow. *Proof:* see Misra [44, p.1629]. Here I will state part of the proof as a proposition:

> **Proposition 2:** Lyapounov variables for K-flows can be actually constructed as follows: Let $h(\lambda)$ be any monotonically decreasing, positive, continuous, and differentiable function (with derivative $h'(\lambda)$ that satisfies the conditions: (i) $h(\lambda)$ is bounded (ii) $h(\lambda)' < 0$ (iii) $\mid h'(\lambda) \mid$ is bounded. Then the operator M
>
> $$M = h(T) + \alpha P_0 \text{ with } \alpha \geq 0 \tag{4.69}$$
>
> has all the properties of a Lyapounov variable. Here $h(T)$ is the operator that corresponds to the function $h(\lambda)$ in the spectral family F_λ of T.

Misra's idea of extending Koopman's framework with a microscopic entropy operator will prove to be very appealing for the Brussels School. Especially the commutation relation 4.68 will be important since the operator T will be interpreted as 'internal age operator' of a system and the construction of the operator M (equation 4.69) will be used as a new way of constructing a nonunitary operator Λ.

4.2.1.2 From Dynamics to Probabilistic Descriptions

The results of Misra were further developed in two articles entitled *From Dynamics to Probabilistic Descriptions* (Misra and Prigogine 1979 [45]) and *On Intrinsic Randomness of Dynamical Systems* (Goldstein, Misra and Courbage 1981 [31]).[15] The extension of the results to Bernouilli systems was accomplished in *On the Equivalence between Bernouilli Dynamical Systems and Stochastic Markov Processes* (Courbage and Misra 1980 [22]).

With every Markov process (with transition probabilities $P(t, \omega, \Delta)$ having an invariant measure μ, one can associate a family W_t of operators defined by

$$W_t f(\omega) = \int f(\omega')P(t, \omega, d\omega') \tag{4.70}$$

for $f \in L^2_\mu$. The Chapman-Kolmogorov equation (see appendix) then implies the semigroup property for W_t. We can derive that if ρ denotes the distribution function describing the initial state of the system, it evolves in time under the Markov process described by the hermitian coniugate W_t^* of W_t as:

$$\rho_t = W_t^* \rho \tag{4.71}$$

The operators W_t form a **contractive Markov semigroup** for $t > 0$ with the following properties

[15] Both articles are partly copy and pastes and heavily intertwined.

- W_t preserves positivity
- $W_t 1 = 1$
- $W_t^a 1 = 1$

where the superscript a denotes adjointness.

Misra and Prigogine state, that it seems natural to consider the stochastic evolution associated with the semigroup W_t 4.2 as 'equivalent' to the deterministic evolution described by the unitary group U_t if there exists a bounded transformation Λ on L^2_μ such that

$$\begin{array}{ll} (i) & \Lambda U_t \rho = W_t^* \Lambda \rho \text{ for } t > 0 \\ (ii) & \Lambda\rho \geq 0, \text{if} \rho \geq 0 \, \text{(positivity)} \\ (iii) & \int_{\Gamma_E} \rho d\mu = \int_{\Gamma_E} \Lambda \rho d\mu \\ (iv) & \Lambda \rho_{equil} = \rho_{equil} \\ (v) & \Lambda \text{ has a density defined inverse } \Lambda^{-1} \end{array} \tag{4.72}$$

A further condition imposed on W_t is that the equilibrium state is an attractor for the process. This further condition is a defining condition of a **monotonic Markov semigroup**. This requirement comes from the fact that the BS want the evolution under W_t to display an irreversible approach to equilibrium expressed as requiring that the norm

$$(vi) \, \|W_t^*(\rho - \rho_{equil})\|^2 = \|W_t^* \rho - \rho_{equil}\| \text{ decreases monotonically as } t \to \infty \tag{4.73}$$

Condition (i) expresses the fact that states transformed under Λ evolve under W_t, an irreversible stochastic Markovian evolution for $t \geq 0$. (ii)-(iv) are necessary if $\Lambda\rho$ is to effect a change of representation: transformed states are states and expectation values in the new representation are equal to expectation values in the old representation. Feature (v) expresses that the passage from deterministic to probabilistic description brought about by Λ involves 'no loss of information' i.e. is not a projection which would correspond to a coarse-graining or contraction of description. Systems are called **intrinsically random** by the Brussels School if a Λ operator with these requirements (i)-(vi) can be constructed.

Requirement (vi) is equivalent to stating that

$$\|W_t^* f\|^2 \equiv \langle W_t^* f, W_t^* f \rangle \text{ decreases monotonically to } 0 \, \forall f \in K_0^\perp \tag{4.74}$$

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The subset $K_0^{\perp}$ is orthogonal to ρ_{equil}. This implies that

$$\begin{gathered}\langle U_t \Lambda^{-1} f, M U_t \Lambda^{-1} f \rangle \text{ with } \Lambda^a \Lambda = M \\ \text{decreases monotonically with t } \forall f \in K_0^{\perp} \cap D_{\Lambda^{-1}} \end{gathered} \tag{4.75}$$

As $K_0^{\perp}$ is invariant under Λ and Λ^{-1} we derive that M is a Lyapounov variable for the dynamical group U_t. This shows that the Λ transformation (when it exists) may be constructed as essentially the square root of a Lyapounov variable. Hence the connection with the results of 1978 (proposition 2) is made.

Requirement (ii) is however not generally guaranteed, although positivity preserving is crucial for Λ to be nonunitary. This will be the topic of investigation in the next paragraph.

4.2.1.3 Preserving Positivity and Symmetry Breaking

Goldstein, Misra and Courbage prove the following theorem:

> **Theorem 4:** Let the abstract dynamical system $\{\Gamma, B, \mu, T_t\}$ be a K-flow, U_t the unitary group induced by T_t. Further, let F_λ and $P_{-\infty}$ be the projections defined earlier, and $h(\lambda)$ a function satisfying the following conditions: (i) $h(\lambda)$ is strictly monotonically decreasing with $\lim_{\lambda\to\infty} = 0$ and $h(-\infty) \leq 1$ (ii) $h(\lambda+s)/h(\lambda) \equiv \tilde{h_s}(\lambda)$ is a monotonically decreasing function of λ for every $s \geq 0$. Suppose Λ is defined by:
>
> $$\Lambda = \int_{-\infty}^{\infty} h(\lambda) dF_\lambda + P_0 \tag{4.76}$$
>
> Then Λ is positivity improving[16], has a densely defined inverse Λ^{-1}, and the operators $\Lambda U_t \Lambda^{-1} = W_t^*$ form a monotonic Markov semigroup for $t \geq 0$.

[16]Which is a stronger result than positivity preserving. In *On Intrinsic Randomness of Dynamical System* Goldstein, Misra and Courbage, while proving the positivity preserving property of Λ cf. Theorem 4, actually prove the slightly stronger result that if $h(\lambda)$ is strictly monotonically decreasing then Λ is 'positivity improving'. i.e. for any nonnegative function $\rho'(\neq 0)$ and *any* measurable set Δ with $\mu(\Delta) > 0$

$$\langle \phi_\Delta, \Lambda\rho' \rangle = \int_\Delta \Lambda\rho' d\mu > 0 \tag{4.77}$$

They interpret this result as follows: "The result shows that the action of Λ is highly 'delocalizing' in the following sense: Even if the support of ρ is confined to an arbitrary small volume of the phase space Γ, the transport of the transformed functions covers the entire phase space. Obviously such an operator cannot be the multiplication operator by a phase space function, nor can it be the induced operator from

The intrinsic randomness of K-flows is therefore established once the class of functions $h(\lambda)$ is nonempty.

We have seen that Λ has an inverse. Why is there then more to this change of representation than a simple change of coordinates? The transformation is more than a simple change of coordinates precisely because Λ defined according to conditions (i) - (vi) is nonunitary. Why, given its definition, is Λ nonunitary? Batterman in *Randomness and Probability in Dynamical Theories* ([11]) calls the answer 'subtle'. While the Brussels School requires that Λ preserves positivity, they do not require that Λ^{-1} is positivity preserving as well. Consider here to proposition 3 (Goldstein, Misra and Courbage):

> **Proposition 3:** Let Λ in addition to satisfying conditions (i)-(ii) also satisfy the following: (i') Λ^{-1} preserves positivity, and (ii') the domain of Λ^{-1} contains all the characteristic functions $\phi_\Delta(\Delta \in B)$. Then there exists a measure preserving point transformation T of Γ onto itself such that
> $$(\Lambda\rho)(\omega) = \rho(T\omega) \tag{4.78}$$

In fact if Λ^{-1} would be positivity preserving, the transformation $\Lambda U_t \Lambda^{-1}$ would be unitary. But then an inconsistency would arise with condition (v) since W_t is a *monotonic* Markov semigroup.

> "The breaking of the time reversal symmetry is located in the failure of $\Lambda U_t \Lambda^{-1}$ to preserve positivity for $t \leq 0$. This is the Prigogine school's account of the symmetry breaking mentioned (..) If the positivity requirement is violated, then the evolution cannot be *physical* since it would involve pathological 'negative' probabilities."(Batterman [11, p.252-3]).

Batterman regards the positivity preserving requirement (for $t > 0$), leading to symmetrybreaking as a natural requirement based on the physical argument that 'negative probabilities' cannot exist. Goldstein, Misra and Courbage make this point as follows:

an underlying point transformation. It is because of such delocalizing action of Λ that one may expect the change of representation $\rho \to \Lambda\rho$ to lead from deterministic to stochastic dynamics" ([31, p.121]). In the formalism that Mackey introduces [42] Λ would be called a smoothing Markov operator i.e. this definition implies that any initial density, even if concentrated on a small region of the phase space X will eventually be smoothed out by P^t and not end up looking like a delta-function ([42, p.70]).

> "One might say that the 'reverse operation' leading to the deterministic description, though mathematically well defined, is a physically inadmissible operation. This is in conformity with the view that intrinsic randomness is a property only of highly unstable dynamical motion and in this situation the deterministic description of dynamic development constitutes an unphysical idealization."([31, p.116]

We will see when discussing the interpretation of the formalism that the inadmissibility is grounded in the fact that the reverse operation would require an infinite amount of information. Physically inadmissible has to be understood therefore as practically impossible to realize. The second point that is made is an important point that is often stressed by the BS, that the deterministic description constitutes an unphysical idealization.

The Λ transformation is often seen as not just an equivalent representation of the system, but as an ontological claim which states that 'trajectories have to be eliminated from physics'. This claim will also be discussed later on. As an illustration of the formalism, I will discuss the baker transformation, which is the main working example of the NC approach as it is shown to be positivity preserving and allows the construction of Λ.

4.2.1.4 The Baker Transformation

In order to demonstrate that it is indeed possible to find intrinsically random systems with a positivity preserving Λ operator Prigogine uses as a working example of a K-flow, the baker transformation, so called because it evokes the image of kneading dough, cf. figure 4.2.1.4.

$$\begin{aligned} \omega &= (p,q) \in [0,1] \times [0,1] \\ B\omega &= (2p, q/2),\ 0 \leq p < 1/2 \\ B\omega &= (2p-1, (q+1)/2),\ 1/2 \leq p < 1 \end{aligned} \tag{4.79}$$

The characteristic property of the baker transformation is that the partition $P = \{\Delta_0, \Delta_0^c\}$ of the unit square in to the right and left halves is 'independent' and 'generating' with respect to B. These properties are exploited in the construction of M and a positivity preserving Λ operator. The discrete group B^n now induces a discrete unitary group U_n on L^2_μ:

$$(U^n\rho)(\omega) = \rho(B^{-n}\omega) \tag{4.80}$$

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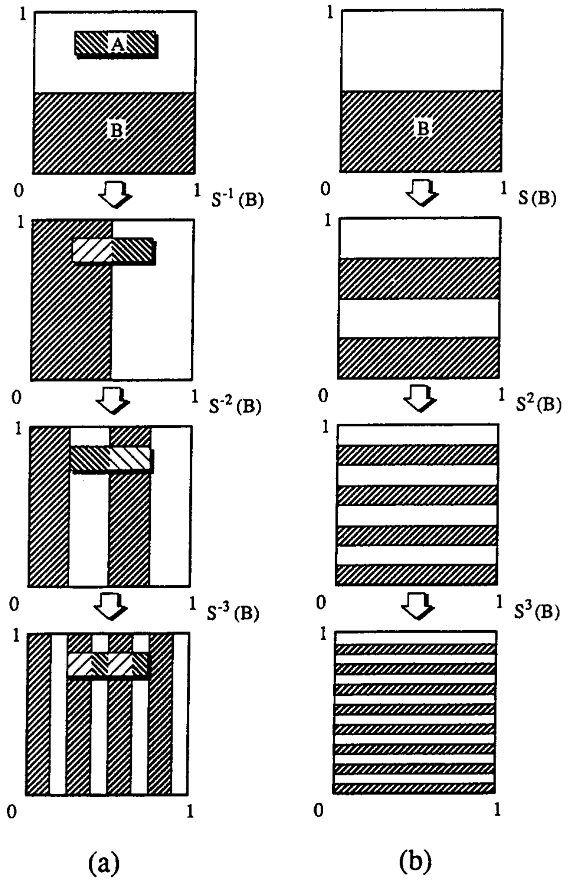

Figure 4.3: *The behavior of an initial set B under backward (a) and forward (b) iteration by the uniformly mixing baker transformation (Mackey [42, p.62])*

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In the case of a continuous parameter group $U_t = e^{-iLt}$, the operator T is, by definition 4.68 a self-adjoint operator that satisfies the canonical commutation relation:

$$i[L, T] = I \tag{4.81}$$

The discrete group U_n induced by the baker transformation corresponds to the discrete subgroup e^{-iLn} (n integers) of the group $U_t = e^{-iLt}$. Due to the existence of a system of imprimitivity for the baker system, the operator T is a self-adjoint operator that satisfies

$$U^{-n} T U^n = T + nI \tag{4.82}$$

on $K_0^{\perp}$. One further forms a spectral expansion of T with the corresponding eigenvalue ('age') n as

$$T = \sum_{n=-\infty}^{n=+\infty} n E_n \tag{4.83}$$

so that

$$U^{-n} E_m U^{-n} = E_{m-n} \tag{4.84}$$

Prigogine now constructs an orthonormal basis in $K_0^{\perp}$. Let χ_0 be the function which assumes the value -1 on the left half of the square and +1 on the right half. Define $\chi_n = U^n \chi_0$. A complete set of eigenfunctions of T is obtained by taking all possible finite products of χ. Such a product belongs to the eigenvalue m of T when m is the maximum of the indice n of χ_n appearing in the product. For example $\chi_{-5}\chi_3, \chi_1\chi_2\chi_3$ are all eigenvectors of T corresponding to the eigenvalue 3. $\phi_{n,i}$ denotes then a complete set of eigenvectors of T with n the eigenvalue and i the degeneracy. The eigenfunctions $\phi_{n,i}$ together with the constant function 1 form a complete set of orthogonal functions. E_n is then the subspace spanned by the eigenfunctions of T corresponding to eigenvalue n.

The construction of Lyapounov variables M now follows directly from proposition 2. They are monotonically decreasing operator functions of T

$$M = \sum_{n=-\infty}^{n=+\infty} \lambda_n^2 E_n + P_0 \tag{4.85}$$

where λ_n^2 denotes a (two-sided) sequence of non-negative numbers bounded by 1 and decreasing monotonically to 0 as $n \to \infty$ The operator M has decreasing eigenvalues, which stems from the shifting action of U. As M is now a function of an operator (T) instead of an ordinary phase variable, the microcanonical ensemble is attracting in the Lyapounov sense. Λ can now be constructed as

$$\Lambda = M^{1/2} = \sum_{-\infty}^{\infty} \lambda_n E_n + P_0 \tag{4.86}$$

If we now choose

$$\lambda_n = \frac{1}{1 + e^n} \tag{4.87}$$

we satisfy the requirements of theorem 4, and we have obtained a positivity preserving operator Λ.

4.2.2 The Interpretation of New Complementarity

4.2.2.1 Two Different Λ's

We have seen that both in the causal dynamics approach and in new complementarity approach an operator Λ is constructed. The question naturally arises whether this operator is the same or not. Prigogine claims in *From Being to Becoming* that they are equivalent. The new complementarity approach is seen by Prigogine as an extension of his work on subdynamics. As he writes: "For our purpose, the main point to retain [in the present article] is the important notion of equivalence, via a nonunitary, Λ between the dynamical and the thermodynamical descriptions. In conformity with this idea, we seek to determine the conditions on dynamics so that it becomes equivalent, via a similarity transformation Λ, to a stochastic Markov process" ([45, p.4]).

Contrary to Prigogine's view I want to argue instead that both Λ operators *are* different and *should* be distinguished properly. First of all, in the CD approach Λ was constructed using the characteristics of the operators in subdynamics, as a function of L. This requirement was necessary in order to define the star-hermiticity of an operator $Q(L)$. Λ in causal dynamics is a star-unitary operator. In the more abstract NC approach, we have no information whatsoever about the dependence of Λ on L. So whereas in the CD dependence on L is required, in the NC Λ can be constructed without any knowledge about L.

My second argument is that the Λ of causal dynamics does not necessarily satisfy the requirements (i - vi) of an intrinsically random system. We have seen (theorem 3) that in order to admit the existence of a Lyapounov variable the system necessarily has to be mixing. A sufficient condition for the existence is that the dynamical evolution satisfies the conditions of a K-flow. The equivalence of both Λ's would imply therefore the equivalence of K-flows and dissipative systems. But this equivalence is hard to establish, since we have to take a thermodynamic limit to obtain a dissipative system. It should be proved therefore that each dissipative system has (at least) the dynamics of a K-flow, which would be surprising since as we have seen dissipative phenomena already for small classical sytems.

My third argument is that Λ in the NC approach is constructed using a theorem of Misra in an extended Koopman formalism as the square root of a Lyapounov variable M. But as we have seen the existence of a Lyapunov variable could not be derived within CD. A final observation is that Λ is implicitly defined with recourse to Markov processes which were rejected in the CD approach. This conlcudes my arguments against the equality of the Λ of CD and NC.

This conflation has lead to a lof of confusion in the literature. Both Obcemea and Brändas and Karakostas for example treat the Λ of CD and of NC as essentially the same. As I conjectured, the result of this confusion has been that CD has often been criticised as a failing attempt at deriving intrinsic irreversibility.

4.2.2.2 The 'Equivalence' Thesis

As we have seen, for intrinsically random systems, there exists a nonunitary Λ transformation which effects a change *from* dynamics *to* probabilistic decriptions. These descriptions are therefore not simply 'equivalent', which in itself of course is already a problematical concept necessitating it to be put between marks, but called similar. It is important to realize that Prigogine interprets intrinsic randomness in an ontological sense. It is because of the existence of a similarity transformation that Prigogine claims that 'trajectories have to be eliminated from physics' and form 'unphysical idealizations' and 'phase space points cannot exist'. These claims have attracted a lot of attention as we have seen in chapter two. The main argument for these ontological conclusions comes from the delocalising properties of Λ.

In the 1983 article entitled *Irreversibility and Nonlocality* ([46]) Prigogine and Misra develop a method of extension of Λ and W_t to singular distributions. The main aim of this extension is to illustrate and discuss the fact that the second law of thermodynamics when formulated as a *dynamical principle*, implies a departure from nonlocality. This interpretation is based on an investigaton of the evolution of phase points (i.e. Dirac δ functions type) under the action of Λ.[17]

[17] Martinez and Tirapegui in an article entitled *A possible Physical Interpretation of the Λ operator in the Prigogine theory of Irreversibility* [43] show that "if observations take a finite time Δt and if one can only distinguish between points in phase space separated by distances greater than ϵ then the

Let us consider the singular distribution $\delta(x,y) = \delta(x-x_0)\delta(y-y_0)$ concentrated on the phase point $\omega_0 = (x_0, y_0)$ within the baker transformation formalism. The phase point may be expanded in terms of the eigenfunctions $\phi_{n,i}$ of Λ, leading to

$$\Lambda\delta_\omega \equiv \hat{\delta}_\omega(x,y) = \sum_{n,i} \lambda_n \phi_{n,i}(x_0,y_0)\phi_{n,i}(x,y) + 1 \tag{4.88}$$

$$\begin{aligned} W_t\hat{\delta}_\omega = &\sum_{n\leq 0,i} \frac{1}{1+e^{n+t}}\phi_{n,i}(x_0,y_0)\phi_{n,i}(x,y) + \\ &\sum_{n\geq 0,i} \frac{1}{1+e^{n+t}}\phi_{n,i}(x_0,y_0)\phi_{n,i}(x,y) + 1 \end{aligned} \tag{4.89}$$

The next long quotation gives the argumentation for the physical inadmissibility of the inverse operation of Λ.

> "Now the second term on the right hand side converges to a square integrable function (note that $\mid \phi_{n,i}(x,y) = 1 \mid$ and $\sum_{n\geq 0,i} \mid \frac{1}{1+e^{n+t}} \mid^2 < \infty$) say $g(x,y)$ satisfying $\mid (g(x,y) \mid\leq ke^{-t}$ with k a constant. Because of this exponential damping the distribution is for large t indistinguishable from the truncated distribution containing only the first and third terms. The transformation Λ applied to this truncated distribution, however, doe not give back a distribution concentrated on a point of the phase space. In other terms, in order to reconstruct the evolution of the phase space point ω one has to have precise knowledge of the contribution represented by the second term. But as this contribution is exponentially dampened with time, the reconstruction of the phase space trajectory is not physically possible for large t."(Misra and Prigogine 1983 [46, p.426])

> "One can, of course, mathematically reconstruct the unitary evolution U_t corresponding to the motion of phase points along trajectories from a physical evolution W_t through the formula $\Lambda^{-1}W_t\Lambda = U_t$. The important point however is that the reconstruction for large *t* requires infinite precision in the observation of physically involving states. For *any given finite precision of observation*, the reconstruction will fail after a suitably large finite time. These considerations show the essentially

Λ operator is given by these conditions.

> nonlocal character of the transformation Λ and of the physical evolution W_t."(ibid p.422)

The main argument is therefore that the reconstruction is inadmissible since it would require an infinite amount of information. Physically impossible has to be understood therefore as practically impossible to realize. It is because of epistemological considerations that we arrive at a departure from nonlocality. But as Bishop points out, this strong ontological conclusion is not warranted. "In my judgment Prigogine and colleagues ought to be arguing that the probability description is the primary physical picture of the behaviour of extremely unstable systems"(Bishop [14, p.20]).[18] Prigogine falls prey to the epistemological fallacy here. On top of that, the strangeness of the argument is that the conclusion only holds *after* we have taken the time limit.

Batterman has also criticized the Prigogine's line of argument. With a sense of humor, he writes, "the 'point' is that the argument against exact states is framed within the Hilbert space formalism of classical mechanics. (Furthermore it is only in this framework that Λ can be defined). Strictly speaking the reference to Λ's application to phase points is nonsense. Phase points are not the kinds of things that can be transformed under Λ. And, Dirac δ distributions concentrated on phase points are not phase points"(Batterman [11, p.260]). Batterman gives two arguments. First of all δ functions are not properly defined within the Koopman formalism[19], since results and inferences carry along the usual rider that they hold almost everywhere, except possibly, on a set of measure zero. "For example, we identify two locally summable functions when they are almost equal everywhere. The results, therefore, really apply to equivalence classes of such functions (modulo zero). This relativation modulo zero applies to singular distributions as well."(ibid p.260)

Batterman's second argument criticizes Prigogine's claim that phase space points cannot exist by pointing that Prigogine considers only properties of the vector space analogs of phase points, that is singular distributions in terms of the orthormormal basis $\phi_{n,i}$ for $L^2_\mu(\Omega)$. "But once again, these are not phase points and as a result, their argument for such a strong ontological conclusion loses much of its force" (ibid p.260).

I do not want to delve deeper in these considerations. For the moment I conclude that the 'equivalence thesis' between deterministic and probabilistic descriptions is

[18] For a discussion on the controversies about Prigogine's claim that "exact deterministic trajectories of unstable dynamical systems are idealizations" see Bricmont [21]; Bishop [13]

[19] Whether the Koopmanian formalism is well-suited to deal with such an extension will be a topic of discussion when dealing with Rigged Hilbert Spaces.

problemetical.

4.2.2.3 Internal Time

The operator T is interpreted by Prigogine (and Misra) for the first time in *From Being to Becoming* ([61]) and in *From Dynamics to Probabilistic Descriptions* (1979 [45, p.25]) as the internal time, or age operator of a system.

Since not all distributions ϱ have a well-defined age, an average age is introduced for the excess $\hat{\varrho} = \varrho - 1$ of ϱ with respect to the equilibrium $\varrho = 1$. The average age is then given by

$$T(\varrho) = \frac{1}{|| \hat{\varrho} ||^2} \langle \hat{\varrho}, T\hat{\varrho} \rangle \tag{4.90}$$

The characteristic property of the internal time operators T_1, T_2 etc. associated with various systems is that (with suitable normalisation)

$$d\langle\langle T_1 \rangle\rangle = d\langle\langle T_2 \rangle\rangle = ... = dt \tag{4.91}$$

where $d\langle\langle T \rangle\rangle$ denotes the average of the internal time operator T in a state ϱ. According to Prigogine, we find that the (change in) external or ordinary time t which labels the dynamical evolution now corresponds simply to the common change in the ‘average age’ shared by all systems.

> “The internal time operator (when it exists) contains, however, additional information about the physical system that concerns the fluctuation or dispersion of the ‘internal age’ around the average value. In both classical and quantum systems, time appears simply as an external parameter to label the dynamical group. In contrast, the internal time operators considered here are new physical observables associated with the irreversible evolution of the system. From this point of view the concept of internal time operator is closer to the concepts of thermodynamic and biological time and may serve as microscopic counterpart of the latter phenomenological concepts”.

A more developed account on the interpretation of the T operator can be found in *Irreversibility and Space-Time Structures* (Prigogine 1985 [64]). Suppose we expand the fine-grained distribution function ϱ in terms of the eigenfunctions of the internal time operator T. We obtain formally

$$\varrho = \sum_{n=-\infty}^{n=\infty} c_n \chi_n \tag{4.92}$$

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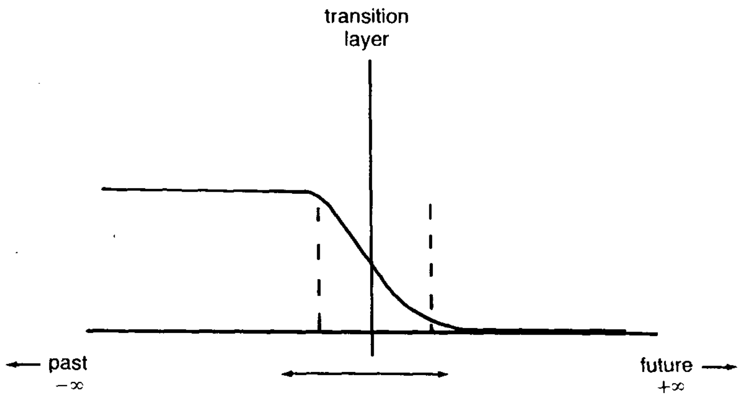

Figure 4.4: *Transition between past and future. λ_n is plotted at the vertical axis.*

Therefore we obtain for $\tilde{\varrho}$

$$\tilde{\varrho} = \Lambda\varrho = \sum_{n=-\infty}^{n=\infty} c_n \lambda_n \chi_n \tag{4.93}$$

The λ_n values are again the eigenvalues of the Λ operator corresponding to the eigenfunction χ_n with the important requirement that λ_n varies from 1 to 0 for $n \to \infty$, where n denotes the eigenvalue of the internal time T. What we now get according to Prigogine can be seen in figure 4.2.2.3. Whereas in the ordinary representation past and future play a symmetrical role, in the physical representation the present contains only the contributions from the nearby future. Past and present are represented by a transition layer of the order of the Lyapounov time. Whereas we are used to represent the present as a point on a straight time axis, Prigogine proposes a view in which the past is separated from the future by an interval (measured by the Lyapounov time) interpreted in Bergsonian terminology as ‘duration’.

The physical meaning of the eigenvalues of the T operator has been questioned by Verstraeten [81]. Verstraeten criticizes the technical validity of construction of T for the Baker transformation. He notices that Prigogine identifies ρ_∞ and $\rho_{-\infty}$ though these points cannot be reached by denumerable applications of the T-operator. Indeed for each integer number n_1 there is always a denumerable number n_1+1 such that $\rho_{n_1+1} = T\rho_{n_1}$. Contrary to Prigogine’s assertions, ρ_∞ and $\rho_{-\infty}$ which represent complete chaos in the respective phase spaces, do not evolve out of equation 4.83, because the shift operator is defined by equation 4.80.

Verstraeten also questions the 'new complementarity' that is expressed in the commutation relations between L, M and T. Stone's theorem which is applied in these commutation relations is only valid for continuous time. Therefore, U_t defined by the Baker's transformation has no physical meaning in the Prigogine framework, as it is only defined for a denumerable number of events, according to Verstraeten. The idea of introducing time operators (also for continuous cases) is further developed by Antoniou et al (1998 [7]).

To be honest I fail to see the connection between the operator T, and Prigogine's interpretation of it as an 'age operator'. Age seems to me to be quite a different concept from psychological time. The former has to do with the rate of biological processes, and should be allowed to show discrepancies with respect to the average time. The latter has to do with our experience of time. One of these experiences is that time is continuous and not discrete as is suggested here.[20]

4.2.2.4 Prigogine's Revolutionary Views on Physics

In the previous sections we came across some rather bald statements. I want to illustrate in this section, what I already indicated in the introductionary chapter, that since the publication of *From being to becoming* Prigogine's has launched some revolutionary ideas about physics. It is not my aim here to discuss these ideas but the section serves as an illustration of Prigogine's associative manner of thinking. This is my top four:

- "The second law of thermodynamics plays the role of a selection principle for space and time concepts as used in general relativity." (Prigogine [64, p.248])
- "The situation that classical mechanics, when the second law is incorporated within the axiomatic scheme of classical dynamics, does not constitute a closed or self-contained scheme, is reminiscent of Gödels conclusion that the axiomatic scheme of arithmetic is incomplete."(Misra and Prigogine [46, p.428])

[20] For sake of completeness I will mention here two more authors that discuss Prigogine's new complementarity approach. Borzeskowski and Wahsner in their article entitled *Nonlinear Irreversible Thermodynamics* ([20]) develop a more philosophical rejection of Prigogine's ideas. Atmanspacher and Scheingraber in their article entitled *A Fundamental Link Between System Theory and Statistical Physics* (1987 [8]) use Prigogine's operator formalism of new complementarity, but they interpret M as an information operator. This gives the following uncertainty relation $i[L, M] \equiv KI$ where K is the Kolomogorov entropy.

- "The impossibility of defining the entropy operator M, the nonexistence of a time operator in QM and the problem of interpreting and justifying the time-energy uncertainty relation are thus linked together."(Prigogine [61, p.244])

- "The inclusion of an entropy operator among the observables thus requires that the pure states lose their priviliged position in theory and that the pure and mixed states be treated on an equal basis. Physically, this means that, for systems having entropy as an observable, the distinction between pure and mixed states ceases to be operationally meaningful and there would be limitations on the possibility of realizing coherent superpositions of quantum states."(Prigogine [61, p.247])

4.2.2.5 The Overlooked Degeneracy

Others have criticized the baker transformation example as well. Bricmont criticizes Prigogine for using the baker transformation as a demonstration of irreversible behaviour.

> "In [the baker transformation] there is no sense of a micro/macro distinction: how could one define the macroscopic variables? To put it otherwise, we can make a movie of the motion of a point in the plane evolving under the baker's map (..) and run it backwards, we shall not be able to tell the difference. There is nothing funny or implausible going on, unlike the backward movie of any real irreversible macroscopic phenomenon. So the first critique of this alleged connection between unstable dynamical systems (i.e. what I call here chaotic systems) and irreversibility is that one 'explains' irreversibility in systems in which nothing irreversible happens, and where therefore is nothing to be explained"(Bricmont [21, p.17])

As we will see in the next chapter, according to Bricmont, the problem of time's arrow has been already solved. An explanation of irreversibility should consist of two necessary ingredients: an appeal to initial conditions and a micro/macro distinction. Bricmont seems to criticize Prigogine's attempt to ground irreversibility on the microscopical level. However I do not think this line of criticism makes Bricmont's position consistent, as we are then committed to the position that also in mixing systems nothing irreversible happens, which is contrary to Bricmont's own position as far as I can see.

But we can interpret Bricmont's criticism differently, as a criticism of Prigogine's use of the word irreversible for a system that is invertible. As we have seen in figure 4.2.1.4 of the baker transformation, it is clearly invertible. Using Mackey's definition of irreversibility the baker transformation is therefore by definition reversible. We are however not obliged to subscribe to Mackey's equivocation of irreversibility with noninvertibility, but we do have to enquire about the status of the invertible map in Prigogine's theory.

As Bishop remarks, "the necessary and sufficient conditions for the existence of the microscopic operator *M* and, hence for Λ admit alternative notions of entropy that have the opposite temporal behaviour to the BAG definition (namely $\Lambda^+ = M_+^{\frac{1}{2}}$)"([14, p.18]). This remark points to the problem Prigogine fails to realize. There exists another semigroup, conditional on his definition of entropy, which reflects the invertibility of the baker transformation. Remember that the monotonic semigroup identified with Λ was derived from the square root of M. But there is a degeneracy here, because we can also identify another semigroup with the other square root. The problem of irreversibility exists in accounting for our choice, not by assuming it.

4.2.3 Analysis within SBSS

In the previous section we have seen how the interpretation of new complementarity ran into difficulties. Especially Prigogine's 'equivalence thesis' between deterministic and probabilistic descriptions proved problematic. In this section I want to summarizse the new complementarity approach within the heuristic of SBSS. The NC approach to irreversibility can be summarized concisely:

- (1) Intrinsically random systems display symmetry breaking, since for these there exists a nonunitary Λ transformation which preserves positivity for $t > 0$, but not for $t < 0$. This positivity preserving argument is motivated by a 'no negative probabilities argument' i.e. that the reverse operation is physically inadmissible.[21]

- (2) The necessity to choose between two possible transformations and two possible semigroups was overlooked. This was illustrated by the invertibility of the Baker's transformation.

[21]"In this way we see that the natural requirement that the transformed evolution $\Lambda U_n \Lambda^{-1}$ should not give rise to 'negative probabilities' inevitably leads to a loss of symmetry between the postive and negative directions in time causing the physical evolution to be described by a semigroup rather than by a group"([46, p.23])

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Concerning the symmetry breaking step in (1) there is a problem. As Bischop remarks (Verstraeten makes a similar remark), the condition that positivity is preserved is not guaranteed by their theory as is admitted by Misra, Prigogine and Courbage (1979 [45, p.12]). This condition must be checked for each physical system to be modeled, requiring a Λ to be constructed for each system.[22] As Bishop states, Λ was *only* constructed for the baker transformation and there is no indication that there are any realistic physical systems for which it could be constructed. Moreover, whether an operator can be defined as a multiplication by a phase space function is representation dependent (consider the momentum operator).

Regarding the selection argument. We saw that in 1981 Courbage, Prigogine, and Misra noticed that we can also require positivity preserving for $t < 0$, but not for both $t < 0$ and $t > 0$. Misra and Prigogine add to this in 1983, "it is of course possible to define *another* transformation $\tilde{\Lambda}$ such that $\tilde{\Lambda} U_n \tilde{\Lambda}^{-1}$ preserve positivity for $t \leq 0$. The important point, however is, that the *same transformed group* $\Lambda U_n \Lambda^{-1}$ cannot correspond to probabilistic processes for both positive and negative time t"([46, p.23]). Courbage is however the first to state that the fact that the symmetry breaking leaves us with a degeneracy, is problematic: "the time symmetry breaking of the dynamical group U_t into W_t and W'_t for $t \geq 0$ and $t \leq 0$, is not sufficient to involve a clear selection between them so that both evolutions seem at first possible"(1983 [23, p.463]).

To lift this degeneracy, in subsequent papers several arguments are given by the BS. This results in a new approach in which a microscopic version of the second law of thermodynamics as a selector of initial conditions is formulated. This approach will be discussed subsequently.

-

4.3 Entropy as a Selector of Initial Conditions

The formalism of what I like to call entropy as a selector of initial conditions (ES in short) is put forward in *Intrinsic Randomness and Intrinsic Irreversibility in Classical Dynamical Systems* (Courbage and Prigogine 1983 [24]) with the mathematical proofs in *Intrinsic Irreversibility of Kolmogorov Systems* (Courbage 1983 [23]).

[22] In *Non-unitary transformation of conservative to dissipative evolutions* (Antoniou and Misra [1] the condition of positivity is relaxed. "In this work the condition of positivity is dropped. This permits us to consider more general functions $\Lambda(\tau)$ which bring about the similarity transformation between conservative and dissipative evolutions" (p.2729). However in return the Λ function is required to be bounded and its inverse unbounded.

Supporting arguments can be found in *Irreversibility and Nonlocality* (Misra and Prigogine 1983 [46]).

The main aim of this approach is formulated by Courbage and Prigogine as lifting the degeneracy that exists between the two possible changes of representation in the NC approach, one leading to a montonically contractive semigroup corresponding to the approach to equilibrium for $t \to \infty$ and the other for $t \to -\infty$, by introducing *a limitation on the physically observable states*, by selecting only the *physically realizable ones*. This approach is sometimes referred to as the dynamical formulation of the second law. We arrive at the following formulation of the approach which is a specification of SBSS:

> The dynamical formulation of the second law of thermodynamics therefore consists of two statements: (1) first, it is the statement that the dynamical system admits two distinct transformations Λ and $\Lambda^{'}$ so that the dynamical group U_t is transformed into two distinct Markov semigroups W_t and $W_t^{'}$ corresponding to the two directions of time. (2) In addition, there is a selection principle according to which only one of these transformations gives rise to physically realizable states and physical evolution. (Misra and Prigogine [46, p.422])

As we have seen previously, the invertibility of the baker transformation is overlooked in the NC approach, as noted by Courbage in 1983. The ES approach incorporates this insight from the start. As is explicitly stated by Courbage and Prigogine "of course the baker's transformation is too symmetrical to allow one to choose one class of states and exclude the other" ([24, p.2412]). A new analysis is hence developed which investigates highly unstable systems with states and measures that have broken time symmetry. The outline of the ES approach is as follows:

- "We can now express the second law of thermodynamics on the microscopic level by requiring that only measures that lead to equilibrium for $t \to +\infty$ are observed in nature or can be prepared in the laboratory (..)

- The role of the entropy functional in this formalism consists in the expression of the amount of information necessary for the preparation of a state (..) The set of initial states having finite entropy represents the set of admissible states (..)

- Now a system is intrinsically irreversible if there exists a subset of physically admissible initial states $\mathcal{C}^+$ tending to equilibrium in the future but not in the past. As a consequence, the velocity inverse $\mathcal{C}^-$ of the states $\mathcal{C}^+$ cannot tend to equilibrium in the future and are therefore excluded by the microscopic formulation of the second principle (..)

- Then, the second principle excludes from the admissible states all probability distributions which do not approach equilibrium in the future". ([24, p.2412])

4.3.1 The Formalism of Entropy as Selector

Consider an abstract classical dynamical system corresponding to a phase space Γ, a normalized stationary measure μ on a Borel σ algebra B of measurable subsets E of Γ and a group of measure preserving transformations S_t. In addition to the measure μ on B, which represents equilibrium, we consider nonequilibrium measures ν that evolve under the automorphisms group α_t, acting on the set of normalized measures of Γ.

$$(\alpha_t\nu)(E) = \nu_t(E) = \nu(S_{-t}E) \tag{4.94}$$

We next introduce the time inversion operator I, which is a one-to-one transformation onto the phase space Γ such that (i) $I^2 = 1$, (ii) $IS_tI = S_t^{-1} = S_{-t}$ (iii) $\mu(IE) = \mu(E)$ for all E. This transformation defines the operator V on the space of measures

$$(V\nu)(E) = \nu(IE) \tag{4.95}$$

The transformation I corresponds in classical systems to velocity inversion. Prigogine now defines the approach to equilibrium by the condition that for all continuous functions f:

$$\int_\Gamma f d\nu_t \rightarrow_{t\rightarrow\pm\infty} \int_\Gamma f d\mu \tag{4.96}$$

If we assume mixing, any initial normalized measure ν_t that is absolutely continuous with respect to μ approaches equilibrium both in the past and in the future. Therefore to distinguish between past and future, one needs to consider probability measures having a nonvanishing singular component with respect to μ. A measure $\tilde{\nu}$ is singular (denoted by a tilde) with respect to μ if $\tilde{\nu}$ and μ have disjoint supports i.e. $\tilde{\nu}$ is concentrated on a set having null μ measure.

Prigogine now considers dynamical systems for which there exists two classes of probability measures $\mathcal{C}^+$ and $\mathcal{C}^-$, where $\tilde{\nu} \in \mathcal{C}^+$ iff $\tilde{\nu}_t \rightarrow \mu$ for $t \rightarrow \infty$ and not for $t \rightarrow -\infty$ and $\tilde{\nu} \in \mathcal{C}^-$ iff $\tilde{\nu}_t \rightarrow \mu$ for $t \rightarrow -\infty$ and not for $t \rightarrow \infty$. It is clear that the velocity inversion is a one-to-one mapping between $\mathcal{C}^+$ and $\mathcal{C}^-$: V $\mathcal{C}^+$. It is also clear that both $\tilde{\mu}$ and its velocity inverse $\tilde{\mu}$' cannot belong to the same class. Finally the class $\mathcal{C}^+$ is propagated by the dynamics –i.e., $\alpha_t \mathcal{C}^+ \subset \mathcal{C}^-$.

It is a feature of a K-system that it is a dynamic system with a distinguished sub-σ algebra B_0 such that:

- $S_t B_0 = B_t \subset B_{t'}$ if $t < t'$
- $\cup_{-\infty}^{\infty} B_t$ generates B
- $\cap_{-\infty}^{\infty} B_t = B_{-\infty}$ is the trivial sub-σ algebra generated by Γ and all sets of null measure.

For any measure ν and any finite partition $\mathcal{F}$ Prigogine defines the entropy of ν relatively to μ and $\mathcal{F}$ by:

$$\Omega(\nu, \mu, \mathcal{F}) = \sum_{Q \in \mathcal{F}} \nu(Q) \log \frac{\nu(Q)}{\mu(Q)} \tag{4.97}$$

A sequence of finite partitions is said to be decreasing if $\mathcal{P}_{n+1}$ is a refinement of $\mathcal{P}_n$. It is well known that if (Γ, B_0, μ) is a Lebesque measure space, there exists a decreasing sequence of finite partitions which tend to the partition ξ_0 that generates B_0. Courbage then proves the following theorem:

Theorem 5:

- $\Omega(\nu, \mathcal{P}_n)$ is a positive and non-decreasing function of n, which tends to $\Omega(\nu, B_0)$ as $n \to \infty$:

  $$\Omega(\nu, B_0) = \lim_{n \to \infty} \Omega(\nu, \mathcal{P}_n) \tag{4.98}$$

- $\Omega(\nu, B_0) = sup_{\mathcal{F} \leq \xi_0} \Omega(\nu, \mathcal{F})$
- If $\Omega(\nu, B_0) < \infty$, then the measure $\nu_{|B_0}$ restriction of ν to B_0 is absolutely continuous with respect to μ. In this case:

  $$\Omega(\nu, B_0) = \int_{\Gamma} \rho_0 \log \rho_0 d\mu \tag{4.99}$$

  where ρ_0 is the Radon-Nykodim density of $\nu_{|B_0}$ with respect to μ.
- Conversely, if $\nu_{|B_0}$ is absolutely continuous w.r.t. μ with density ρ_0 and if $\int \rho_0 \log \rho_0 d\mu < \infty$ then $\Omega(\nu, B_0) = \int_{\Gamma} \rho_0 \log \rho_0 d\mu$

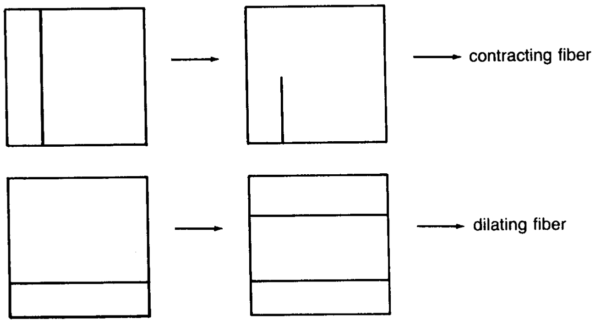

Figure 4.5: *Baker Transformation: contracting and dilating fiber.*

This theorem follows according to Courbage straightforwardly from Martingales theory.[23] The importance of the theorem is that a measure ν can be singular with respect to μ on B while its restriction to B_0 can be absolutely continuous with respect to the restriction $\mu_{|B_0}$. In the case of distributions ν_ρ satisfying $\int \rho^2 d\mu < \infty$, $\Omega(\nu_\rho, B_0)$ is identical to the entropy introduced earlier in the work of Misra and Prigogine. The NC approach was originally formulated in L_2 spaces, but is now extended to L_1 spaces.

4.3.1.1 Measures and Fibers

What is a fiber? Consider the baker transformation. In this case contracting and dilating fibers are horizontal and vertical lines of of the square. Measures concentrated on contracting fibers do not tend to the uniform measure for $t \to \infty$. They can be represented by a Dirac delta function concentrated on a point, whereas measures concentrated on dilating fibers will tend to the uniform measure, but for $t \to -\infty$ they do not. As a result we have two classes of states that are not invariant with respect to time inversion.

The entropy is now a selector of the initial conditions as it satisfies the following requirements, with μ dependence suppressed ([24, p.2414]):

[23] I refer to [24] for the details of the proof. Suchanecki et al (1989 [76]) give a reformulation of 'the Prigogine theory of irreversible dynamical systems' using a stochastic integral with respect operator valued martingales.

Proposition 4

- (i) $\Omega(\nu, B_0) = +\infty$ for all distributions ν concentrated on the contracting fibers and phase points
- (ii) $\Omega(\nu, B_0) < +\infty$ for distributions concentrated on the dilating fibers
- (iii) $\Omega(\nu, B_0)$ is decreasing to zero for any ν_0 such that $\Omega(\nu, B_0) < +\infty$
- (iv) if $\Omega(\nu, B_0) < +\infty$, then ν_t tends to equilibrium μ as $t \to +\infty$

Prigogine then defines when a system is called **intrinsically irreversible** (which is a stronger notion than intrinsically randomness because it restricts the physically observable states) as follows. Consider a classical dynamical system (Γ, B, S_T, μ) realized as a group of automorphisms α_t on the set of all probability measures (or states) on (Γ, B). Such as system is called intrinsically irreversible if it meets the following conditions:

- (A) There exists a class of states $\mathcal{C}^+$ that has the properties (i) $\mathcal{C}^+$ is physically realizable, (ii) the velocity inverse $\mathcal{C}^- = V\mathcal{C}^+$ is not physically realizable, (iii) $\alpha_t\mathcal{C}^+ \subset \mathcal{C}^+$ and (iv) $\alpha_t\mathcal{C}^+$ tends to μ for $t \to \infty$, but $\alpha_t\mathcal{C}^-$ does not

- (B) There exists a one-to-one affine transformation $\tilde{\Lambda}$ on a subset $\mathcal{D}^+$ of states that contains $\mathcal{C}^+$ but may not contain $\mathcal{C}^-$ with the following properties: (i) $\tilde{\Lambda}$ maps a state into a state (ii) $\tilde{\Lambda}$ is one-to-one (iii) $\alpha_t\mathcal{D}^+ \subset \mathcal{D}^+$ tends to μ for $t \to \infty$ (iv) $\tilde{\Lambda}\alpha_t\nu = \tilde{W_t^*}\Lambda\nu$ for any $\nu \in \mathcal{D}^+$, where W_t^* is a semigroup of a Markov process associated with a transition probability $P(t, \omega, E)$–i.e. if $\tilde{\nu} = \tilde{\Lambda}\nu$ then
$$(W_t^*\tilde{\nu})(E) = \int_\Gamma d\tilde{\nu}(\omega)P(t, \omega, E) \tag{4.100}$$
for any $E \in B$ and (v) $\tilde{\Lambda}\nu_t$ is absolutely continuous with a density $\tilde{\rho}_t$ verifying a H-theorem–i.e.
$$\Omega(\tilde{\nu}_t) = \int_\Gamma \tilde{\rho}_t \log \tilde{\rho}_t d\mu \text{ decreases monotonically and tends to zero as } t \to \infty \tag{4.101}$$

4.3.2 The Interpretation of Entropy as Selector

One might ask the question whether ES is a different approach or just an extension of the NC approach. The BS sees ES as an extension of the NC approach:

> "The $\tilde{\Lambda}$ transformation is essentially the extension of the transformation Λ previously constructed in L^2_μ."([24, p.2414])
>
> "We recall the 'canonical method to construct the operator Λ which introduces Λ as a function of an internal time operator (..) here we present directly $\tilde{\Lambda}$ and show that all states of $\mathcal{D}^+$ transformed by $\tilde{\Lambda}$ evolve under a Markov process."([23, p.466])

However, my own opinion is that there are important mathematical changes in formalism between the second and third approach which warrant a distinction. Whereas the NC approach defined intrinsically random systems, the ES approach defines intrinsically irreversible systems as systems which are intrinsically random and for which a physical distinction between contracting and dilating fibers can be made.

But there is also a tremendous change in perspective compared to the NC approach. The ES approach is explicitly developed as a means to argue for one semigroup above the other, by excluding certain types of initial conditions as being unrealizable, since their preparation would require infinite information (or negative entropy). Whereas in the NC approach physical inadmissibility was used to argue against the possibility of the Λ^{-1} transformation, in the ES approach the impossibility of realizing certain initial conditions is incorporated from the start in the definition of an intrinsically irreversible system. This is my main reason for considering the microscopic formulation of the second law as a different approach.

My second observation in favor of a distinction concerns the importance and interpretation of the (nonequilibrium) entropy. In the CD approach, entropy was seen as a concrete experimentally relevant quantity, in which entropy was a property of the state of the system in the physical representation. In the NC approach the entropy was interpreted as an operator, which is obviously not a property of a state. The existence of such an operator was imposed from the start. In the ES approach, entropy is defined as a selector of initial conditions within measure theory. Entropy is perceived as (the) information needed for realizing a certain initial state. We see therefore differences in the role and definition of entropy in these three approaches. This concludes my argument for classifying the ES as a different approach.

I will now turn my attention to the question whether ES is a viable approach. Batterman mentions the selection argument but only superficially:

> "Some singular initial distributions are 'regularized' under Λ so that they become *absolutely continuous* with the Lebesgue measure on Ω.

> This plays a crucial role in their theory of irreversibility. It supposedly allows them to determine which direction of time is preferred, that is which contraction group of operators is realized"(Batterman [11, p.258]).

Sklar's main criticism of the selection priciple can be seen to be directed against the heuristic of SBSS itself which I will take up in chapter five.

> "But there are singular initial distributions whose representation in the new form spread out in the future to fill the available phase space, yet they do not spread out in the other time direction. What Prigogine claims is that only those singular initial states which show this asymmetric limiting behaviour in the new representation are admissible as representing genuine physical states of systems. This is backed up by a definition of entropy and a demonstration that with this sense of entropy the admissible states can be prepared using only a finite amount of 'information', whereas their time inverses can only be prepared utilizing an infinite amount of information. The entropy definition uses dilating and contracting fibers, which are also used in other models, such as the Lorentz gas (..) But it is the old story: we have introduced a selection principle which works. We have not explained why such a uniform selection principle correctly describes the world"(Sklar [75, p.215]).

Both Batterman and Sklar assume that the selection principle works, but is this so? To answer this question it is important to distinguish between two questions:

- Q_1: What is precisely the selection principle?
- Q_2: Does the principle succeed in establishing intrinsic irreversibility?

Regarding Q_1, what we expect is illustrated in the following scheme:

NC	**ES**
intrinsically random systems	intrinsically irreversible systems
$W_t^* = \Lambda U_t \Lambda^{-1}$ for $t > 0$	$\tilde{\Lambda}_+ \alpha_t \nu = \tilde{W}_t^* \nu$ for $\nu \in \mathcal{C}^+$
$W_t^* = \Lambda' U_t \Lambda^{-1}$' for $t < 0$	$\tilde{\Lambda}_- \alpha_t \nu = \tilde{W}_t^* \nu$ for $\nu \in \mathcal{C}^-$
$\varrho(t) = W_t^* \varrho(0)$	

The degenaracy of the NC approach would be resolved because the two transformations would be no longer acting on the same time-symmetric set of states, but

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on two distinct set of states $\mathcal{C}^+$ and $\mathcal{C}^-$. The selection principle$_1$ would then require us to choose the $\mathcal{C}^+$as these are the physically realizable initial states, which would lead to the $t > 0$ semigroup.

However, we have seen in the formalism that there is also a second sense in which we can perceive the principle$_2$: certain entropy expressions are required to be finite. The infinite expressions, measures concentrated on contracting fibers for $t > 0$ are excluded. But the problem of the second interpretation of the principle is that it is purely mathematical, and as a result of that it is always possible to mathematically define two semigroups. We can illustrate this with the baker transformation (which is confusingly given again as main working example). What the Brussel School demonstrates is that we have both for $t > 0$ and for $t < 0$ contracting and dilating fibers:

> "Had we started with the 'time-inverted' transformation resulting in a Markov semigroup for $t \leq 0$ then the situation would have been reversed. In this case the entropy (..) associated with singular measures concentrated on a contracting fiber would be finite, whereas the entropy associated with measures concentrated on dilating fibers would necessarily be infinite"(Misra and Prigogine 1983 [46, p.427]).

But this still leaves us a degeneracy to account for, and it cannot lead to a selection of semigroup. What we would need is therefore a meta-selection principle which tells us which selection principle to select.

It is clear that principle$_1$ does resolve the degeneracy between the two transformed semigroups, but this raises of course Q_2: Does principle$_1$ succeed in establishing intrinsic irreversibility? What we therefore need according to SBSS is two independent arguments.

A necessary condition for symmetrybreaking is that we are dealing with intrinsically irreversible systems. One of their defining conditions is that there exists a class of states that is physically realizable. Selection principle$_1$ demands to choose these physically realizable systems.

But physical realizability is then used twice. First it is a necessary defining condition for an intrinsic irreversible system in order to break symmetry. As Courbage notes "the admissible states are defined eventually as a requirement of the physical system". Secondly, the realizability is used as selection argument. This is circular since we do not have two independent arguments and the answer to Q_2 is obviously no. The circularity arises because the BS explicitly refers to 'realizability' in a mathematical formalism, which is of course questionable in itself. Prigogine should give a definition of an intrinsic irreversible system without referring to the empirical reality.

It is therefore clear that we have reached a point here far removed from Prigogine's original attempt at deriving irreversibility due to the dynamics alone, since he writes:

> "It should be noted that in our formulation, the second law is not a consequence of dynamics. It involves a supplementary phenomenological element (the impossibility of realizing a set of converging trajectories)."(Courbage and Prigogine [24, p.2412])

In this passage Popper's spirit in order to explain irreversibility resounds. But was not Prigogine's original attempt aimed at intrinsic irreversibility due to the dynamics alone, without taking recourse to a form of course graining or a loss of information.

It is even a bit ironical that in the approach where we have drifted away most from Prigogine's original argumentation, the name of the concept defined as intrinsically irreversible, reflects this motivation maximally.

4.4 The Later Years

It is (unfortunately) beyond the scope of this paper to delve very deep in the later approaches that were developed by the BS. But as we will see the SBSS mechanism lies at the heart of the later approaches to irreversibility as well. A complicating factor is that later works often refer to the earlier results, as if all results are enhanced and supported by one another. But this may seem dubious having seen the considerable differences in the early approaches. I will distinguish again three different approaches, and characterize the formalism and interpretation of each of them briefly with a focus on the way semigroups derived and mentioned.

4.4.1 An Alternative to Quantum Theory

Two articles published in 1988 entitled *Poincaré's Theorem and Unitary Transformations for Classical and Quantum Systems* (Petrosky and Prigogine [50]) and *An Alternative to Quantum Theory* (Prigogine and Petrosky [65]) mark the beginning of a new approach formulated directly in QM, no longer using the Koopman formalism.

- The quantum mechanical version of Poincaré's celebrated theorem (on the nonexistence of analytical invariants of motion) states that a unitary transformation diagonalizing the Hamiltonian diverges, whenever the collision operator (the same as in CD) does not vanish.

- The asymptotic collision operator diverges because in the limit of large systems continuous sets of resonances appear.[24]

- A generalization of QM is proposed based on an extension of the traditional diagonalization procedure of operators. Instead the weaker requirement of block diagonalization of superoperators is used, which leads to the construction of a nonfactorizable superoperator Λ with broken timesymmetry.

- "A striking consequence of the broken time symmetry is that we can distinguish between a semigroup in which equilibrium is reached in the future and a semigroup oriented towards the past on the level of observables"([65, p.480]) as expressed by
$$H^{\Lambda'} \neq H^{\Lambda} \tag{4.102}$$

It is clear that the new approach is heavily based upon the CD approach. The non-vanishing of the collision operator for example is the same condition as the dissipativity condition. Λ is again a star-unitary superoperator. "However, we believe that the present paper goes far beyond the earlier work, as we present here a complete recursive construction of the non-unitary transformation Λ valid for all orders in the coupling constant through an appropriate generalization of the unitary transformation theory.."([65, p.462]). The main difference is that Λ is no longer constructed using 'supplementary assumptions' (about the resolvent of L), but by a recursive construction in which it is assumed that Λ reduces to the unitary transformation for systems with a discrete spectrum. As a result Λ is now used to define the Π operators (and not vice versa).

The interpretation of the approach is reminiscent and analogous to the one we encountered in the NC approach.

> "The analogy between lifetimes and positive Lyapounov exponents in classical mechanics is striking. In both cases they limit the use of idealizations such as trajectories or wave functions to describe the time evolution of classical or quantum systems"(ibid p.481)
>
> "A direct consequence of the block diagonality of H^{Λ} is the time-energy uncertainty relation"(ibid p.476).

[24] I refer to Bishop ([14]) for the details.

> "As we are describing the time change in the form of a semigroup, there is no longer any need for the collapse of the wave function"(ibid p.482)
>
> "The new approach outlined in this paper takes into account the *intrinsic* stochasticity and irreversibility of quantum processes, which appears when unstable states are involved"(ibid p.481)
>
> "The unification between QM and dissipation leads to the conclusion that the 'arrow of time' can be detected even on the microscopical level by the observation of the energy levels of quantum states"(ibid p.481)
>
> "As the direction of time is a global property of our universe, in agreement with the present outlook in cosmology (..) a quantum system (..) would be embedded in the semigroup description of the evolution of our universe as a whole"(ibid p.469)

The selection argument for the future oriented semigroup of ES approach is clearly surpassed. As Prigogine from now on writes: "Irreversibility emerges naturally as the selection of the semigroup compatible with our future observations"(Antoniou and Prigogine [3, p.443]).

4.4.2 Rigged Hilbert Spaces

Since 1991 the BS uses the so-called Rigged Hilbert Space formulation of QM (originally due to Bohm and Gadella *Dirac Kets, Gamov Vectors and Gel'fand Triplets* 1989 [15]) for their analysis of irreversibility. The RHS approach is put forward in *Intrinsic Irreversibility and Integrability of Dynamics* (Antoniou and Prigogine 1993 [3]). A short introduction to RHS is given in the appendix. This work has been analyzed by Bishop. His main point has been to argue that the choice for RHS was motivated by (i) technical necessity because of the problems that existed for the definition of Dirac delta functions in the Koopmanian superspace (see Batterman's criticism on the delocalising property of Λ) (ii) RHS provides a representation of dynamics in which the arrow of time is intrinsically manifested.

- Again, in line with the '88 approach of the previous section, there arise difficulties for non-trivial physically interesting systems (such as scattering / interacting fields), which display a continuous spectrum, due to the earlier mentioned resonances. These systems are now explicitly called **large Poincaré systems**.

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- These difficulties are manifestations of the fact that the eigenvalue problem for unbounded self-adjoint operators is not computable in Hilbert Space (as shown by Pour-el and Richards). However in suitable RHS a so-called **time ordering rule** (TOR) provides a constructive algorithm for the eigenvalue problem of unbounded operators.

- The time-ordering rule is seen as a generalized boundary condition necessary to uniquely determine the solution of the dynamical equation. Terms corresponding to excitation processes are oriented towards the past and advanced propagators are used, while terms corresponding to de-excitations and emission of radiation are future-oriented through retarded propagators. This generalized boundary condition results in time-oriented solutions of the reversible system.

- The Friedrichs model (coupling of a particle to a scalar field) becomes now the main working example, comparable to the baker transformation in the early years. A new spectral decomposition of the Hamiltonian operator is derived for the model, in which the eigenvalue problem is extended from the real axis to the complex plane, which is seen as a qualifying feature of unstable dynamical systems which reveals their intrinsically irreversible character.

- The unitary group solution of the Schrödinger equation when extended to the new representation splits into two semigroups, one decaying in the future and the other in the past.

The formalism is interpreted as follows:

> "This split shows clearly the significance of irreversibility as selection of the semigroup compatible with future observations."([3, p.445])
>
> "The analogy of the two semigroups of the Friedrichs model the two semigroups of Kolmogorov systems is striking."(ibid p.462)[25]
>
> "The only possibility for a coherent world view is to have both irreversbility and probability at the fundamental level (..) We obtain eigendensities not reducible to trajectories and wavefunctions, living in extended Liouville spaces which provide an intrinsic irreversible and probabilistic representation of dynamics."(ibid p.462)

[25] In *Generalized spectral decomposition of the β-adic Baker's Transformation and Intrinsic Irreversibility* (Antoniou and Tasaki 1992 [2]) a generalized spectral decomposition of the Frobenius-Perron operator is obtained. The unitary FP operator when extended split sinto two semigroups.

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The BAG seem to justify the selection of the future oriented semigroup by means of the TOR. In that sense the TOR can be compared with the CD approach in which we choose retarded instead of advanced solutions.

4.4.3 Embedding

Already in 1980 interest arose of the BS in the inverse (or converse) problem of statistical physics: how to embed probabilistic processes into larger deterministic dynamics. In an article entitled *On Converse to Koopman's Lemma* ([32]) Goodrich *et al* show that under certain (physically motivated) conditions a unitary operator family U_t can be made to generate a corresponding underlying family T_t of point transformations. In *From Probabilistic Descriptions to Deterministic Dynamics* ([4])[26] the physically significant converse question is addressed whether the MPC-theory can explain the observed phenomenological Markov semigroups like the kinetic or diffusion processes, as projections of Kolmogorov or mixing dynamical systems. More specifically, when can a Markov semigroup be 'lifted' to a unitary 'superdynamics' induced by an underlying point transformation?

In a later article entitled *On the Inverse Problem of Statistical Physics: from Irreversible Semigroups to Chaotic Dynamics* (1998 [6]) it is shown that all measure preserving stationary Markov processes arise as projections of Kolmogorov dynamical systems which have positive entropy production and are prototypes of chaos. However, this leaves the general problem unsolved of intertwining by a Λ transformation an arbitrary Markov semigroup with some determinate dynamics. Because of that the formalism developed by Mackey in *Time's Arrow: the Origin of Thermodynamic Behavior* (1993 [42]) is more promising and elucidating.

Mackey explores as an alternative to coarse graining the consequences of having an invertible dynamics in which not all variables are observable.[27] This means essentially that we have a dynamical system operating in an *m*-dimensional space, but are able to observe only $n < m$ of these variables. That is, we observe only a *trace* of the dynamical system in an *n*-dimensional space because (m - n) of the variables are hidden to us, e.g. because either we do not know about them, or do not have the technology to measure them. Mackey proves that *every continuous trajectory (function) in a phase space X is the trace of a single dynamical system operating in*

[26] Antoniou and Gustafson choose this title as the converse to the title of the basic paper of Misra, Courbage and Prigogine *From Dynamics to Probabilistic Descriptions* 1979.

[27] Coarse graining is according to Mackey not a viable route because it induces the entropy of the coarse grained density to approach the equilibrium entropy for both the positive and negative times ('irrespective of the direction of time')

$$
\begin{tikzcd}
Y \arrow[r, "S_t"] \arrow[d, "F"'] & Y \arrow[d, "F"] \\
X \arrow[r, "T_t"'] & X
\end{tikzcd} .
$$

a higher dimensional phase space Y. This has important consequences for the conditional entropy, because for every invertible system the entropy is always constant, but this need not be the case for the entropy of a density evolving under the action of the trace of a dynamical system. Mackey defines accordingly the notion of a factor of a transformation as follows (see figure 4.4.3):

> **Definition:** Let X and Y be two different phase spaces with normalized measures μ_f and μ_g and associated uniform densities f and g respectively, and $T_t : X \to X$ and $S_t : Y \to Y$ be two measure preserving transformations. If there is a transformation $F : Y \to X$ that is also measure preserving i.e. if $\mu_g(F^{-1}(A)) = \mu_f(A)$ for all subsets A of the pase space X, and such that $T_t \circ F = F \circ S_t$, then T_t is called a factor of S_t. Therefore *a trace of the system S_t is a trajectory of the factor T_t.* [28]

The following theorem by Rochlin 1964 expresses formally the connection between these concepts and the behaviour of entropy ([42, p.110-4]), since Mackey has shown that the entropy of an f_* exact transformation[29] increases smoothly to its maximum value of zero, irrespective of the initial density with which the system was prepared.

> **Theorem Rochlin**: Every f_* exact transformation is the factor of a K automorphism.

[28] Cf. the baker transformation: the trace of the baker transformation is the dyadic transformation. Since this transformation is not invertible we have no problem with the aforementioned approach to equilibrium for both time directions.

[29] i.e. if $\lim_{t\to\infty} \mu_f(S_t(A)) = 1$

The relation between probabilistic descriptions and deterministic dynamics is now elucidated by Mackey as the former is a factor of the latter effected by a projection:

> "The transformation F involved in our discussion of factors and traces is precisely what Misra *et al* (1979) and Misra and Prigogine (1981) refer to as a projection operator in their work on the generation of noninvertible behavior from invertible dynamics. The Rochlin Theorem serves as the analytic link in their work between invertible K automorphisms and f_* exact transformations (or strong or monotonic Markov operators.)"(Mackey [42, p.113])

But Mackey accomplishes more. Given any Markov operator P it is always possible to construct a sequence of deterministic transformations $\{S_n\}$ such that the limiting value of $P^t_{S_n} f$ approximates $P^t f$ as closely as one likes.

- **Theorem Mackey**: Let P be a given Markov operator in a finite normalized space X = [0,1). Then there is a sequence of transformations $S_n X \to X$ with corresponding Frobenius-Perron operators P_{S_n} such that

$$\lim_{n\to\infty} \| P_{S_n} f - Pf \| = 0 \tag{4.103}$$

According to Mackey (ibid p.138), 'the consequences of this theorem are extremely far reaching, for it tells us that *any Markov operator*, whether it arises from the influence of random or deterministic perturbations on a totally deterministic system, can always be approximated by a totally deterministic system to any degree of accuracy. In fact, the proof of the theorem offers a constructive way of finding this approximating system'.

Mackey's formalism has elucidated the NC approach and the relation between probabilistic descriptions and deterministic dynamics. The embedding approach might be seen indeed as an alternative for coarse graining, but from an interpretational point of view it seems very far removed from the original intention of the BS to ground intrinsic irreversibility in the dynamics. First of all, in the embedding approach we explicitly demand that we are only able to observe a subset of all variables, hence we loose information due to epistemological constraints. Secondly, it becomes even harder to maintain that the probabilistic description is the primary physical picture, if we prove that we can embed any Markov process in a deterministic dynamics.

4.5 Conclusions

In this chapter I have given a chronological overview of the relation between semigroups, symmetry and irreversibility in Prigogine's theories, focusing on the 1969-1985 time frame, called by Bishop the early years. My conclusions are as follows:

- I have argued that we have to distinguish in the early time frame between at least three different approaches: causal dynamics; new complementarity; entropy as a selector. In later years I have briefly characterized another three approaches: an alternative to quantum mechanics; rigged Hilbert spaces; embedding.
- These different approaches are often conflated in the literature. We have seen that Karakostas and O&B did not properly distinguish between the CD and NC approaches, hereby conflating the different definitions of Λ. Because of that, they failed to discuss the retarded - advanced picture. Bishop does not properly distinguish between the NC and ES approach. This conflation is encouraged because the BS itself pretends that all approaches are strengthening another and intertwined. We have seen however that there is not only a considerable change in formalism, but also in interpretation of these different approaches.
- We have seen that SBSS is an appropriate heuristic in understanding the work of the BS. In the first four approaches the symmetrybreaking is provided by a nonunitary transformation. We see however a clear development in thinking about the selection argument. In the CD approach retarded solutions are chosen by various reasons, which I argued to be fallacious. The NC approach itself is not sufficient to involve a clear selection as stated by Courbage in 1983. The ES approach is therefore explicitly developed as means to overcome the degeneracy, which I argued to be circular.
- From ES onwards, the selection argument consists of an appeal to phenomenological observations about time asymmetry. The future oriented semigroup is naturally selected. The most persistent problem is to give a satisfying selection argument for the future oriented group. This type of argument is still lacking in all the BS approaches.
- Problematic as well in all approaches is that it is not easy to demonstrate that real physical systems possess the properties required for their theory such as; DC, 'intrinsic randomness'. These conditions require checking procedures for

every real physical system (which is practically impossible), and were only demonstrated for models such as the baker transformation and the Friedrich's model.

- Results regarding embedding and the delocalizing properties of Λ were reproduced by Mackey. Mackey however deals extensively with the question, not only how to account for the existence of a unique state of equilibrium, but also whether the equilibrium state will be reached, and moreover, whether it is a unique minimal state and not a meta-stable state. The Brussels School never tried to argue for the uniqueness of the state of equilibrium. Mackey's formalism is in that respect more promising.

- I conclude that the Brussels School has not given us a convincing account of intrinsic irreversibility. Prigogine's interpretation of CD was argued to be effectively a form of coarse graining. In the ES approach intrinsic irreversibility is explicitly defined with an appeal to phenomenological facts. In the embedding approach epistemological constraints play an important role. As Bishop writes: "Appealing to experimental observations of time asymmetry, whereby taking phenomenological laws as fundamental and thereby excluding all definitions of entropy that license anti-thermodynamic behavior, seems to beg the question".

Indeed, if we use phenomenological observations in order to 'derive' intrinsic irreversibility, one wonders why Prigogine not simply argues for the fundamentality of the phenomenological laws. We arrive again at the tension that was observed in chapter two between the physical and philosophical perspective on revalidating time. An interpretation of Prigogine's work in which no longer a selection argument is given, will be developed in the next chapter and its plausibility discussed.

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Chapter 5

Puzzling Prigogine

> Indeed, it seems not a very great exaggeration to say that the main problem with 'the problem of the direction of time' is to figure out what the problem is supposed to be!
>
> – Earman 1974 [28]

In this chapter I will discuss philosophical criticisms directed at the validity of SBSS itself. As an appetizer I will briefly describe the position of Bricmont, who claims that the problem of time's arrow has been already solved by a proper appeal to initial conditions and a micro-macro distinction. It will be argued that Bricmont's understanding of time's arrow problem is too simple: appealing to initial conditions without arguing for the specific character of them merely shifts the problem. More important is that Bricmont fails to realize the symmetry of statistical explanations.

Next I will discuss criticism of Sklar. Sklar dismisses the SBSS mechanism as a viable approach when he writes: "we have introduced a selection principle which works. We have not explained why such a principle correctly describes the world" ([75, p.215]). Sklar and Prigogine differ in opinion about what is puzzling about the arrow of time. Accordingly I will discuss criticism of Karakostas and Price on the BS that they deliver a piece of mathematical magic, in deriving asymmetry from symmetrical premisses alone. I will demonstrate that this mispresents Prigogine's (later) position. Prigogine agrees to what I will identify as a *conservation of symmetry in reasoning* principle. Price later refines his criticism towards the sharpened point that Prigogine needs two arguments (asymmetries in Price's words) to account for irreversibility. Price's counterproposal consists of using only one asymmetry (one argument). Prigogine might react however that Price's position is based on see-

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ing symmetrical laws as fundamental. If we claim in contrast a fundamental status for irreversible processes the selection argument becomes redundant, which might explain why Prigogine never tries to provide one in later years. Price and Prigogine are thus disagreeing about what ought to be explained.

Prigogine's proposal of crediting irreversible processes with a fundamental status is connected to the philosophical question that was raised in chapter two, namely, whether Prigogine adheres to an anti-reductionist position. Conservation of symmetry in reasoning leaves us at least two possible routes. The monistic route consists in perceiving reality as asymmetrical and therefore criticizes the limitid theoretical practice of physics. It is not directly related to reductionism. The second route has a dualistic ontology consisting either of reversible and irreversible processes or of unstable and stable systems. The second ontology is more in agreement with the aim of deriving intrinsic irreversibility for unstable systems. In what ways the dualistic route implies anti-reductionism will be discussed. Conflicting statements of Prigogine make it impossible to argue in favor of any of these routes. Hence the title of the chapter will be justified.

5.1 The Discussion between Bricmont and Prigogine

Bricmont in *Science of Chaos or Chaos in Science* (1995 [21]) criticizes Prigogine's views on irreversibility. Especially Prigogine's books aimed at the popularisation of science are attacked. Bricmont's basic point is:

> "it is perfectly possible to give a natural account of irreversible phenomena on the basis of reversible fundamental laws, and of suitable assumptions about initial conditions. This was done a century ago by Boltzmann, and despite numerous misunderstandings and misguided objections (some of them coming from famous scientists, such as Zermelo or Poincaré), his explanation still holds today "([21, p.11]).

Beside an appeal to initial conditions, there is another basic ingredient for the classical explanation of irreversibility: "we have to distinguish between microscopic and macroscopic variables."(ibid p.12). Thus Bricmont rejects Prigogine's aim of developing intrisic irreversibility as an alternative to the subjectivity involved in coarse graining:

> "There is no subjectivism here: the evolution of the macroscopic variables is objectively determined by the microscopic ones, and they behave as they do whether we look at them or not. In that sense they are

> completely objective. But it is true that, if we look at a single molecule, or at a collection of molecules represented by a point in phase space, there is no sense in which they evolve 'irreversible', if we are not willing to consider some of the macroscopic variables that they determine"([21, p.18]).

Subjectivity has been much overrated according to Bricmont, and the charge of subjectivity should be completely reversed: "to 'explain' irreversibility through the behaviour of probability distributions (which *are* describing our ignorance), as Prigogine does, is to proceed as if the limitations of human knowledge played a fundamental physical role" (ibid p.19). Bricmont therefore claims that Prigogine misunderstands what irreversible phenomena really are.

But this completely misconstrues Prigogines position. Prigogine does not want to 'explain' irreversibility by 'introducing' probability distributions, instead he wants to argue that it is an inherent property of certain microscopic systems that they are 'intrinsically random". For Prigogine, irreversibility is not rooted in, or a consequence of, the micro/macro distinction as Bricmont argues.

I want to turn my attention now to the first ingredient which is necessary according to Bricmont for the explanation of irreversibility, an appeal to initial conditions. "The real problem is not to explain why one goes to equilibrium, but why there are systems out of equilibrium in the first place (..) the only real problem with irreversibility is not to explain irreversible behaviour in the future, but to account for the 'exceptional' conditions of the universe in the past" (ibid p.16). Is appealing to initial conditions as Bricmont argues a sufficient way of explaining irreversibility?

That depends of course on how we perceive the problem that has to be solved. If we have a time symmetrical microscopic law and we want to explain the present asymmetric behaviour (as a result of earlier behaviour) appealing to initial conditions is fine. But if the problem is to explain why there is asymmetry at all, appealing to initial conditions instead of appealing to final conditions, merely shifts the problem. The choice for initial conditions already presupposes a form of time asymmetry because this appeal overtly reasons from only the past (and thus not the future). In other words, for this perception of the problem, it is not enough as Bricmont argues to refer to initial conditions. We have to give an argument why we choose that particular initial or in Price's words, one-end-boundary-condition. Bricmont fails to account for the exceptional initial conditions and therefore only shifts the problem without

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solving it.[1]

A more fundamental problem with Bricmont's position is that the asymmetrical law we are talking about, the second law of thermodynamics, is interpreted by him in a statistical manner as the number of possible microstates consistent with a certain macrostate. But statistical explanations are necessarily time symmetrical. If we infer that the probability of arriving at a certain future state is great, by the same reasoning we have to conclude that, the same probability holds that it was the preceding state. As we have seen when discussing the Kac ring model, having an unprobable initial state, statistical arguments predict entropy increase in either time direction.[2] I conclude therefore that the modern problem of time's arrow (luckily) has not been solved yet by Bricmont, so let's proceed with our discussion on the validity of the SBSS mechanism.

5.2 (A)Symmetrical Principles

I have argued that for Prigogine SBSS is the framework within which we can in principle give a satisfactory explanation of irreversibilty. Sklar questions this validity since he writes: "we have introduced a selection principle which works. We have not explained why such a principle correctly describes the world"([75, p.215]). Sklar is very pessimistic about ever being able to answer the problem of time's arrow due to the so-called *parity of reasoning problem*:

> "[1] All recent attempts to explain the origin of the thermodynamic arrow fail, because they introduce overtly or covertly a time asymmetric postulate, which overtly or covertly begs the question at issue, or [2] because it fails to recognize that its rationale for the increase of entropy of a system in the future direction would also serve as a rationale for the entropy increase in the past time direction."(Sklar [75])

Let me first discuss the second statement [2]. Framed as a criticism on the SBSS approach the parity of reasoning problem would be: for the scheme of SBSS there exists also a scheme which may be called SBOSS, in which the other semigroup

[1] Price mentions for example two possible arguments to explain the one-end-boundary-condition (i) anthropic reasoning à la Boltzmann (ii) cosmological principle à la Penrose and Hawking.

[2] The symmetry is often overlooked as in the Encyclopaedia Britannica: "It is true that the second law of thermodynamics is time asymmetrical; but this law is not strictly true (..) and it is now regarded as a statistical derivative of the fundamental laws together with certain boundary conditions".

would have been selected. Namely due to the time-reversed argument of the original argument by which we had selected one of the semigroups. Valid as Sklar's argument may be, it leads us nowhere except perhaps into an unfalsifiable skeptical position with respect to ever properly explaining time's arrow. Attempts to explain time asymmetric behavior necessitate the use of asymmetrical assumptions. We have to get the machine from the ground somewhere. I will now turn my attention to [1] in the above quotation.

The content of this objection is spelled out particularly clearly in relation to the BS in certain criticisms of Karakostas and Price. This will lead to the identification of what I will subsequently refer to as the *conservation of symmetry in reasoning* principle. Karakostas (1996 [37]) advances the following general argument which shows, independently of the mathematical formalism of the theory concerned, that asymmetrical conclusions cannot be derived from symmetrical premisses, and hence that the Brussels School approach is question begging.

> "To see this, let *T* stand for the Brussels Schools's subdynamics approach or any other theoretical scheme whose purpose is to describe the time evolution of physical systems towards equilibrium. Hence, by definition, *T* deals with an initial nonequilibrium state *W(t)* which is confined to a certain region of its phase space at time *t*. The approach then of the system to equilibrium at any time *t'* is determined by *W(t')*. The decisive question is whether, in order to predict *W(t')*, *T* needs to take into account whether *t'* is actually later or earlier than *t*, in the ordinary sense of macroscopic time. If not, then *T* is symmetric with respect to time reversal, allows *W(t')* to reach equilibrium either in the far future or in the far past alike, and therefore incorporates no preferred direction of time. If on the other hand, *T* depends upon the further information of time ordering, then this can only be as a consequence of the fact that *T* already treated the future and past directions of time differently –for otherwise the extra information would be redundant. As in this case therefore any preferred direction of time would have been built into the structure of *T* right from the beginning as it is the case with the Brussels school's microscopic theory of irreversible processes."([37, p.397-8])

The basic point Karakostas makes here is, that obtaining asymmetry from symmetry is logically impossible. Any theory that attempts to derive asymmetry for symmetrical premisses must involve an asymmetrical premisse somewhere and it that sense begs the question. A very similar type of argument is given by Huw Price.

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In the chapter *The lessons of the second law* ([55]) Price criticizes what he calls the application of non-linear methods developed by the Brussels School to solve the conflict between thermodynamics and physics. Price's point is that the Brussels School fails to understand the real puzzle of thermodynamics. As I have already discussed in chapter two, while elaborating on Boltzmann's H-theorem, Price's point of view is that the puzzling asymmetry of thermodynamics consists in the fact that entropy is low in the past, not in the fact that there is a law which states that entropy increases.[3]

> "The nonlinear approach may tell us how certain nonequilibrium systems behave, *given that there are some*, but it doesn't explain how the gradient comes to exist in the first place. This is the great mystery of the subject, and the theory of nonequilibrium systems simply does not touch it. Suppose that the proponents of the nonlinear dynamical method - or any other dynamical method, for that matter -claim that despite the fact that it is a symmetric theory, it produces asymmetric consequences in thermodynamics, and hence avoids the old paradoxes of reversibility. To undermine their claim, we describe an example of a physical system to which the new method is supposed to apply, specifying its state at some time t. We then ask our opponents to tell us the state of the system at another time, say $t + 1$, *without being told whether $t + 1$ is actually earlier or later than t.* (That is, without being told whether a positive time interval in our description corresponds to a later or an earlier time in the real world.) If our opponents are able to produce an answer without this extra information, then their theory must be time-symmetric, for it generates the same results in both temporal directions. If they need the extra information, on the other hand, this can only be because at some point their theory treats the two directions of time differently – like Boltzmann's original H-theorem, in effect it slips in some asymmetry at the beginning. So in neither case do we get what the advocates of this approach call 'symmetry-breaking': a temporal asymmetry which arises where there was none before. Either there is no temporal asymmetry at any stage, or it is there from the beginning." ([55, p.43])

Both Karakostas and Price refer to what I will call a logical 'law'. I will refer to this as

[3] Price's point is similar to Bricmont's. Bricmont however fails to realize the symmetry of statistical explanations.

- **CSR** Conservation of Symmetry in Reasoning principle: Either there is no temporal asymmetry in the conclusion, or it is there in the assumptions

As simple as Karakostas and Price's line of reasoning may appear, the conclusion of their argument leads to an impossibillity argument for any theory whose purpose it is to explain asymmetry by symmetrical premisses alone. Their accusation is that that is precisely Prigogine's intention. Is it? A discussion that was held between Prigogine and Price will hopefully shed some light on this issue.

5.3 The Discussion between Price and Prigogine

In this section I will reconstruct a discussion in which Prigogine and Price (though indirectly) engaged, that will hopefully shed some light on Prigogine's philosophical position. Price argues for a new Archimedean point from which to evaluate philosophical positions in the philosophy of time, called by Price 'the view from nowhen' reminiscent on Thomas Nagel's 'view from nowhere'. Most philosophical debates are thoroughly blurred according to Price, because the vocabulary that is being used already contains time asymmetrical notions as 'future, 'earlier' etc. His 'view from nowhen' can be interpreted as a methodological principle aimed at freeing philosophy from these mystifications. Whether he succeeds in doing so, is not the topic I want to consider, the exchange of arguments is what I am primarily interested in.

What is the discussion about? First of all in a book review of Price's book Prigogine remarks:

> "It does not seem an exaggeration to say that recent years have witnessed a radical change of perspective in science. Classical science emphasized stability and equilibrium - now we see instabilities ranging from cosmology and atomic physics through fluid dynamics, chemistry and biology (..) in all these processes distance from equilibrium and therefore the arrow of time, has an essential role, somewhat like temperature in equilibrium physics (..) life is associated with non equilibrium processes (..) Irreversibility is not an universal phenomenon; the two-body problem (of the harmonic oscillator) are well-described by the time-reversible laws of dynamics." ([67])

Price's position is called by Priogine a self-defeating position, because it amounts to going back to the atemporal perspective of Parmenides. If there is no flow of time, how about life and human existence? asks Prigogine rhetorically. Prigogine's argues for a different reading of Price's book. He agrees with Price's analysis of the frequent use of double standards in physics i.e. applying an argument selectively while it could equally well be used in either temporal direction. This use of a double standard constitutes the heart of Price criticism of the BAG approach i.e. "attempting a piece of mathematical magic: to derive an asymmetric result from symmetric mechanical theories (without putting the symmetry in at the beginning, primitive and unexplained"(Price [86]). Yet, although Prigogine agrees with the above stated analysis by Price, he criticizes Price instead for considering

> "only the first of these two possibilities which leads him to his radical conclusion that the flow of time is associated with an anthropomorphic view of the universe. If instead we consider the second possibility, the time-reversible laws of classical and quantum mechanics have to be extended when we deal with situations in which we expect to find time asymmetry. While the standard formulation of classical or QM consists of dynamical groups involving time symmetry, in important situations associated with chaos or in large systems formed by many interacting units, we find that we obtain semigroups that include the distinction between past and future".([68])

Prigogine herewith seems to disqualify Price's criticism as misunderstanding the claims of the Brussels School. The Brussels School does not attempt to derive asymmetry from symmetrical premisses but 'extends' the time-reversible laws when asymmetry is to be expected. Prigogine explains this point of view as follows:

> "In nature there are two types of processes: reversible processes described by a group without temporal asymmetry and irreversible ones described by semigroups where the asymmetry is already there from the beginning" (..) "Price's attempt to eradicate the flow of time does not seem to lead to any new insights into the difficulties of modern physics."(ibid) [4]

This suggestion by Prigogine of an ontology based on both types of groups is elaborated in the following manner:

> "In chapter two Price considers 'The lessons of the second law'. He describes the numerous attempts to derive the increase of entropy despite

[4]Which can be called ironical because as we have seen must authors criticize the Brussels School exactly for the same reason.

> the time reversibility of the laws of dynamics. As already stated, his critique is right. But is this the whole story? Transport equations involving diffusion or heat conduction are irreversible processes in which past and future play different roles. They are as 'real' as, say, planetary motion; for example the diffusion coefficient can be measured to as many places of accuracy as experimental devices permit. Moreover, we know that irreversible processes play an essential role in life. Is life then the result of an anthropomorphic illusion? The difficulty is to relate these irreversible processes to the time symmetrical laws of dynamics. To dismiss them seems a strange attitude. Physics as a whole just is not time symmetric as assumed by Price."(ibid)

Price has however a more fundamental reason for dismissing the Brussel School approach: "its more basic mistake is to misunderstand what is actually puzzling about the thermodynamic arrow. The odd thing is not that entropy is high in the future, but that it is not high at all times. The Brussels School methods do a fine job of telling us how nonequilibrium systems behave, *given that there are some*. However, they do not tell us anything about why such systems exist in the first place (and all oriented in the same time direction)".([55, p.43])[5]

Price later refined his criticism on the Brussels School. His main problem with their approach is that they use two asymmetries to explain irreversibility. They use an asymmetric law (the second law) which states that entropy increases. They also need an asymmetrical boundary condition (a time ordering rule) which fixes the entropy at a certain point in time. Price's own approach would be to advocate a one-asymmetry explanation of time's arrow consisting of using just an one-end-boundary condition which states that entropy is low in the past, and then resorting to the time symmetrical

[5]This last phrase within parentheses stems from Price's review (Nature 348 22-11-1990 p.336) of the book *The Arrow of Time* (Coveney and Highfield [25]), which has a forword by Prigogine. Coveney and Highfield in their book soon make clear that they want to argue that the key to the puzzle of the arrow of time lies in some of the insights the Brussels School. However according to Price 'the book's basic mistake is to fail to properly distinguish several quite separate issues about time. For a start the authors overlook an ambiguity in the arrow metaphor, and hence confuse the issue of the directionality of time (i.e. the question whether the universe is symmetric in time) with the question as to whether time flows, when they remark that 'time travels like an arrow". Price: "the arrow is best kept for simple directionality (..) It does not provide a key to the resolution of the conflict between thermodynamics and mechanics. The real puzzle is why there is an arrow of time at all. The theory of non-equilibrium positions may tell us how non-equilibrium systems behave given that there are some; but it doesn't explain how they come to be so common in the first place and all oriented in the same temporal direction".

statistical explanation given by the later Boltzmann $(S = k \log W)$ which would explain the increase of entropy conditional on this low entropy boundary condition as very probable. The symmetry of statistical explanations forces Price to conclude that a low entropy condition could equally apply to the future.

Framed within the SBSS mechanism, Price criticizes the mechanism for needing two (asymmetric) independent arguments for explaining asymmetry. Prigogine might react to Price's position as follows. Price presupposes that the fundamental laws of nature are time symmetric expressed as 'physics as a whole is not time symmetric as assumed by Price'. This remark is directly related to the reductionist assumption, which sees symmetrical laws as more fundamental than asymmetrical laws and hence that the latter ought to be reduced to the former. Prigogine thus criticizes Price for presupposing RA. If we attribute a fundamental status to irreversible processes the selection argument would be no longer necessary. This line of reasoning might explain why Prigogine never tries to provide us a selection argument in later years. The debate between Price and Prigogine is thus transformed into an debate about the validity of RA.

For the moment it suffices to conclude that Price and Prigogine differ in opinion on what is puzzling on the thermodynamic arrow. For Price the puzzle is why there is any macroscopic asymmetry given the fundamental symmetrical microscopic laws. For Prigogine this is no more puzzling than asking why there are symmetrical laws at all. Prigogine criticizes Price for preferring symmetry, while physics as a whole is just not symmetric. Does a denial of RA make Prigogine necessarily an anti-reductionist? Can Prigogine develop a consistent position in claiming a fundamental status for irreversible processes as well? These questions will be pursued in the next section.

5.4 Conflicting Interpretations

What we do know is that Prigogine adheres to CSR. Prigogine's fundamental premisse is as we have seen that there *is* asymmetry in reality (for the moment I leave unspecified of what it is predicated). There are then at least two possible routes: (1) reality is asymmetrical at all levels (monistic) or (2) reality is dualistic i.e. consisting of symmetrical and asymmetrical domains. At the same time we have a reductionist question whether macroscopic phenomena are reducible to microscopic ones. The first route is not directly related to the question of reductionism. In the second route depending on the ontology we choose reductionism becomes more important. What I want to show in this section is basically that both routes are possible interpretations of Prigogine's thinking and how they are related to the red./anti-reductionist issue.

The monistic route stresses the fundamental character of irreversible processes in reality in the sense that reversible processes are mere idealizations. This interpretation of Prigogine's position is in line with Prigogine's criticism of physics for its pertinent idealization and abstraction tendencies. We have already seen the narrative about Zeitvergessenheit in chapter two. But there are of course many ways in which physics treats idealized situations. An example given by Prigogine is that of an isolated system: "a true closed system, on the other hand, is an ideal concept - as unattainable as a perpetual motion machine. A terrarium or a space colony could be considered close approximations, but these, too, rely on external energy from the sun"([84]). Prigogine formulates his position quite clearly as: "what has changed is the perception of the relative importance of these [deterministic and indeterministic] processes. We begin to see now that the deterministic processes can be seen only in isolated, artificial systems. The natural world, on the other hand, is a world of irreversible processes, of self-organization."([84]). Beside a misidentification of deterministic processes with reversible processes, the quotation is an ontological claim that reality consists only of irreversible processes.

A monistic interpretation of Prigogine is also given by Griffin in his introduction *Time and the Fallacy of Misplaced Concreteness* of the book *Physics and the Ultimate Significance of Time: Bohm, Prigogine and Process Philosophy* ([35]).[6]

> "The fundamental character of irreversible processes is Prigogine's key thought; only this kind of process gives science the kind of time that is presupposed in all human experiences, and hence in philosophy and the humanities in general. Prigogine's program is to reveal the existence of irreversibillity at all levels, including and especially the microscopic level: there must be something at the microscopic level that provides the root of irreversibility at the macroscopic level. This latter irreversibility cannot intelligibly be thought to emerge, as a miracle, from fully reversible processes" ([35, p.17])[7]

There are a few interesting things mentioned in the quote. Griffin states that 'only an irreversible process gives science the kind of time that is presupposed in all human

[6]The book grew out of the conference held in 1984 between Priogine, David Bohm and various advocates of the school of process philosophy inspired primarily by Whitehead. Griffin attempts to interpret Prigogine's philosophy of time against a background that challengess the fundamental premise of the timelessness of physics.

[7]Based on *Order out of Chaos* ([62, p.285-98]). All references in the quotations of Griffin in this section are to *Order out of Chaos*.

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experiences'. In other words, Prigogine's view would not be that it is science's task to provide an adequate concept of time, but that science must become consistent with the primitive common sense notion of time we have from our experience. It is not the task of science to establish the meaning of time, nor give a definition, but to unify our understanding of time i.e. render our experience consistent with the treatment of time in physics.[8] The opposition that is sought is between the theoretical approach of physics on the one hand, and a non-physicist phenomenological and experimental point of view on the other.

The second point we can distill from this quote is that Griffin clearly opposes an anti-reductionist interpretation of Prigogine in which irreversibility is claimed to emerge from reversible dynamics, if for example the dynamics is complex enough. Such an interpretation seems to be advocated by for example Bishop. "Prigogine's position is anti-reductionistic in the sense that collective effects cannot be mathematically reduced to a description in terms of individual particle trajectories. A reductionist would argue that individual trajectories are both necessary and sufficient for determining the probabilistic behaviour of unstable systems."[9] But the reductionism–anti-reductionism discussion cuts across the asymmetry-symmetry discussion of section 5.2, since an anti-reductionist position in Bishop's sense does not imply that we derive an asymmetrical conclusion from symmetrical premisses.

But at other points Prigogine seems to oppose the monistic interpretation sketched above. Conflicting statements can be found. Whereas in *An Alternative to Quantum Theory* Prigogine writes "as the direction of time is a global property of our universe in agreement with the present outlook in cosmology"(Prigogine and Petrosky 1988 [65, p.461]), at other points Prigogine states that "irreversibility is not an universal phenomenon; the two-body problem (of the harmonic oscillator) are well-described by the time-reversible laws of dynamics" ([67]) and in *Order out of chaos* Prigogine and Stengers state, that "irreversibility is not a universal property"([62, p.239]). I will now turn my attention to the dualistic alternative.

From the discussion between Prigogine and Price we might conclude that Prigogine opts for a 'dualistic interpretation', in which reality consists of an ontological mixture of reversible and irreversible processes described by respectively groups and semigroups. There is a strong argument in favor of such an interpretation. Indeed

[8]Griffin calls this the Whiteheadian interpretation of Prigogine, which he owes to Stengers. "Under this interpretation Prigogine's 'artificial' would be the same as Whitehead's 'abstract'. That is, 'artificial' systems would be those which had been *conceptually* simplified to make them conform to the conceptual tools of a reversible dynamics." ([35] p.19) This would mean that even in artificial situations there is no reversibility due to the fallacy of the misplaced concreteness.

[9]Private communication.

as Uffink for example writes, "there exists another classical tradition in the study of heat phenomena, the heat equation of Fourier, the diffusion equation of Fick or transport equations. They all form a *bona fide* form of classical physics"([78, p.58]). As Prigogine writes: 'transport equations are as 'real' as, say, planetary motion; for example the diffusion coefficient can be measured to as many places of accuracy as experimental devices permit'. The dualist could argue that different branches of physics deal with different sorts of problems. So he could argue that the burden of proof lies by the reductionist if he has a problem with this dualism. The reductionist could argue instead that the question is relevant and important, how the two types of processes are interrelated. The dualistic interpretation is just denying interpretational problems instead of solving them.

There is I think another candidate for this dualism. Beside the above mentioned ontology consisting of reversible and irreversible processes one can think of an ontology based on stable and unstable systems. This ontology is less a matter of category but a matter of degree (think of the ergodic hierarchy) and accusations of anti-reductionism can be easier rebutted. This position is compatible with the physical work of Prigogine in deriving intrinsic irreversibility as irreversibility due to the dynamics alone.

Prigogine hints at a dualistic ontology consisting of unstable and stable systems, when he speaks about age operators. Prigogine alludes to Eddington's entropic clocks when he writes that, "a watch has no time in our sense. Everyday it goes back into its own past. It is we who have time, and this expresses the fact that, like all chemical systems, we belong to the category of highly unstable dynamical systems for which an object such as T can be defined"([64, p.244]). There is then as Eddington noted, on the one hand the objective time of physics of stable systems, while on the other our 'subjective' time based on entropy clocks i.e. based on the fact that we ourselves our unstable systems. This ontology would hence greatly differ from the former since it would not presuppose a primitive concept of time, but would try to define time.

However as Griffin states: "The crucial question in regard to the correctness of the above interpretation of Prigogine's position is just this, i.e. whether he means that the far-from-equilibrium processes provide the basis for a *definition* of time. If he does, then, even though he says that the entropy barrier[10] is infinite so that reversibility could never *in fact* occur, his position would suffer from the paradoxes afflicting all the other attempts to define time in terms of physical processes"([35, p.19]). Griffin alludes here to Reichenbach's proposal to identify the direction of

[10] The longer we wait the more information we need in returning to our original state.

time with the entropy ingredient, leading to absurd conclusion that when entropy decreases also time decreases. Hence I conclude along with Griffin that the dualistic interpretation consisting of (un)stable systems is not an unproblematical approach either.

The arguments in favor and against the monistic and dualistic interpretation of Prigogine's position reflect our analysis of the problem of time's arrow in chapter two. Whereas for Eddington time's arrow denoted a complex of things amongst others metaphysical questions about time, since Popper and Grünbaum the problem of time's arrow is a well-described physical problem in which a primitive concept of time is presupposed. We arrive again at the tension already noted in chapter two between the physical and philosophical perspective on revalidating time.

5.5 Conclusions

In this chapter we saw that 'oversymmetrizing' as Sklar and Price seem to do when criticizing SBSS, does not bring us much further in solving the modern puzzle regarding the arrow of time. The conservation of symmetry in reasoning principle demands that to arrive at an asymmetrical conclusion asymmetry has to be put in somewhere. It was shown that both Price and Karakostas and Prigogine adhere to this conservation principle, but differ in opinion in what is actually the problem of time's arrow that has to be solved. Whereas for Price the problem is why there is an arrow of time in the first place and hence to reconcile the observed macroscopic asymmetry with the microscopic symmetrical laws, Prigogine is primarily concerned with deriving intrinsic irreversibility (the approach to equilibrium) and hence reconciling physics with our own perception of time asymmetry. Prigogine claims a fundamental status of irreversible processes.

This lead to two possible routes: reality is asymmetrical (monistic) or is dualistic. The former route hinged on the limited practice of physics and is more or less in agreement with the Whiteheadians and Bergsonians among us in the sense that a primitive notion of time is assumed.

The second route allowed two different ontologies. An ontology based on (ir)reversible systems transformed into a burden of proof argument regarding reductionism. An ontology based on (un)stable systems seemed to be more in agreement with the phsyical work of Prigogine in which intrinsic irreversibility is derived for unstable (K) systems and could easier be defended against anti-reductionist accusations.

However, as we have seen conflicting statements can be found of Prigogine making it mere speculation to argue in favor of any of these positions. What could be the case of course is that Prigogine's thinking about revalidating time has gradually shifted over the years from a more monistic point of view towards a dualistic position (hence solving the inconsistencies). But it is impossible to make this precise with the material I have found up till now.

A lot of confusion in the literature has arisen simply because others have taken Prigogine to mean with concepts as becoming, irreversibility and time's arrow precisely what they themselves mean with it. But as I have hoped to argue in this paper, this assumption is not easy to maintain. Until Prigogine clarifies his position it remains an incomplete puzzle.

Chapter 6

Appendix

6.1 Master Equations and Markov Processes

[1] A principal tool in the study of NESM is the master equation. This is one of a family of closely related equations; the Fokker-Planck equation, the Chapman-Kolmogorov equation, as well as various transport equations. The framework of all these equations is that of a Markov process. The guiding thought of a Markov process can be illustrated as follows.

Suppose that we are not longer interested in the microscopic, deterministic evolution, but want to investigate instead the evolution of macroscopic states. On a macroscopic level the evolution is no longer deterministic. We have to resort therefore to transition probabilities, from a given begin state to a certain end state. As it is generally impossible to characterize this evolution, we resort to a distribution of these states, which is called a stochastic process. An important example of a stochastic process can be derived if we consider a process in which the conditional transitional probabilities $P_{(1|n)}$ only depend on the condition on the latest instance. This constitutes a Markov process:

$$P_{(1|n)}(y_n, t_n \mid y_{n-1}, t_{n-1} : ... : y_n, t_n) = P_{(1|1)}(y_n, t_n \mid y_{n-1}, t_{n-1}) \tag{6.1}$$

A Markov process can be interpreted as a process without memory, because extra information about the more distant past does not increase our future estimates. Important to realize is that the definition of a Markov process is not strictly asymmetrical. The choice to conditionalize on *earlier* values is special. We could equally

[1] This section is based upon the syllabus *Grondslagen van de thermische en statistische fysica* [79].

well enquire about the chance of obtaining a certain value in the past, given that *later* behaviour is given.

The transition probability $P_{(1|1)}$ obeys the so-called Chapman-Kolmogorov equation:

$$P_{(1|1)}(y_3, t_3 \mid y_1, t_1) = \sum_{y_2} P_{(1|1)}(y_3, t_3 \mid y_2, t_2) P_{(1|1)}(y_2, t_2 \mid y_1, t_1),\ t_1 < t_2 < t_3 \tag{6.2}$$

A special assumption is *homogeneity*: a Markov process is called homogeneous iff its transition probability does not depend explicitly on t_1 and t_2 but only on their difference $t = t_2 - t_1$. We may then write

$$P_{(1|1)}(y_2, t_2 \mid y_1, t_1) = T_t(y_2, y_1) \tag{6.3}$$

The Chapman-Kolmogorov equation can then be interpreted as the semigroup property of these evolution operators T_t.

6.2 The $\tilde{\Sigma}$ Operator

In this section I will follow the '69 ([58]) notation in deriving the expression for the $\tilde{\Sigma}$ operator (4.24) in terms of the creation and destruction operators. Secondly, the semigroup property of $\tilde{\Sigma}$ will be derived.

First of all Prigogine decomposes the Liouville-von Neumann operator in an unperturbed part L_0 and a perturbation δL

$$L = L_0 + \delta L \tag{6.4}$$

L_0 is diagonal in the representation we wok in, and to the vacuum state corresponds the zero eigenvalue of L_0. In terms of L_0 we introduce the unperturbed resolvent operator

$$R^0(z) = \frac{1}{z - L_0} \tag{6.5}$$

Prigogine defines the following irreducible operators:

$$\psi(z) = \langle o \mid \delta L \sum_{n=1}^{\infty} (R^o(z)\delta L)^n \mid o\rangle_{irr} \text{ collision operator} \tag{6.6}$$

$$\mathcal{D}_c(z) = \langle o \mid \sum_{n=1}^{\infty} (\delta L R^0(z))^n \mid c\rangle_{irr} \text{ destruction operator} \tag{6.7}$$

$$\mathcal{C}_c(z) = \langle c \mid \sum_{n=1}^{\infty} (R^0(z)\delta L)^n \mid o\rangle_{irr} \text{ creation operator} \tag{6.8}$$

It is evident from these definitions that

$$\psi(z) = \sum_c \langle o \mid \delta L \mid c\rangle \mathcal{C}_c(z) = \sum_c \mathcal{D}_c(z)\langle c \mid \delta L \mid o\rangle \tag{6.9}$$

All these operators are submitted to an **irreducibility condition**: the vacuum to vacuum correlation cannot appear as an intermediate state. As a consequence of their irreducibility, all the operators are regular functions of z in the neighbourhood of $z = 0$ and do not vanish in $z = 0$, which is called the **regularity assumption**.[2]

The matrix elements are now expressed in these operators. For example, since the vacuum to vacuum transition in $\langle o \mid R^0(z) \mid o\rangle$ may be performed in an arbitrary number of collisions (hence the name collision operator), this leads to

$$\langle o \mid R^0(z) \mid o\rangle = \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} [\psi_z]^n \tag{6.10}$$

Inserting these compositions we now derive:

$$\varrho_o(t) = (2\pi i)^{-1} \int dz e^{-izt} \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} [\psi_z]^n [\varrho_0(0) + \sum_c \mathcal{D}_c(z)\varrho_c(0)] \tag{6.11}$$

$$\begin{aligned} \varrho_c(t) = (2\pi i)^{-1} \int dz e^{-izt} \Big\{ &\sum_{c'} P_{cc'}(z)\varrho_{c'}(0) + \\ \mathcal{C}_c(z) \sum_{n=0}^{\infty} \frac{1}{z^{n+1}} [\psi_z]^n &[\varrho_o(0) + \sum_{c'} \mathcal{D}_{c'}(z)\varrho_{c'}(0)] \Big\} \end{aligned} \tag{6.12}$$

Prigogine notes that a derivation with respect to time would give the master equation for $\varrho_o(t)$. The correlations in 6.12 may be split into two parts:

$$\varrho_c(t) = \varrho_c(t)' + \varrho_c(t)'' \tag{6.13}$$

Only in the asymptotic limit the evolution of $\varrho_o(t)$ becomes independent of the correlations. $\varrho_c(t)'$ then vanishes and $\varrho_c(t)''$ becomes a well-defined functional of $\varrho_o(t)$ at the same time.

[2](Balescu [10, p.529] Prigogine, George, Henin [58, p.421]). To be honest I do not understand the precise meaning of the first assumption.

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But as the BS writes "we go far beyond these results"(p.419) by analyzing the structure of the resolvent. "To perform the asymptotic limit we have simply to separate, in the contour integration, the singularity at $z = 0$". Assuming as a consequence of their irreducibility, that all the operators are regular in the neighborhood of $z = 0$, Prigogine writes for sufficiently long times:

$$\varrho_o(t) = e^{-i\Omega\psi t} A[\varrho_o(0) + \sum_c D_c \varrho_c(0)] \tag{6.14}$$

$$\varrho_c(t) = C_c \varrho_o(t) \tag{6.15}$$

where the operators Ω, A, D_c, C_c are all functionals of ψ and its derivatives at $z = +i0$ with $\Delta \equiv \frac{\partial}{\partial z}$:

$$A = \sum_{n=0}^{\infty} \frac{1}{n!} \Delta^n \psi^n \tag{6.16}$$

$$\Omega\psi = \sum_{n=0}^{\infty} \frac{1}{n!} (\Delta^n \psi)(\Omega\psi)^n \tag{6.17}$$

$$C_c = \sum_{n=0}^{\infty} \frac{1}{n!} (\Delta^n \mathcal{C}_c)(\Omega\psi)^n \tag{6.18}$$

$$D_c = \sum_{n=0}^{\infty} (\overline{\Omega\psi})^n \frac{1}{n!} (\Delta^n \mathcal{D}_c) \tag{6.19}$$

$$\overline{\Omega\psi}^n = \sum_{n=0}^{\infty} (\overline{\Omega\psi})^n \frac{1}{n!} (\Delta^n \psi) \tag{6.20}$$

This can be written in the more convenient notation:

$$\begin{pmatrix} \tilde{\varrho}_\mathbf{o}(\mathbf{t}) \\ \tilde{\varrho}_\mathbf{c}(\mathbf{t}) \end{pmatrix} = \begin{pmatrix} e^{-i\Omega\psi t} A & e^{-i\Omega\psi t} A D_c \\ C_c e^{-i\Omega\psi t} A & C_c e^{-i\Omega\psi t} A D_c \end{pmatrix} \begin{pmatrix} \varrho_\mathbf{o}(\mathbf{0}) \\ \varrho_\mathbf{c}(\mathbf{0}) \end{pmatrix} \tag{6.21}$$

If we now identify $\Omega\psi$ with θ and A with $(1+DC)^{-1}$ (suppressing the c-dependence) we arrive at equation 4.24. Since in the asymptotic limit a result of Turner (1968)[3] is

$$A^2 + A \sum_c D_c C_c A = A \tag{6.22}$$

"it is easy to verify that $\tilde{\Sigma}(t)$ satisfies the semigroup property"(p.422).

[3] I could not recover this article.

6.3 Rigged Hilbert Space

[4]The standard way in which (extrinsic) irreversibility is introduced in quantum theory is through the Kossakowski-Lindblad semigroup, which consists of a phenomenological approach because non quantum mechanical terms are added without an explanation of their dynamical origin. As the intrinsic evolution of standard QM is given by a unitary group, HS quantum theory does not allow for intrinsic irreversibility. HS quantum theory does not adequately describe decaying states and resonances. Within the HS formulation resonances are thus denied the status of autonomous microphysical entities; the exponentially decaying wave function is not an element of $L^2[0,\infty]$.

The HS formulation is however not the only mathematical idealization that can be obtained from the emperical foundations of quantum mechanics. Whereas the state vector of a stationary state is characterized by its energy E_s, the phenomenological characteristics of a resonance or decaying state are its energy E_R (resonance energy) and its lifetime τ_R or width $\Gamma = \frac{\hbar}{\tau_R}$, which are combined into a complex energy: $z_R = E_R - i\Gamma/2$. We wish to describe a mathematical theory that contains besides stationary states also vectors that describe exponentially decaying states, called Gamow vectors. These Gamow vectors are generalized eigenvectors of a *self-adjoint* Hamiltonian with complex eigenvalues. It turned out that these mathematical entities are already contained in the Rigged Hilbert Space (RHS) formulation, which had been introduced into quantum mechanics previously in order to make the Dirac formalism of bra's and kets mathematically rigorous.

From a philosophical viewpoint the choice between different spaces is underdetermined. According to Bohm, the mapping of structures in the domain of physical reality onto mathematical structures always contains some idealization. "What one can justify from more physical notions, is something like the algebraic structure of a linear space with scalar product and linear operators. The topological structure however cannot be 'derived from experiments'- the mathematical theory of physical reality always contains some idealizations." The choice for RHS can be motivated when one has processes with a preferred direction of time. To show this, it is not important which particular interpretation of QM one adheres as long as there is a distinction between observables and states.

As autonomous entities both stable and unstable particles are described in RHS on the same footing and given the same ontological status. The argument put forward

[4]This introduction to RHS is a summary of Bohm *et al* (*Quantum Mechanical Irreversibility* 1997 [18, p.485-498])

by Bohm is that unstable particles are not qualitatively different but only quantitatively i.e. by a zero or very small value of Γ.

In the RHS, intrinsic irreversibility is due to the new initial-boundary conditions allowed by the RHS and is accomplished by changing not the dynamical laws, but by enlarging the admissible solutions of the LvN equation, which in the RHS also includes exponentially decaying Gamow states. Within the RHS formulation the arrow of time can be translated into a QM arrow of Time, tracing the origins of irreversibility back to causality and initial (boundary) conditions. This is the task Bohm has set for himself.

In both RHS and HS the observationally defined set of states K is mapped onto the set of statistical operators W, and the set of observables is mapped onto the set of essentially self-adjoined operators A. The difference between HS and RHS can now be illustrated. In HS there is a one-to-one correspondence between the set of states and the set of statistical operators in H as well as a one-to-one correspondence between observables and self-adjoint operators. A RHS consists of a triplet of spaces, commonly referred to as the rigged Hilbert space or the Gel'fand triplet

$$\Phi \subset H = H^{\times} \subset \Phi^{\times} \tag{6.23}$$

As mentioned previously the main difference between HS and RHS is topological. The (normal) HS is obtained from a linear scalar product space Ψ by completing it with respect to the topology induced by the norm $\| \ h \ \| = \sqrt{(h,h)}$, where (h,h) denotes the scalar product. In the RHS the same linear space Ψ is completed with respect to a nuclear topology given by a countable number of norms. This topology is stronger than that of the HS. The completion with respect to this nuclear topology gives the space which we denote by Φ, with $\Phi \in H$. Since the dual $H^{\times}$ of H can be identified with H itself i.e. $H^{\times} = H$, the topological dual $\Phi^{\times}$ of Φ is larger than H and contains both H and Φ as subsets dense with respect to $\tau^{\times}$, the weak topology in $\Phi^{\times}$. The topology of the test space Φ is stronger so that "interesting" linear functionals and operators, not continuous with the Hilbert space topology become continuous.

The upshot is at follows. Because the dual of H and H itself are identical, the HS can only accomodate self-adjoint operators with real eigenvalues. Entities connected with microphysical systems that are not directly accessible to observations like the Gamow vectors, having complex eigenvalues, can now be described in $\Phi^{\times}$ within the RHS formalism.[5]

[5]If we compare the results of the BS with the Bohm and Gadella group (*Microphysical Irreversibil-*

6.4 Internet-references

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- http://order.ph.utexas.edu (The Ilya Prigogine Center of Studies in Statistical Mechanics and Complex Systems)
- http://solvayins.ulb.ac.be (Solvay Institute)
- http://www.crs4.it/CISST/ (Instituto di Documentazione e Ricerca sull'Opera di Prigogine: all publications listed)
- http://www.nobel.se/chemistry/laureates/1977 (Web site of the Nobel Foundation on Prigogine)
- http://www.ac.by/members/academicians/prigogine.html (Prigogine interviewed by Tukker)
- http://www.chaosforum.com/denkers/column7.html (Prigogine interviewed by H.Nunnink)
- http://www.physics.nyu.edu/faculty/sokal/ (Sokal Bricmont Affair)
- http://www.mdpi.org/entropy/entropyweb/prigogine.htm (Site with critical comments)
- http://www.uni-jena.de/ms/ms_time.html (Sandbothe: the temporalisation of time)
- http://www.santafe.edu/ shalizi/notebooks/prigogine.html (Criticisms om BAG)
- http://plato.stanford.edu/ price/TAAPreviews.html (Reviews of Huw Price's *Arrow of Time*/discussion with Coveney)
- http://plato.stanford.edu/price/coveney-letter.html (Letter to Coveney)

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ity and the Time-reversal Operation (Bohm 1995 [16]); *The Time Operator for Semigroup Evolutions* (Bohm and Wickramasekara 1997 [17])) which has been investigating the most basic features of scattering (considering the S-matrix element between a pure in-state and a pure out-state), we see remarkably a contradiction between the results obtained by the two groups for the $t < 0$ semigroup.

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6.5 Abbreviations

- NESM: non-equilibrium statistical mechanics
- QM: quantum mechanics
- LvN: Liouville von Neumann equation
- HS: Hilbert space
- RHS: rigged Hilbert space
- RA: reductionist assumption
- SBSS: Symmetry breaking and semigroup selection
- GME: generalized master equation
- CD: causal dynamics
- DC: dissipativity condition
- NC: new complementarity
- ES: entropy as selector of initial conditions
- TOR: time ordering rule
- CSR: conservation of symmetry in reasoning principle

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Epilogue

Time present and time past
Are both perhaps present in time future,
And time future contained in time past.
If all time is eternally present
All time is redeemable
– T.S. Eliot, *Burnt Norton*

Het studeren van zowel natuurkunde en wijsbegeerte heeft als voornaamste voordeel de mogelijkheid gesprekken te manipuleren, daar beiden aan de nodige stereotypen onderhevig zijn. Helaas voltrekken deze gesprekken zich niet zelden volgens de lijn: 'natuurkunde, knap hoor. Op de midelbare school vond ik het al lastig..' danwel 'filosofie: interessant! Zeg, wat vond je van de wereld van Sofie? ..' Beide lijnen lopen ogenblikkelijk dood daar ik natuurkunde ben gaan studeren omdat dit juist bekend stond als de moeilijkste studie en ik de wereld van Sofie altijd vermeden heb te lezen. Waarom dan niet zeggen dat je ze allebei doet? Omdat het antwoord met een aan één grenzende waarschijnlijkheid luidt: 'Tjee, wat een interessante combi' (een kolfje naar de hand van een behaviorist).

Om voorgoed uit deze malaise te geraken, was het zaak een sciptieonderwerp te kiezen waar ik vele kanten mee uitkon. De keuze voor Ilya Prigogine bleek een schot in de roos. Bij natuurkundigen onstaan leuke gesprekken als je de 'charlatan' Prigogine probeert te verdedigen, wijsgerigen zijn te desillusioneren door de wetenschappelijke merites van Prigogine te betwijfelen, en eenieder heeft wel een idee over tijd. Zo was het Ruth Will, docerend aan de University of Western Cape, die ogenblikkelijk begon over Eliot, en Robin die alles wat hij gelezen had in fantasy boeken begon te bepleiten. Tijdens deze talrijke gesprekken werd mijn enthousiasme voor 'tijd' alsmaar aangewakkerd.

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Tegelijkertijd was het bestuderen van Prigogine een voortdurende strijd. Er is enorm veel tijd gaan zitten in het krijgen van een degelijk overzicht, daar dit tot nog toe ontbreekt. De holistische zienswijze van Prigogine op zijn eigen werk aan de ene kant en het eclecticimse van zijn criticasters aan de andere, maken Prigogine obscuur. Lastig ook is dat Prigogine's werk gestoeld blijkt te zijn op (voor mij) gecompliceerde wiskunde. Niet alleen heb ik veel geleerd over Prigogine, maar in een breder kader ook over het interessant probleem dat vele wetenschapsdisciplines lijnrecht tegenoverelkaar staan in zijn werk.

Daarnaast is het scriptie proces ook een academisch ter wereld komen. Ik ben Jos Uffink zeer erkentelijk voor het verrichten van deze keizersnede: Jos, jouw idiosyncratische scherpzinnige intelligentie heeft me wel eens tot wanhoop gedreven, maar uiteindelijk heb ik er niet alleen respect voor verkregen, maar ook enorm veel van heb geleerd. Igor Douven, ook al beperkte jouw commentaar zich tot hoofdlijnen, dit was niet minder belangrijk.

Gelukkig bleek na verloop van tijd dat ik niet de enige was die zich met Prigogine bezig hield. Robert Bishop uit Austin ben ik veel dank verschuldigd voor zijn opbouwende kritiek op mijn scriptie en de mogelijkheid die hij me gaf om zijn drafts te becommentarieren. Jan Bor dank ik hartelijk voor zijn aanstekelijk enthousisasme over Bergson en Prigogine, en de gelegenheid om mijn ideeën over Prigogine in zijn college over het voetlicht te mogen brengen. Tot slot wil ik Maaike, Sam, Henk, Marnix en Stefan bedanken voor hun nimmer aflatende morele steun.