Reconsidering Bohr's Reply to EPR*

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Abstract

Although Bohr's reply to the EPR argument is supposed to be a watershed moment in the development of his philosophy of quantum theory, it is difficult to find a clear statement of the reply's philosophical point. Moreover, some have claimed that the point is simply that Bohr is a radical positivist. In this paper, we show that such claims are unfounded. In particular, we give a mathematically rigorous reconstruction of Bohr's reply to the *original* EPR argument that clarifies its logical structure, and which shows that it does not rest on questionable philosophical assumptions. Rather, Bohr's reply is dictated by his commitment to provide "classical" and "objective" descriptions of experimental phenomena.

1 Introduction

The past few decades have seen tremendous growth in our understanding of interpretations of quantum mechanics. For example, a number of "no-go" results have been obtained which show that some or other interpretation violates constraints that we would expect any plausible interpretation of quantum mechanics to satisfy. Thus, although there is no immediate hope of convergence of opinion on interpretive issues, we certainly have an increased understanding of the technical and conceptual issues at stake. Perhaps, then, we can make use of this increased technical awareness to shed some new light on the great old episodes in the conceptual development of quantum mechanics.

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One historical episode of enduring philosophical interest is the debate between Bohr and Einstein (along with Podolsky and Rosen) over the completeness of quantum mechanics. Although folklore has it that Bohr was the victor in this debate, Fine and Beller [13] have recently claimed that Bohr's reply to the EPR argument of 1935 is basically a failure. In particular, Fine and Beller claim that "...as a result of EPR, Bohr eventually turned from his original concept of disturbance, to make a final — and somewhat forced — landing in positivism" [13, p. 29]. They also make the stronger philosophical claim that "...a positivistic shift is the only salvageable version of Bohr's reply" [13, p. 9]. Unfortunately, Fine and Beller did not see the need to provide an argument for this claim. And, although we are willing to concede — for purposes of argument — that the later Bohr embraced positivism, we are *not* willing to concede that he was rationally compelled to do so. In fact, we will argue that Bohr's defense of the completeness of quantum mechanics does not depend in any way on suspect philosophical doctrines. To this end, we will supply a formal reconstruction of Bohr's reply to EPR, showing that his reply is dictated by the dual requirements that any description of experimental data must be *classical* and *objective*.

The structure of this paper is as follows. In Section 2, we provide an informal preliminary account of the EPR argument and of Bohr's reply. In Section 3, we consider some salient features of Bohr's general outlook on quantum theory. We then return to Bohr's reply to EPR in Sections 4 and 5. In Section 4, we reconstruct Bohr's reply to EPR in the case of Bohm's simplified spin version of the EPR experiment. Finally, in Section 5, we reconstruct Bohr's reply to EPR in the case of the original (position-momentum) version of the EPR experiment.

2 Informal Preview

In classical mechanics, a state description for a point particle includes a precise specification of both its position and its momentum. In contrast, a quantum-mechanical state description supplies only a statistical distribution over various position and momentum values. It would be quite natural, then, to regard the quantum-mechanical description as *incomplete* — i.e. as providing less than the full amount of information about the particle. Bohr, however, insists that the imprecision in the quantum-mechanical state description reflects a fundamental indeterminacy in nature rather than the incompleteness of the theory. The EPR argument attempts to directly rebut this completeness claim by showing that quantum mechanics (in conjunction with plausible extra-theoretical constraints) entails that particles always have both a precise position and a precise momentum.

EPR ask us to consider a system consisting of a pair of spacelike separated particles. They then note that, according to quantum mechanics, there is a state $\psi_{\rm epr}$ in which the positions of the two particles are strictly correlated, *and* the momenta of the two particles are strictly correlated. It follows then that if we were to measure the position of the first particle, we could predict with certainty the outcome of a position measurement on the second particle; *and* if we were to measure the momentum of the first particle, we could predict with certainty the outcome of a momentum measurement on the second particle.

EPR then claim that our ability to predict with certainty the outcomes of these measurements on the second particle shows that each such measurement reveals a pre-existing "element of reality." In what has come to be know as the "EPR reality criterion," they say:

> If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity. [11, p. 77]

In particular, if we determine the position of the first particle in this strictly correlated state, then we can conclude that the second particle also has a definite position. And if we determine the momentum of the first particle in this strictly correlated state, then the second particle must also have a definite momentum.

Of course, it does not immediately follow that there is any single situation in which both the position and the momentum of the second particle are elements of reality. However, EPR also make the (*prima facie* plausible) assumption that what counts as an element of reality for the second particle should be independent of which measurement is performed on the first particle. In other words, a measurement on the first particle can play a *probative*, but not a *constitutive*, role with respect to the elements of reality for the second particle. Consequently, EPR conclude that both the position and the momentum of the second particle are elements of reality, regardless of which measurement is performed on the first particle.

2.1 Bohr's reply

According to Bohr, the EPR argument somehow misses the point about the nature of quantum-mechanical description. Unfortunately, though, not much scholarly work has been done attempting to reconstruct Bohr's reply in a cogent fashion.

We should begin by noting that Bohr most certainly does not maintain the "hyperpositivist" position according to which no possessed properties or reality should be attributed to an unmeasured system. (For example, Ruark claims that, for Bohr, "a given system has reality only when it is actually measured" [24, 466].) Quite to the contrary, Bohr explicitly claims that when the position of the first particle is measured, "...we obtain a basis for conclusions about the initial position of the other particle relative to the rest of the apparatus" [1, p. 148]. Thus, Bohr agrees with EPR that once the position (respectively, momentum) of the first particle is actually measured, the position of the second particle is an element of reality — *whether or not* its position is actually empirically determined. In other words, Bohr accepts the outcome of an application of the EPR reality criterion, so long as its application is restricted to individual measurement contexts (i.e. the results of its application in different contexts are not combined).

In order, then, to rationally reject EPR's conclusion, Bohr must reject the claim that elements of reality for the second particle cannot be constituted by measurements carried out on the first particle. In other words, Bohr believes that a measurement on the first particle *can* serve to constitute elements of reality for the second, spacelike separated, particle.

To this point, we have not said anything particularly novel about Bohr's reply to EPR. It is relatively well-known that his reply amounts to claiming — what EPR thought was absurd [11, p. 480] — that what is real with respect to the second particle can depend in a nontrivial way on which measurement is performed on the first particle. However, where previous defenders of Bohr have uniformly stumbled is in giving a coherent account of *how* a measurement on one system can influence what is real for some spacelike separated system.

Unfortunately, Bohr's statements on this issue are brief and obscure. For example, he says,

> It is true that in the measurements under consideration any direct mechanical interaction of the [second] system and the measuring agencies is excluded, but a closer examination reveals that the procedure of measurement has an essential influence on the conditions on which the very definition of the physical quantities in question rests. [2, p. 65]

That is, a measurement on the first system influences the conditions which must obtain in order for us to "define" elements of reality for the second system. Moreover, this influence is of such a sort that a position (momentum) measurement on the first particle supplies the conditions needed to define the position (momentum) of the second particle.

Before we proceed to our positive account, we need first to dismiss one *prima facie* plausible, but nonetheless mistaken, explication of Bohr's notion of defining a quantity. In particular, some have claimed that, according to Bohr, an observable of a system comes to have a definite value when the wavefunction of the system collapses onto one of that observable's eigenstates. This amounts to attributing to Bohr the claim that:

Eigenstate-Eigenvalue Link: A quantity Q is defined in state ψ iff. ψ is an eigenvector for Q;

along with the claim that by measuring an observable, we can cause the quantum state to collapse onto an eigenstate of that observable. In that case, Bohr would claim that by measuring the position of the first particle, we collapse the EPR state onto an eigenstate of position for the second particle — and thereby "cause" the second particle to have a definite position. Similarly, if we were to measure the momentum of the first particle, we would "cause" the second particle to have a definite momentum. In either case, the measurement on the first particle would be the cause of the reality associated with the second particle.

However, there are at least two good reasons for rejecting this reading of Bohr. First, Bohr explicitly claims that a measurement of the first particle cannot bring about a "mechanical" change in the second particle. In philosophical terms, we might say that Bohr does not believe that the position measurement on the first particle *causes* the second particle to have a position, at least not in the same sense that a brick can *cause* a window to shatter. Thus, if Bohr does believe in a collapse the wavefunction, it is as some sort of *non-physical* (perhaps epistemic) process. However, it is our firm opinion that, unless the quantum state can be taken to represent our ignorance of the "true" hidden state of the system, there is no coherent non-physical interpretation of collapse. (We doubt the coherence of recent attempts to maintain both a subjectivist interpretation of quantum probabilities, and the claim that "there are no unknown quantum states" [7].) Thus, if Bohr endorses collapse, then he is already committed to the incompleteness of quantum mechanics, and the EPR argument is superfluous.

The second, and more important, reason for resisting this reading of Bohr is the complete lack of textual evidence supporting the claim that Bohr believed in wavefunction collapse (see [18]). Thus, there is no good reason to think that Bohr's reply to the EPR argument depends in any way on the notion of wavefunction collapse.

3 Classical Description and Appropriate Mixtures

In order to do justice to Bohr's reply to EPR, it is essential that we avoid caricatured views of Bohr's general philosophical outlook, and of his interpretation of quantum mechanics. This is particularly difficult, because there has been a long history of misinterpretation of Bohr. For example, in terms of general philosophical themes, one might find Bohr associated with anti-realism, idealism, and subjectivism. Moreover, in terms of the specific features of an interpretation of quantum mechanics, Bohr is often associated with wavefunction collapse, creation of properties/attributes upon measurement, and "cuts" between the microscopic and macroscopic realms. However, these characterizations of Bohr are pure distortion, and can find no justification in his published work. Indeed, Bohr's philosophical commitments, and the picture of quantum mechanics that arises from these commitments, are radically different from the mythical version that we have received from his critics and from his well-intended (but mistaken) followers. (Our own understanding of Bohr has its most immediate precedent in recent work on "no collapse" interpretations of quantum mechanics [4, 5, 6, 15]. However, this sort of analysis of Bohr's interpretation was suggested independently, and much earlier, by Don Howard [16]. See also [17, 18].)

According to Bohr, the phenomena investigated by quantum theory cannot be accounted for within the confines of classical physics. Nonetheless, he claims that "…however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms" [3, p. 209]. That is, classical physics embodies a standard of intelligibility that should be exemplified by any description of the empirical evidence. In particular, although the various sources of evidence cannot be reconciled into a single classical picture, the description of any single source of evidence must be classical.

Bohr's statements about the notion of "classical description" have been horribly misunderstood. For a catalog of these misunderstandings and for evidence that they are indeed mistaken, we refer the reader to [16, 17, 18]. On the positive side, we will follow Howard [17] in the claim that the notion of classical description is best explicated via the notion of an "appropriate mixture."

> …we make the clearest sense out of Bohr's stress on the impor-

> tance of a classical account of experimental arrangements and of the results of observation, if we understand a classical description to be one in terms of appropriate mixtures. [17, p. 222]

As Howard [16] shows, the notion of an appropriate mixture can be developed in such a way that Bohr's (sometimes obscure) statements about the possibilities of classical description become mathematically clear statements about the possibility of treating the quantum state as a classical probability measure. In order to see this, we first collect some terminology.

Let $\mathcal{H}$ be a finite-dimensional vector space with inner-product $\langle\cdot,\cdot\rangle$, and let $\mathbf{B}(\mathcal{H})$ denote the family of linear operators on $\mathcal{H}$. We say that a self-adjoint operator W on $\mathcal{H}$ is a *density operator* just in case W has non-negative eigenvalues that sum to 1. If ψ is a vector in $\mathcal{H}$, we let $|\psi\rangle\langle\psi|$ denote the projection onto the ray in $\mathcal{H}$ generated by ψ. Thus, if Tr denotes the trace on $\mathbf{B}(\mathcal{H})$, then $\mathrm{Tr}(|\psi\rangle\langle\psi|A)=\langle\psi,A\psi\rangle$ for any operator A on $\mathcal{H}$. A *measurement context* can be represented by a pair (ψ,R), where ψ is a unit vector (representing the quantum state), and R is a self-adjoint operator (representing the measured observable).

Following Howard [16], we say that a "mixture," represented by a density operator W, is *appropriate* for (ψ,R) just in case $W=\sum_{i=1}^{n}\lambda_i|\phi_i\rangle\langle\phi_i|$, $(n\leq \dim\mathcal{H})$, where each ϕ_i is an eigenvector for R, and $\lambda_i=|\langle\psi,\phi_i\rangle|^2$ for $i=1,\dots,n$. In other words, W is a mixture of eigenstates for R, and it reproduces the probability distribution that ψ assigns to the values of R. Thus, an appropriate mixture for (ψ,R) can be taken to represent our ignorance of the value of R in the state ψ.

Once again, we emphasize that Bohr never explicitly invokes wavefunction collapse, nor does he need to. Indeed, the idea of a "measurement problem" was foreign to Bohr, who seems to take it as a brute empirical fact — needing no further explanation from within quantum theory — that an observable possesses a value when it is measured. Of course, we now know that if Bohr rejects collapse, then he would also have to reject the claim that an observable possesses a value only if the system is in an eigenstate for that observable (i.e., the eigenstate←eigenvalue link) [9]. But there is little reason to believe that Bohr would have been tempted to endorse this suspect claim in the first place.

3.1 Appropriate Mixtures and Elements of Reality

An appropriate mixture is supposed to give a description in which the measured observable is an "element of reality." However, the connection between an appropriate mixture (i.e. some density operator) and elements of reality is not completely clear. Clearly, the intent of writing the appropriate mixture as $W = \sum_{i=1}^{n} \lambda_i |\phi_i\rangle\langle\phi_i|$ is that each "proposition" $|\phi_i\rangle\langle\phi_i|$ has a truth value. However, if W is degenerate then W has infinitely many distinct expansions as a linear combination of orthogonal one-dimensional projections. Thus, W itself does not determine the elements of reality; rather, it is some expansion of W into a linear combination of one-dimensional projections that determines the elements of reality.

In this case, however, we might as well focus on the one-dimensional projections themselves. Thus, we will say that the set $S = \{|\phi_i\rangle\langle\phi_i| : i = 1,\ldots,n\}$ is an *appropriate event space* for the measurement context (ψ, R) just in case S is maximal relative to the following three conditions: (1.) Each ϕ_i is an eigenvector of R; (2.) If $i \neq j$ then ϕ_i and ϕ_j are orthogonal; (3.) Each ϕ_i is *non*orthogonal to ψ. Each of these conditions has a natural interpretation. The first condition states that each proposition in S attributes some value to R (viz. the eigenvalue r_i satisfying $R\phi_i = r_i\phi_i$); the second condition states that the propositions in S are mutually exclusive; and the third condition states that each proposition in S is possible relative to ψ. Note, moreover, that every appropriate event space S can be obtained by taking the projection operators in some orthogonal expansion $W = \sum_{i=1}^{n} \lambda_i |\phi_i\rangle\langle\phi_i|$ of an appropriate mixture for (ψ, R), and then eliminating those projections with coefficient 0.

If we suppose that R possesses a definite value in the context (ψ, R) (and that it is *False* that it possesses any other value) then an appropriate event space S gives a *minimal* list of truth-valued propositions in the context (ψ, R). However, Bohr himself is *not* an ontological minimalist; rather, he claims that "we must strive continually to extend the scope of our description, but in such a way that our messages do not thereby lose their objective and unambiguous character" [22, p. 10]. Thus, we should look for the *maximal* set of propositions that can be consistently supposed to have a truth-value in the context (ψ, R).

It has been pointed out (in relation to the modal interpretation of quantum mechanics [8]) that we can consistently assume that all projections in $S^{\perp}$ are *False*. Moreover, if we do so, then our lattice of truth-valued propositions will be maximal; i.e. we cannot add further elements of reality without violating the requirement of classical description. Thus, given an appropriate event space S, we will take the full family of truth-valued propositions to be those in the set (cf. [8]):

$$\mathbf{Def}(S) := \left\{ P^2 = P = P^* : \forall Q \in S \, [Q \leq P \text{ or } QP = 0] \right\}.$$

It is straightforward to verify that $\mathbf{Def}(S)$ is a sublattice of the lattice of all projection operators on $\mathcal{H}$. Moreover, it can be shown that $\mathbf{Def}(S)$ is maximal in the following sense: If $\mathcal{L}$ is a lattice of projections such that $\mathbf{Def}(S) \subset \mathcal{L}$, then ψ *cannot* be represented as a classical probability distribution over all elements in $\mathcal{L}$. (In this section and the next, we state results without proof. Each of these results is a corollary of the results proved in [15].)

4 Bohr's Reply: Spin Case

We can now make use of the appropriate mixtures account to reconstruct Bohr's reply to EPR. For the sake of mathematical simplicity, we first consider Bohm's spin version of the EPR experiment. We return to the original EPR experiment in the final section.

Suppose that we have prepared an ensemble of spin-1/2 particles in the singlet state:

$$\psi \;=\; \frac{1}{\sqrt{2}}(\,|x+\rangle|x-\rangle - |x-\rangle|x+\rangle\,), \tag{1}$$

where $\sigma_x|x\pm\rangle = \pm|x\pm\rangle$. Then, $\sigma_x \otimes I$ is strictly anticorrelated with $I \otimes \sigma_x$, and $\sigma_y \otimes I$ is strictly anticorrelated with $I \otimes \sigma_y$. Thus, the outcome of a measurement of $\sigma_x \otimes I$ would permit us to predict with certainty the outcome of a measurement of $I \otimes \sigma_x$; and the outcome of a measurement of $\sigma_y \otimes I$ would permit us to predict with certainty the outcome of a measurement of $I \otimes \sigma_y$.

For any orthonormal basis $\{\phi_i\}$ of eigenvectors for $\sigma_x \otimes I$, the event space

$$\{\,|\phi_i\rangle\langle\phi_i| \;:\; |\langle\psi,\phi_i\rangle|^2 \neq 0\}, \tag{2}$$

is appropriate for $(\psi, \sigma_x \otimes I)$. However, since $\sigma_x \otimes I$ is degenerate, there are infinitely many distinct orthonormal bases of eigenvectors for $\sigma_x \otimes I$. Moreover, each basis gives rise to a distinct event space, and each distinct event space permits us to attribute *different* elements of reality to the second (unmeasured) particle.

More concretely, let $P^x_\pm$ denote the projection onto the ray generated by $|x\pm\rangle$, and similarly for $P^y_\pm$ and $P^z_\pm$. Then, each of the following event spaces is appropriate for $(\psi, \sigma_x \otimes I)$:

$$S_{xx} = \{P^x_+ \otimes P^x_-\;,\; P^x_- \otimes P^x_+\}.$$
$$S_{xy} = \{P^x_+ \otimes P^y_+\;,\; P^x_+ \otimes P^y_-\;,\; P^x_- \otimes P^y_+\;,\; P^x_- \otimes P^y_-\}.$$
$$S_{xz} = \{P^x_+ \otimes P^z_+\;,\; P^x_+ \otimes P^z_-\;,\; P^x_- \otimes P^z_+\;,\; P^x_- \otimes P^z_-\}.$$

Clearly, though, these event spaces give *theoretically inequivalent* descriptions of the measurement context. While S_{xx} gives a description in which the second particle has spin-x values that are perfectly anticorrelated with the spin-x values of the first particle, S_{xy} gives a description in which the second particle has spin-y values that are uncorrelated with the spin-x values of the first particle. How do we determine which description is the *correct* one?

One might be inclined to argue that it is an advantage to have more than one "interpretation" (i.e. empirically adequate description) of the same measurement context. That is, one might argue that there is no single correct description of the second particle in this context; rather, there are several incompatible, but individually acceptable, descriptions of the second particle. However — despite his otherwise unorthodox philosophical stance — Bohr is not a pluralist about descriptions of measurement contexts. Indeed, he claims that a measurement context uniquely dictates an interpretation.

> ...we are not dealing with an incomplete description characterized by the arbitrary picking out of different elements of reality at the cost of sacrificing other such elements, but with a rational discrimination between essentially different experimental arrangements and procedures.... [1, p. 148]

Thus, the theorist is not free to make a willy-nilly choice of which elements of reality to ascribe to the second particle; rather, her choice is to be fixed (in some, yet to be explicated, way) by the measurement context.

For Bohr, the correct description of the present measurement context (in which spin-x is measured on the first particle and no measurement is performed on the second particle) is S_{xx}, where the two particles have perfectly anticorrelated spin-x values. However, we do not yet have any explanation for *why* Bohr thinks that this description is privileged. In the next two sections, we shall provide an explanation for Bohr's preference.

4.1 The EPR reality criterion

Isn't it *obvious* that S_{xx} is the correct description of the context in which $\sigma_x \otimes I$ is measured in the EPR state? In particular, if we know that $\sigma_x \otimes I$ has some value (either $+1$ or -1) can we not infer immediately that $I \otimes \sigma_x$ has the opposite value? But what reason do we have to think that $I \otimes \sigma_x$ has any value at all? Since we are refusing to invoke wavefunction collapse, it does not help to note that Lüders' rule entails that a measurement of $\sigma_x \otimes I$ collapses ψ onto either $|x+\rangle|x-\rangle$ or $|x-\rangle|x+\rangle$. Perhaps then our intuition that $I \otimes \sigma_x$ has a value is based on some variant of the EPR reality criterion: If we can predict with certainty the outcome of a measurement of $I \otimes \sigma_x$, then it must possess a value.

According to Howard, it is a contextualized version of the EPR reality criterion that dictates which properties Bohr attributes to the second (unmeasured) particle. Howard says, "...there is no obvious reason why, with the added necessary condition of a restriction to specific experimental contexts, [Bohr] could not accept the EPR reality criterion as it stands" [16, p. 256]. He then spells out concretely what such a contextualized version of the reality criterion would require.

> Once the experimental context is stipulated, which amounts to the specification of the candidates for real status, our decision as to which particular properties to consider as real will turn on the question of predictability with certainty. [16, p. 256]

We will now give a formal description of this notion of a contextualized reality criterion.

First, when Howard says that the experimental context specifies the "candidates" for real status, he presumably means that an observable must be compatible with the measured observable in order to be such a candidate. For example, if we measure $\sigma_x \otimes I$, then $\sigma_y \otimes I$ is not even a candidate for real status. However, in order for the quantum state ψ to be representable as a classical probability distribution over two projections P and P', it is *not* necessary for P and P' to be compatible. Rather, ψ can be represented as a classical probability distribution over P and P' if and only if $[P, P']\psi = 0$. Thus, since we wish to maintain that each spectral projection of R has a truth-value in the context (ψ, R), we will say that a property P is a *candidate for real status* just in case $[P, P']\psi = 0$ for every spectral projection P' of R.

However, since compatibility (or compatibility relative to a state) is not transitive, not every observable that is compatible with the measured observable can be an element of reality. For example, both $I \otimes \sigma_x$ and $I \otimes \sigma_y$ are compatible with $\sigma_x \otimes I$, but it is not possible for both $I \otimes \sigma_x$ and $I \otimes \sigma_y$ to be elements of reality. Thus, we need a criterion that will permit us to choose among the candidates for real status in such a way that we do not end up with a set of properties that cannot be described classically.

According to Howard, "our decision as to which particular properties to consider as real will turn on the question of predictability with certainty." In other words, P is real only if it is strictly correlated with one of the possible outcomes of a measurement of R; i.e. there is some spectral projection P'

of R such that P and P' are strictly correlated in the state ψ. That is, $\langle \psi, (P-P')^2\psi\rangle = 0$, which is equivalent to $P\psi = P'\psi$.

Let $\mathcal{R}$ denote the family of spectral projections for R. Then Howard's proposal amounts to attributing reality to the following set of properties in the context (ψ, R):

$$\mathcal{L}(\psi, R) := \Big\{ P^2 = P = P^* : [P,R]\psi = 0 \ \ \& \ \ \exists P' \in \mathcal{R} \text{ s.t. } P'\psi = P\psi \Big\}.$$

We leave the following straightforward verifications to the reader: (1.) $\mathcal{L}(\psi, R)$ is a sublattice of the lattice of all projections on $\mathcal{H}$. (2.) The quantum state $|\psi\rangle\langle\psi|$ is a mixture of dispersion-free states on $\mathcal{L}(\psi, R)$. (For this, recall that it is sufficient to show that $[P,Q]\psi = 0$ for all $P, Q \in \mathcal{L}(\psi, R)$.) (3.) $I \otimes P^x_{\pm} \in \mathcal{L}(\psi, \sigma_x \otimes I)$ and $I \otimes P^y_{\pm} \notin \mathcal{L}(\psi, \sigma_x \otimes I)$; and similarly with the roles of x and y interchanged.

Thus, the contextualized reality criterion accurately reproduces Bohr's pronouncements on the EPR experiment. However, there is a serious difficulty with this analysis of Bohr's reply. In particular, the EPR reality criterion is best construed as a version of "inference to the best explanation" (cf. [23, p. 72]): The best explanation of our ability to predict the outcome of a measurement with certainty is that the system has some preexisting feature that we are detecting. However, since Bohr is not a classical scientific realist (see, e.g., [16]), we cannot expect him to be persuaded by such inferences to the best explanation. Thus, although Howard's contextualized reality criterion gives the right answers, it fails to give a plausible explanation of why Bohr gave the answers he did.

4.2 Objectivity and invariance

Despite Bohr's rejection of classical scientific realism, he maintains that our descriptions of experimental phenomena must be "objective." Presumably, Bohr's notion of objectivity is to some extent derivative from the idealist philosophical tradition, and therefore has philosophical subtleties that go far beyond the scope of this paper. For our present purposes, however, it will suffice to use a straightforward and clear notion of objectivity that Bohr might have endorsed: For a feature of a system to be objective, that feature must be invariant under the "relevant" group of symmetries. We now explicate this notion, and we show that it dictates a unique classical description of the EPR experiment.

Recall that the event space $S_{xy} = \{P^x_+ \otimes P^y_+, P^x_+ \otimes P^y_-, P^x_- \otimes P^y_+, P^x_- \otimes P^y_-\}$ allows us to describe an ensemble in which the first particle has spin-x values, and second particle has (uncorrelated) spin-y values. Now, consider the symmetry U of the system defined by the following mapping of orthonormal bases:

$$\begin{array}{ccc} |y+\rangle|y+\rangle & \longmapsto & +|z-\rangle|z-\rangle \\ |y+\rangle|y-\rangle & \longmapsto & -|z-\rangle|z+\rangle \\ |y+\rangle|y-\rangle & \longmapsto & -|z-\rangle|z+\rangle \\ |y-\rangle|y-\rangle & \longmapsto & +|z+\rangle|z+\rangle. \end{array}$$

Then, $U^*(\sigma_x \otimes I)U = \sigma_x \otimes I$, and $U\psi = \psi$. That is, U leaves both the state and the measured observable of the context invariant. However,

$$U^*(P^x_\pm \otimes P^y_\pm)U \;=\; P^x_\pm \otimes P^z_\pm.$$

That is, U does not leave the individual elements of S_{xy}, nor even the set as a whole, invariant. In fact, there is no quantum state that is dispersion-free on both $P^x_+ \otimes P^y_+$ and on its transform $U^*(P^x_+ \otimes P^y_+)U = P^x_+ \otimes P^z_+$. Thus, the candidate elements of reality in S_{xy} are not left invariant by the relevant class of symmetries.

In general, let us say that a set S of projections on $\mathcal{H}$ is *definable* in terms of ψ and R just in case: For any unitary operator U on $\mathcal{H}$, if $U\psi = \psi$ and $U^*RU = R$ then $U^*PU = P$ for all $P \in S$. It is straightforward to verify that the set $S_{xx} = \{P^x_+ \otimes P^x_-\,,\, P^x_- \otimes P^x_+\}$ is definable in terms of ψ and R. In fact, it is the only such appropriate event space for this context.

Theorem 1. *$\{P^x_+ \otimes P^x_-\,,\, P^x_- \otimes P^x_+\}$ is the unique appropriate event space for $(\psi, \sigma_x \otimes I)$ that is definable in terms of ψ and $\sigma_x \otimes I$.*

Proof. Suppose that S is an appropriate event space for (ψ, R) that is definable in terms of ψ and R, and let $|\phi\rangle\langle\phi| \in S$. Let $P_1 = (P^x_+ \otimes P^x_-) + (P^x_- \otimes P^x_+)$ and let $P_2 = (P^x_+ \otimes P^x_+) + (P^x_- \otimes P^x_-)$. Then $U := P_1 - P_2$ is a unitary operator. It is obvious that $U^*(\sigma_x \otimes I)U = \sigma_x \otimes I$ and $U\psi = \psi$. Thus, definability entails that $|\phi\rangle\langle\phi|$ commutes with U; and therefore $|\phi\rangle\langle\phi|$ is either a subprojection of P_1 or is a subprojection of P_2. However, the latter is not possible since $P_2\psi = 0$. Thus, $|\phi\rangle\langle\phi|$ is a subprojection of P_1. However, there are only two one-dimensional subprojections of P_1 that are compatible with R, namely $P^x_+ \otimes P^x_-$ and $P^x_- \otimes P^x_+$. Since $|\phi\rangle\langle\phi|$ must be compatible with R it follows that either $|\phi\rangle\langle\phi| = P^x_+ \otimes P^x_-$ or $|\phi\rangle\langle\phi| = P^x_- \otimes P^x_+$. $\square$

Thus, we have a situation analogous to simultaneity relative to an inertial frame in relativity theory. In that case, there is only one simultaneity relation that is invariant under all symmetries that preserve an inertial observer's worldline [20]. Thus, we might wish to regard this simultaneity relation as the correct one relative to that observer, and the others as spurious. In the quantum-mechanical case, there is only one set of properties that is invariant under all the symmetries that preserve the quantum state and the measured observable. So, we should regard these properties as those that possess values relative to that measurement context.

It is easy to see that $\mathcal{L}(\psi, \sigma_x \otimes I) = \mathbf{Def}(S_{xx})$. Thus Howard's suggestion of applying a contextualized reality criterion turns out to be (extensionally) equivalent to requiring that the elements of reality be definable in terms of ψ and R. It follows that those attracted by Howard's analysis of Bohr's response to EPR now have independent grounds to think that $\mathcal{L}(\psi, \sigma_x \otimes I)$ gives the correct list of elements of reality in the context $(\psi, \sigma_x \otimes I)$.

5 Bohr's Reply: Position-Momentum Case

There are a couple of formal obstacles that we encounter in attempting to reconstruct Bohr's reply to the *original* EPR argument. First, there is an obstacle in describing the EPR experiment itself: The EPR state supposedly assigns dispersion-free values to the relative position $Q_1 - Q_2$ and to the total momentum $P_1 + P_2$ of the two particles. However, $Q_1 - Q_2$ and $P_1 + P_2$ are continuous spectrum observables, and no standard quantum state (i.e., density operator) assigns a dispersion-free value to a continuous spectrum observable. Thus, in terms of the standard mathematical formalism for quantum mechanics, *the EPR state does not exist.* Second, there is an obstacle in applying the account of appropriate mixtures to the EPR experiment: Since the position (or momentum) observable of the first particle has a continuous spectrum, no density operator W is a convex combination of dispersion-free states of the measured observable. Thus, there are no appropriate mixtures (in our earlier sense) for this measurement context.

We can overcome both of these obstacles by expanding the state space of our system so that it includes eigenstates for continuous spectrum observables. To do this rigorously, we will employ the C^*-algebraic formalism of quantum theory. We first recall the basic elements of this formalism.

A C^*-algebra $\mathcal{A}$ is a complex Banach space with norm $A \mapsto \|A\|$, involution $A \mapsto A^*$, and a product $A, B \mapsto AB$ satisfying:

$$(AB)^* = B^*A^*, \qquad \|A^*A\| = \|A\|^2, \qquad \|AB\| \leq \|A\|\|B\|. \tag{3}$$

We assume that $\mathcal{A}$ has a two-sided identity I. Let ω be a linear functional on $\mathcal{A}$. We say that ω is a *state* just in case ω is positive [i.e. $\omega(A^*A) \geq 0$ for all $A \in \mathcal{A}$], and ω is normalized [i.e. $\omega(I) = 1$]. A state ω is said to be *pure* just in case: If $\omega = \lambda\rho + (1-\lambda)\tau$ where ρ, τ are states of $\mathcal{A}$ and $\lambda \in (0,1)$, then $\omega = \rho = \tau$. A state ω is said to be *dispersion-free* on $A \in \mathcal{A}$ just in case $\omega(A^*A) = |\omega(A)|^2$. If ω is dispersion-free on A for all $A \in \mathcal{A}$, we say that ω is *dispersion-free* on the algebra $\mathcal{A}$.

We represent a measurement context by a pair $(\omega, \mathcal{R})$, where ω is a state of $\mathcal{A}$, and $\mathcal{R}$ is a mutually commuting family of operators in $\mathcal{A}$ (representing the measured observables). We are interested now in determining which families of observables can be described classically as possessing values in the state ω. Thus, if $\mathcal{B}$ is a C^*-subalgebra of $\mathcal{A}$, we say that $\omega|_{\mathcal{B}}$ (i.e., the restriction of ω to $\mathcal{B}$) is a *classical probability measure* (or more briefly, *classical*) just in case

$$\omega(A) = \int \omega_\lambda(A) d\mu(\lambda), \qquad A \in \mathcal{B}, \tag{4}$$

where each ω_λ is a dispersion-free state of $\mathcal{B}$.

We now construct a specific C^*-algebra that provides the model for a single particle with one degree of freedom. In the standard Hilbert space description of a single particle, we can take our state space to be the Hilbert space $L_2(\mathbb{R})$ of (equivalence classes of) square-integrable functions from $\mathbb{R}$ into $\mathbb{C}$. The position observable can be represented by the self-adjoint operator Q defined by $Q\psi(x) = x \cdot \psi(x)$ (on a dense domain in $L_2(\mathbb{R})$), and the momentum observable can be represented by the self-adjoint operator $P = -i(d/dx)$ (also defined on a dense domain in $L_2(\mathbb{R})$). (We set $\hbar = 1$ throughout.) We may then define one-parameter unitary groups by setting $U_a := \exp\{iaQ\}$ for $a \in \mathbb{R}$, and $V_b := \exp\{ibP\}$ for $b \in \mathbb{R}$. Let $\mathcal{A}[\mathbb{R}^2]$ denote the C^*-subalgebra of operators on $L_2(\mathbb{R})$ generated by $\{U_a : a \in \mathbb{R}\} \cup \{V_b : b \in \mathbb{R}\}$. We call $\mathcal{A}[\mathbb{R}^2]$ the *Weyl algebra* for one degree of freedom.

Of course, $\mathcal{A}[\mathbb{R}^2]$ itself does not contain either Q or P. However, the group $\{U_a : a \in \mathbb{R}\}$ can be thought of as a surrogate for Q, in the sense that a state ω of $\mathcal{A}[\mathbb{R}^2]$ should be thought of as an "eigenstate" for Q just in case ω is dispersion-free on the set $\{U_a : a \in \mathbb{R}\}$. Similarly, the group $\{V_b : b \in \mathbb{R}\}$ can be thought of as a surrogate for P. (Moreover, the indeterminacy relation between Q and P can be formulated rigorously as follows: There is no state of $\mathcal{A}[\mathbb{R}^2]$ that is simultaneously dispersion-free on both $\{U_a : a \in \mathbb{R}\}$ and $\{V_b : b \in \mathbb{R}\}$ [10, p. 455].)

5.1 Formal model of the EPR experiment

In the standard formalism, the state space of a pair of particles (each with one degree of freedom) can be taken as the tensor product Hilbert space $L_2(\mathbb{R}) \otimes L_2(\mathbb{R})$. Similarly, in the C^*-algebraic formalism, the algebra of observables for a pair of particles (each with one degree of freedom) can be represented as the tensor product $\mathcal{A}[\mathbb{R}^2] \otimes \mathcal{A}[\mathbb{R}^2]$.

The EPR state is supposed to be that state in which $Q_1 - Q_2$ has the value λ, and $P_1 + P_2$ has the value μ. (At present, we have no guarantee of either existence or uniqueness.) Since $\exp\{ia(Q_1 - Q_2)\} = U_a \otimes U_{-a}$ and $\exp\{ib(P_1 + P_2)\} = V_b \otimes V_b$, and since dispersion-free states preserve functional relations, the EPR state should assign the (dispersion-free) value $e^{ia\lambda}$ to $U_a \otimes U_{-a}$ and the value $e^{ib\mu}$ to $V_b \otimes V_b$. Fortunately, for a fixed pair (λ, μ) of real numbers, there is a unique pure state ω of $\mathcal{A}[\mathbb{R}^2] \otimes \mathcal{A}[\mathbb{R}^2]$ that satisfies these two conditions [14, Theorem 1]. We will simply call ω *the EPR state.*

Suppose then that we are in a context in which all elements in $\mathcal{Q}_1 := \{U_a \otimes I : a \in \mathbb{R}\}$ can be assigned definite numerical (complex) values (e.g., a context in which the position of the first particle has been determined). We can then ask: Which observables can be consistently described, along with the elements of $\mathcal{Q}_1$, as possessing values in the state ψ? Since the elements of $\mathcal{Q}_2 := \{I \otimes U_a : a \in \mathbb{R}\}$ commute pairwise with the elements of $\mathcal{Q}_1$, we could provide a consistent description in which the second particle has a definite position (that is strictly correlated with the first particle's position). However, since the elements of $\mathcal{P}_2 := \{I \otimes V_a : a \in \mathbb{R}\}$ also commute pairwise with the elements of $\mathcal{Q}_1$, we could provide a consistent description in which the second particle has a definite momentum (which is uncorrelated with the position of the first particle). The requirement of consistency does not itself tell us which of these descriptions is the correct one. In order to find a basis for choosing between the descriptions, we turn again to symmetry considerations.

Let $\mathcal{A}, \mathcal{B}$ be C^*-algebras, and let π be a mapping of $\mathcal{A}$ into $\mathcal{B}$. We say that π is a $*$-*homomorphism* just in case π is linear, multiplicative, and preserves adjoints. If π is also a bijection, we say that π is a $*$-*isomorphism*; and we say that π is a $*$-*automorphism* when we wish to indicate that $\mathcal{B}$ was already assumed to be isomorphic to $\mathcal{A}$. Finally, let ω be a state of $\mathcal{A}$, let $\mathcal{R}$ be a mutually commuting family of operators in $\mathcal{A}$, and let $\mathcal{B}$ be a C^*-subalgebra of $\mathcal{A}$. We that $\mathcal{B}$ is *definable* in terms of ω and $\mathcal{R}$ just in case: For any $*$-automorphism α of $\mathcal{A}$, if $\alpha(\mathcal{R}) = \mathcal{R}$ and $\omega \circ \alpha = \omega$, then $\alpha(\mathcal{B}) = \mathcal{B}$. Thus, in our present circumstance, we wish to determine which (if any) of the candidate algebras of "elements of reality" identified above is definable in terms of the EPR state and the measured observables $\mathcal{Q}_1$.

5.2 The uniqueness theorem

We turn now to the main technical result of our paper. Our main result shows that there is a unique (subject to the constraint of maximality) algebra of observables $\mathcal{B}$ such that: (1.) It is consistent to suppose that all elements of $\mathcal{B}$ possess a definite value in the EPR state; (2.) The position observable of the first particle (more precisely: its surrogate unitary group) lies in $\mathcal{B}$; and (3.) $\mathcal{B}$ is left invariant by all symmetries that leave the EPR state and the position of the first particle invariant. Furthermore, we show that *these requirements alone entail that the second particle also has a definite position.* We take this result as demonstrating that if the position of the first particle is assumed to be definite, then the *only* invariant classical description is one in which the second particle also has a definite position.

Theorem 2. *Let ω be the EPR state. There is a unique subalgebra $\mathcal{B}$ of $\mathcal{A}[\mathbb{R}^2]\otimes\mathcal{A}[\mathbb{R}^2]$ that is maximal with respect to the three conditions:*

1. *$\omega|_{\mathcal{B}}$ is a classical probability distribution;*
2. *$\mathcal{Q}_1 \subseteq \mathcal{B}$;*
3. *$\mathcal{B}$ is definable in terms of ω and $\mathcal{Q}_1$.*

Moreover, it follows that:

4. *$\mathcal{Q}_2 \subseteq \mathcal{B}$.*

By symmetry, the result also holds if we replace $\mathcal{Q}_i$ with $\mathcal{P}_i$ throughout the statement of the theorem.

For the proof of the theorem, we will need to invoke two technical lemmas. First, let $\mathbf{B}(\mathcal{H})$ denote the algebra of bounded linear operators on the Hilbert space $\mathcal{H}$. If $\mathcal{B}$ is a subset of $\mathbf{B}(\mathcal{H})$, we let $\mathcal{B}'$ denote the set of all operators in $\mathbf{B}(\mathcal{H})$ that commute with each operator in $\mathcal{B}$, and we let $\mathcal{B}''=(\mathcal{B}')'$.

Lemma 1. *Let $\mathcal{B}$ be a C^*-algebra of operators acting on $\mathcal{H}$. Let $U_t = \exp\{-itH\}$, where H is a bounded self-adjoint operator acting on $\mathcal{H}$. If $U_t\mathcal{B}U_{-t} = \mathcal{B}$ for all $t\in\mathbb{R}$, then there is a one-parameter unitary group $\{V_t : t\in\mathbb{R}\}\subseteq\mathcal{B}''$ such that $U_tAU_{-t} = V_tAV_{-t}$ for all $A\in\mathcal{B}$ and $t\in\mathbb{R}$.*

Proof. See Theorem 4.1.15 of [25]. □

Lemma 2. *Let $\mathcal{A}$ be a C^*-algebra, let ω be a state of $\mathcal{A}$, and let $(\pi, \mathcal{H}, \Omega)$ be the GNS representation of $\mathcal{A}$ induced by ω [19, p. 279]. Suppose that α is a $*$-automorphism of $\mathcal{A}$ such that $\omega \circ \alpha = \omega$. Then there is a unitary operator U on $\mathcal{H}$ such that $U\Omega = \Omega$ and $U\pi(A)U^* = \pi(\alpha(A))$ for all $A \in \mathcal{A}$.*

Proof. See Proposition 7.4.12 of [21]. □

Proof. Proof of the Theorem Let $\mathcal{A} = \mathcal{A}[\mathbb{R}^2] \otimes \mathcal{A}[\mathbb{R}^2]$. By the GNS construction (see [19, Thm. 4.5.2]), there is a Hilbert space $\mathcal{H}$, a unit vector $\Omega \in \mathcal{H}$, and a $*$-homomorphism π from $\mathcal{A}$ into $\mathbf{B}(\mathcal{H})$ such that

$$\omega(A) = \langle \Omega, \pi(A)\Omega \rangle, \qquad A \in \mathcal{A}. \tag{5}$$

Since $\mathcal{A}$ is simple, π is a $*$-isomorphism. Thus, we can suppress reference to π, and suppose that $\mathcal{A}$ is given as a C^*-algebra of operators acting on $\mathcal{H}$. We will need to make frequent use of the following result: For any subalgebra $\mathcal{B}$ of $\mathcal{A}$, $\omega|_{\mathcal{B}}$ is a classical probability distribution if and only if $[A, B]\Omega = 0$ for all $A, B \in \mathcal{B}$ [15, Prop. 2.2].

Our proof now splits into two parts: (I.) We show that if a subalgebra of $\mathcal{A}$ maximally satisfies conditions 1.–3. of the theorem, then it also satisfies condition 4. (II.) We show that there is a unique subalgebra of $\mathcal{A}$, viz., $\mathcal{F}^\Omega \cap \mathcal{A}$ (to be defined later), that maximally satisfies 1.–4. To finish off the argument, we note that if $\mathcal{B}$ is any subalgebra of $\mathcal{A}$ satisfying 1.–3., then (by an application of Zorn's lemma) it is contained in an algebra $\mathcal{C}$ that maximally satisfies 1.–3., and hence also maximally satisfies 1.–4., in virtue of part (I.). Thus, by part (II.), $\mathcal{C} = \mathcal{F}^\Omega \cap \mathcal{A}$, and $\mathcal{B} \subseteq \mathcal{F}^\Omega \cap \mathcal{A}$, establishing that the latter is also the unique subalgebra of $\mathcal{A}$ maximally satisfying just 1.–3.

(I.) Suppose that $\mathcal{B}$ is a C^*-subalgebra of $\mathcal{A}$ that maximally satisfies conditions 1.–3. of the theorem. We wish to show that $I \otimes U_a \in \mathcal{B}$ for all $a \in \mathbb{R}$. If we set

$$A \;:=\; (1/2)[(I \otimes U_a) + (I \otimes U_{-a})], \tag{6}$$

$$B \;:=\; (i/2)[(I \otimes U_{-a}) - (I \otimes U_a)], \tag{7}$$

then $A + iB = I \otimes U_a$. Thus, it will suffice to show that $A, B \in \mathcal{B}$. We will treat the case of A; the case of B can be dealt with by a similar argument. Let

$$A' \;=\; (1/2)[e^{-ia\lambda}(U_a \otimes I) + e^{ia\lambda}(U_{-a} \otimes I)]. \tag{8}$$

A straightforward calculation (using the definition of the EPR state ω and the fact $\omega(U_a \otimes U_b) = 0$ if $b \neq -a$ [14, Eqn. 19]) shows that $\omega((A'-A)^*(A'-A)) = 0$. Thus, in the GNS representation,

$$\begin{aligned} \|(A'-A)\Omega\|^2 &= \langle (A'-A)\Omega, (A'-A)\Omega\rangle &(9)\\ &= \langle \Omega, (A'-A)^*(A'-A)\Omega\rangle \;=\; 0. &(10)\end{aligned}$$

Let $H = A' - A$ and let $U_t = \exp\{-itH\}$ for all $t \in \mathbb{R}$. We claim now that if $U_t \in \mathcal{B}$ for all $t \in \mathbb{R}$ then $A \in \mathcal{B}$. Indeed, suppose that $U_t \in \mathcal{B}$ for all $t \in \mathbb{R}$. Since $\lim_{t\to 0} \|iH - t^{-1}(U_t - I)\| = 0$, and since $\mathcal{A}$ is closed in the norm topology, it follows that $H \in \mathcal{B}$. Moreover, since $A = H - A'$ and $A' \in \mathcal{B}$, it follows that $A \in \mathcal{B}$. Thus, it will suffice to show that $U_t \in \mathcal{B}$ for all $t \in \mathbb{R}$.

For each $t \in \mathbb{R}$, define a $*$-automorphism α_t of $\mathcal{A}$ by setting $\alpha_t(Z) = U_t Z U_{-t}$ for all $Z \in \mathcal{A}$. Since $H\Omega = (A'-A)\Omega = 0$, it follows that $U_t\Omega = \exp\{-itH\}\Omega = \Omega$ for all $t \in \mathbb{R}$. Thus,

$$\omega(\alpha_t(Z)) = \langle \Omega, U_t Z U_{-t}\Omega\rangle = \langle \Omega, Z\Omega\rangle = \omega(Z), \tag{11}$$

for all $Z \in \mathcal{A}$. Moreover, $\alpha_t(X) = U_t X U_{-t} = X$ for all $X \in \mathcal{R}$. Since $\mathcal{B}$ is definable in terms of $\mathcal{R}$ and ω, it follows that $U_t \mathcal{B} U_{-t} = \alpha_t(\mathcal{B}) = \mathcal{B}$ for all $t \in \mathbb{R}$. Thus, Lemma 1 entails that there is a unitary group $\{V_t : t \in \mathbb{R}\} \subseteq \mathcal{B}''$ such that $U_t Z U_{-t} = V_t Z V_{-t}$ for all $Z \in \mathcal{B}$ and $t \in \mathbb{R}$. Since $\omega|_{\mathcal{B}''}$ is classical (see [15, Cor. 2.9]), and since $Z, V_{-t} \in \mathcal{B}''$, we have

$$U_t Z U_{-t}\Omega = V_t Z V_{-t}\Omega = V_t V_{-t} Z\Omega = Z\Omega. \tag{12}$$

Thus, $[U_t, Z]\Omega = 0$ for all $t \in \mathbb{R}$.

Let $[\mathcal{B}\Omega]$ denote the closed linear span of $\mathcal{B}\Omega = \{Y\Omega : Y \in \mathcal{B}\}$, and let P denote the orthogonal projection onto $[\mathcal{B}\Omega]$. Then $P \in \mathcal{B}' = (\mathcal{B}'')'$, and $\mathcal{B}''P$ is a von Neumann algebra acting on $[\mathcal{B}\Omega]$ [19, Prop. 5.5.6]. Let $\mathcal{B}^\Omega$ denote the subalgebra of $\mathbf{B}(\mathcal{H})$ given by

$$\mathcal{B}^\Omega = (I-P)\mathbf{B}(\mathcal{H})(I-P) \oplus \mathcal{B}''P. \tag{13}$$

In order to complete the first part of the proof, we show (a.) $\mathcal{B} = \mathcal{B}^\Omega \cap \mathcal{A}$, and (b.) $U_t \in \mathcal{B}^\Omega \cap \mathcal{A}$ for all $t \in \mathbb{R}$.

(a.) Recall that $\mathcal{B}$ was assumed to be maximal with respect to conditions 1.–3. of the theorem. Since $\mathcal{B} \subseteq \mathcal{B}^\Omega \cap \mathcal{A}$, it will follow that $\mathcal{B} = \mathcal{B}^\Omega \cap \mathcal{A}$ if it can be shown that $\mathcal{B}^\Omega \cap \mathcal{A}$ satisfies conditions 1. and 3.

We first show that $\omega|_{\mathcal{B}^\Omega \cap \mathcal{A}}$ is a classical probability distribution. Since $\omega|_{\mathcal{B}}$ is classical, and ω is a normal state in the representation, it follows that

$\omega|_{\mathcal{B}''}$ is classical. Thus, $\mathcal{B}''P$ is abelian [15, Prop. 2.2], and $\omega|_{\mathcal{B}^\Omega}$ is classical [15, Thm. 2.8]. Therefore $\omega|_{\mathcal{B}^\Omega \cap \mathcal{A}}$ is classical.

We now show that $\mathcal{B}^\Omega \cap \mathcal{A}$ is definable in terms of $\mathcal{R}$ and ω. Let α be a $*$-automorphism of $\mathcal{A}$ such that $\alpha(\mathcal{R}) = \mathcal{R}$ and $\omega \circ \alpha = \omega$. Since $\mathcal{B}$ is definable in terms of $\mathcal{R}$ and ω, $\alpha(\mathcal{B}) = \mathcal{B}$. By Lemma 2, there is a unitary operator U on $\mathcal{H}$ such that $U\Omega = \Omega$ and $\alpha(X) = UXU^*$ for all $X \in \mathcal{A}$. In particular, $U\mathcal{B}U^* = \mathcal{B}$ and by continuity $U\mathcal{B}''U^* = \mathcal{B}''$. For any $Z \in \mathcal{B}$, $U(Z\Omega) = UZU^*\Omega \in [\mathcal{B}\Omega]$ and therefore $[U, P] = 0$. Thus, $U(ZP)U^* = (UZU^*)P \in \mathcal{B}''P$ for any $Z \in \mathcal{B}''$. Thus, $U\mathcal{B}^\Omega U^* = \mathcal{B}^\Omega$, and $\alpha(\mathcal{B}^\Omega \cap \mathcal{A}) = U(\mathcal{B}^\Omega \cap \mathcal{A})U^* = \mathcal{B}^\Omega \cap \mathcal{A}$. Therefore $\mathcal{B}^\Omega \cap \mathcal{A}$ is definable in terms of $\mathcal{R}$ and ω.

(b.) We now show that $U_t \in \mathcal{B}^\Omega \cap \mathcal{A}$ for all $t \in \mathbb{R}$. Clearly $U_t \in \mathcal{A}$ since $U_t = \exp\{-itH\}$ and H is a finite linear combination of elements in $\{U_a \otimes U_b : a, b \in \mathbb{R}\}$. Now, for any $Z \in \mathcal{B}$, we have shown that $U_t Z\Omega = ZU_t\Omega = Z\Omega$. Thus, U_t acts like the identity on the subspace $[\mathcal{B}\Omega]$ of $\mathcal{H}$. If P is again used to denote the orthogonal projection onto $[\mathcal{B}\Omega]$, then $U_tP = P \in \mathcal{B}''P$; and therefore $U_t \in \mathcal{B}^\Omega$.

(II.) We prove that there is a unique subalgebra of $\mathcal{A}$ that maximally satisfies conditions 1.–4. of the theorem. This result turns on the following key fact: For *any* representation $(\pi, \mathcal{H})$ of $\mathcal{A}$, the von Neumann algebra $\pi(\{U_a \otimes U_b : a, b \in \mathbb{R}\})''$ is maximal abelian in $\mathbf{B}(\mathcal{H})$ [12, Thm. I.6]. Thus, in our present notation (suppressing the representation mapping), $\{U_a \otimes U_b : a, b \in \mathbb{R}\}''$ is maximal abelian.

Let $\mathcal{F} = \{U_a \otimes U_b : a, b \in \mathbb{R}\}$, and let P denote the orthogonal projection onto $[\mathcal{F}\Omega]$. Since $\mathcal{F}$ leaves $[\mathcal{F}\Omega]$ invariant, $P \in \mathcal{F}'$. Let $\mathcal{F}^\Omega$ denote the subalgebra of $\mathbf{B}(\mathcal{H})$ given by

$$\mathcal{F}^\Omega = (I - P)\mathbf{B}(\mathcal{H})(I - P) \oplus \mathcal{F}''P. \tag{14}$$

It is clear that $\mathcal{F} \subseteq \mathcal{F}^\Omega \cap \mathcal{A}$. Thus, $\mathcal{F}^\Omega \cap \mathcal{A}$ satisfies conditions 2. and 4. of the theorem. Since $\mathcal{F}''P$ is abelian, $\mathcal{F}^\Omega \cap \mathcal{A}$ satisfies 1. [15, Prop. 2.2]. And, since $\mathcal{F}^\Omega \cap \mathcal{A}$ is constructed out of elements invariant under automorphisms that preserve the EPR state and $\mathcal{Q}_1$, it satisfies 3. For maximality, we must show that $\mathcal{F}^\Omega \cap \mathcal{A}$ contains any other subalgebra $\mathcal{B} \subseteq \mathcal{A}$ satisfying 1.–4.

By hypothesis, $\mathcal{B}$ is a subalgebra of $\mathcal{A}$ such that $\omega|_{\mathcal{B}}$ is classical and $\mathcal{F} \subseteq \mathcal{B}$. (Henceforth, we shall not actually need $\mathcal{B}$'s satisfaction of condition 3.) Then, $\omega|_{\mathcal{B}''}$ is classical and $\mathcal{F}'' \subseteq \mathcal{B}''$. Since $\mathcal{F}''$ is maximal abelian, $\mathcal{F}'' = \mathcal{F}'$. Thus, $P \in \mathcal{F}' = \mathcal{F}'' \subseteq \mathcal{B}''$. Let A be an arbitrary element of $\mathcal{B}$. Then, $A\Omega = AP\Omega = PA\Omega$ since $A, P \in \mathcal{B}''$ and since $\omega|_{\mathcal{B}''}$ is classical. Thus, A leaves $[\mathcal{F}\Omega]$ invariant, and $A = (I - P)A(I - P) + AP$. Furthermore, for any $R \in \mathcal{F}''$, $[AP, RP] = 0$. Since $\mathcal{F}''P$ is a maximal abelian subalgebra of $P\mathbf{B}(\mathcal{H})P$, it follows that $AP \in \mathcal{F}''P$. Therefore, $A \in \mathcal{F}^{\Omega} \cap \mathcal{A}$. Since A was an arbitrary element of $\mathcal{B}$, it follows that $\mathcal{B} \subseteq \mathcal{F}^{\Omega} \cap \mathcal{A}$. $\square$

6 Conclusion

We have shown that Bohr's reply to EPR is a logical consequence of four requirements: (1.) *Empirical Adequacy:* When an observable is measured, it possesses some value in accordance with the probabilities determined by the quantum state. (2.) *Classical Description:* Properties P and P' can be simultaneously real in a quantum state only if that state can be represented as a joint classical probability distribution over P and P'. (3.) *Objectivity:* Elements of reality must be invariants of those symmetries that preserve the defining features of the measurement context. (4.) *Maximality:* Our description should be maximal, subject to the prior three constraints. Obviously, these requirements have nothing to do with the verifiability criterion of meaning or with other central positivistic doctrines. Thus, Bohr's reply to EPR does not require a shift towards positivism.

Nonetheless, our reconstruction of Bohr's reply does not in itself constitute an argument for the superiority of Bohr's point of view over EPR's more "realist" point of view, which rejects the claim that the reality of a system can be constituted "from a distance." However, we wish to emphasize that Bohr is not so much concerned with what is *truly* real for the distant system as he is with the question of what we would be *warranted in asserting* about the distant system from the standpoint of classical description. In particular, Bohr argues that in certain measurement contexts we are warranted in attributing certain elements of reality to distant (unmeasured) systems. He also claims, however, that if we attempt to make *context-independent* attributions of reality to these distant systems, then we will come into conflict with the experimental record.

Moreover, as Bohr himself might have claimed, a similar sort of context-dependence already arises in special relativity. In particular, an inertial observer is warranted in saying that any two events that are orthogonal to his worldline at some worldpoint are simultaneous. However, if we attempt to make *context-independent* attributions of simultaneity to distant events — where the "context" is now set by the observer's frame of reference — then we will run into conflicts with the experimental record.

Of course, a proper defense of Bohr's point of view would require much more space than we have here. However, we have supplied ample justification for the claim that Bohr's reply to EPR — and his philosophy of quantum theory in general — deserves a more fair treatment than it has recently received.

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