Chasing Chimeras

Wayne C.Myrvold
Department of Philosophy
University of Western Ontario

To appear in *The British Journal for the Philosophy of Science*

Abstract

Earman and Ruetsche ([2005]) have cast their gaze upon existing no-go theorems for relativistic modal interpretations, and have found them inconclusive. They suggest that it would be more fruitful to investigate modal interpretations proposed for "really relativistic theories," that is, algebraic relativistic quantum field theories. They investigate the proposal of Clifton ([2000]), and extend Clifton's result that, for a host of states, his proposal yields no definite observables other than multiples of the identity. This leads Earman and Ruetsche to a suspicion that troubles for modal interpretations of such relativistic theories "are due less to the Poincaré invariance of relativistic QFT vs. the Galilean invariance of ordinary nonrelativistic QM than to the infinite number of degrees of freedom of former vs. the finite number of degrees of freedom of the latter" (577–78). I am skeptical of this suggestion. Though there are troubles for modal interpretations a relativistic quantum field theory that are due to its being a field theory—that is, due to infinitude of the degrees of freedom—they are not the *only* troubles faced by modal interpretations of quantum theories set in relativistic spacetime; there are also troubles traceable to relativistic causal structure.

1 Introduction

In a recent paper, John Earman and Laura Ruetsche ([2005]) have taken to task a number of no-go theorems concerning relativistic modal interpretations of quantum mechanics,[1] on the grounds that the target of such results "is not a full-blown manifestly relativistic quantum theory, but a mixture of bits and pieces of ordinary nonrelativistic quantum mechanics and relativity theory" (560). They suggest that a more fruitful avenue of investigation is to explore modal interpretations in the context of fully relativistic quantum theories. Rob Clifton ([2000]) proposed a natural extension to this context of the usual modal rule for picking out sets of definite observables. As Clifton pointed out ([2000], Prop. 3), this has the unfortunate consequence that there is a norm-dense set of states for which the proposal picks out as definite no observables other than multiples of the identity. The bulk of Earman and Ruetsche's paper is devoted to extending Clifton's result and to providing concrete examples of this trivialization. As Clifton, and, following him, Earman and Ruetsche, have rightly emphasized, these trivialization results pose a serious problem for the project of constructing a modal interpretation of a relativistic quantum field theory. For Earman and Ruetsche, this leads to a suspicion that the troubles for relativistic modal interpretations "are due less to the Poincaré invariance of relativistic QFT vs. the Galilean invariance of ordinary nonrelativistic QM than to the infinite number of degrees of freedom of former vs. the finite number of degrees of freedom of the latter" (577–78). This seems to me to misrepresent the situation. There certainly are difficulties for a modal interpretation of a relativistic quantum field theory associated with infinity of degrees of freedom, as exhibited by the trivialization results. But there are also difficulties associated with relativistic causal structure, as exhibited by the above-mentioned no-go results. This matters because, faced with the trivialization theorems, a would-be relativistic modal interpreter might be motivated to bite the bullet and move to a theory in which only finitely many degrees of freedom are associated with bounded regions of spacetime, a move that finds independent motivation in claims from workers in quantum gravity that a successful theory of quantum gravity will have this property.[2] The question then arises whether it is possible to do so while retaining a relativistic causal structure, in the sense explained in §2, below.

The no-go theorems that Earman and Ruetsche criticize make free use of evolving states associated with spacelike hypersurfaces, of states represented by density operators, and of one-dimensional projections onto eigenspaces of observables. By contrast, the "really relativistic" theories with which Earman and Ruetsche are concerned represent observables by Heisenberg-picture operators; states on this picture are not states-at-a-time but are states defined on the entire quasi-local algebra of operators associated with spacetime regions. Moreover, in such theories, an algebra associated with a bounded spacetime region will typically be a type III factor, containing no one-dimensional projections, and states on such an algebra will not be representable by density operators belonging to the algebra.

The point that the no-go theorems avail themselves of apparatus that is either out of place or unfamiliar in a relativistic setting is well taken. Nevertheless, it seems that there is a fairly simple point revealed by the theorems criticized by Earman and Ruetsche, a point that survives the transition to the framework of algebraic relativistic quantum theories. The theorems *do* in fact show that there is a tension between modal interpretations and relativistic causal structure, a tension that is independent of the problems for modal interpretations of quantum field theories that stem from dealing with an infinity of degrees of freedom. Further, the use of "chimerical" theories by the authors of the cited no-go theorems can be defended. Such theories involve a low-velocity approximation to special relativity that retains relativity of simultaneity while dispensing with much of the other apparatus of special relativity. It is possible to get clear the rules by which pieces of relativity and non-relativistic quantum theories are stitched together, and, having done so, we find that the no-go theorems go through in pretty much their original form. The chimeras are quite tame, and useful servants in getting clear about the passage from a really relativistic theory to a non-relativistic limit.

2 Relativity and causal structure

We are concerned with the question of whether modal interpretations, formulated originally for nonrelativistic quantum mechanics, can be rendered compatible with the special theory of relativity. A few words are therefore appropriate about what is at stake.

The special theory of relativity is a theory of spacetime structure; it is the theory that says that physical spacetime has the structure of Minkowski spacetime, whose symmetry group is the Poincaré group. We need not get into an involved discussion of what it means to say that this structure is the structure of physical spacetime, but at minimum it should mean that the causal structure is given by the light-cone structure—two events are causally connectible if and only if one is in the past light-cone of the other. This is a striking difference between Minkowski spacetime and Galilean spacetime. It is this feature that gives rise to the relativity of simultaneity; given an event p, there will, in Minkowski spacetime, be more than one maximal achronal surface that includes p, that is, more than one hypersurface of simultaneity of which p is an element.

It is this feature of Minkowski spacetime that poses a *prima facie* threat of conflict with quantum nonlocality. It is also a feature that survives the transition to the curved spacetimes of general relativity, and which exists also in the discrete causal sets favoured by some researchers into quantum gravity.

If it is causal structure that is of interest, let us focus on that. Assume that we have a spacetime on which is defined a relation $\prec$ of causal precedence; '$x \prec y$' is to be read as 'x is in the causal past of y'. This will be assumed to be transitive and antisymmetric (no closed causal loops). We define the relation of causal inconnectibility as

$$x \sim y :\equiv \neg(x \prec y \vee y \prec x).$$

This relation is symmetric by construction and reflexive by virtue of the antisymmetry of $\prec$. x will be said to be *spacelike separated* from y if and only if $x \sim y$ and $x \neq y$.

In Galilean spacetime, if z is in the causal past (future) of x, it is also in the causal past (future) of any event y that is spacelike separated from x. These two conditions are together equivalent to the transitivity of $\sim$. For a Galilean spacetime, therefore, $\sim$ is an equivalence relation, and the quotient of the spacetime by this relation is a foliation of spacetime into hypersurfaces of simultaneity. In relativistic spacetimes, on the other hand, $\sim$ is not transitive, and, in fact, for any event x, there exist y, z such that $x \sim y$ and $x \sim z$ but $y \prec z$.

The question of interest, or at least one question of interest, is whether the interpretations we are concerned with can respect such a causal structure, or whether, like the Bohm theory, they are compelled to invoke a distinguished foliation and thereby a transitive relation of causal inconnectibility. This, it seems, is the motivating question behind the no-go theorems mentioned in the first paragraph.

3 The theorems purified

Modal interpretations, as originally formulated for non-relativistic quantum mechanics, pick out at each time, for each system, a set of observables that are taken as having definite values. These, typically, will extend beyond those made definite by the quantum state of the system, but must nevertheless be sufficiently constrained so as to avoid a Kochen-Specker obstruction, which would stand in the way of regarding the quantum probabilities as mixtures over assignments of definite values to these observables. Depending on the particular version of modal interpretation considered, the set of definite observables will either be a commuting set or—what amounts essentially to the same thing—a set whose commutators have null value on the quantum state of the system.

Suppose, now, that we want to extend the modal interpretation to a relativistic setting. Relativistic quantum theories are typically formulated in terms of the Heisenberg picture (though, as we shall see in section 4, below, it is possible to introduce a relativistic analog of Schrödinger-picture evolving states). A modal interpretation can be expected to pick, from these Heisenberg operators, a subset to be assigned definite values.

Let us first consider what happens if we try to adapt a Bub-style modal interpretation to a relativistic context. Such an interpretation, as formulated for non-relativistic quantum theory, singles out some observable to have definite values at all times. That is, at each time, the definite observable will be represented by the same Schrödinger-picture operator. Translated into the the Heisenberg picture, observables definite at different times will be represented by operators related by temporal evolution.

We associate with a bounded, open spacetime region X an algebra $\mathcal{A}(X)$, whose self-adjoint part represents observables measurable in X. Let A, B, C, D be mutually disjoint, bounded open regions of spacetime, with $A \sim B$, $A \sim D$, $C \sim B$, $C \sim D$, and suppose there is a timelike translation T that takes A into C and B into D. Let ρ be a state on $\mathcal{A}(A \cup B \cup C \cup D)$. Consider operators $R_A \in \mathcal{A}(A)$, $R_B \in \mathcal{A}(B)$, $R_C \in \mathcal{A}(C)$, and $R_D \in \mathcal{A}(D)$ as candidates for observables that are definite in state ρ. Let us suppose that we expect our modal interpretation to recover, not only ρ's expectation values for R_A, R_B, R_C, and R_D, but also the correlations between the spacelike separated pairs.

> If $X \sim Y$, and $R_X \in \mathcal{A}(X)$, $R_Y \in \mathcal{A}(Y)$ are projections with definite values, then the probability in state ρ that they both take on the value 1 is $\rho(R_X\, R_Y)$.

This is the condition called the "Relativistic Born Rule" in Myrvold ([2002]), minus the unnecessary mention of spacelike hypersurfaces. It can be satisfied only if there is a joint distribution over definite values of $\{R_A, R_B, R_C, R_D\}$ that yields ρ's expectations as marginal probabilities for spacelike separated pairs.

If, now, R_C is the the time evolute of R_A—relative to the time translation T, the "same" Schrödinger-picture observable—and R_D is the time evolute of R_B, these will typically not commute, and there will be a host of states for which an assumption of definite values for all of these observables will run afoul of a Bell inequality. Moreover, depending on the evolution along T, it may be the case that the set of definite observables yields a Kochen-Specker obstruction. This, stripped of apparatus foreign to algebraic relativistic quantum theories, is the content of the no-go theorem of Myrvold ([2002]). In the example used therein, the evolution imposed is such as to make the timelike related pairs of observables $\langle S_A, S_C \rangle, \langle S_B, S_D \rangle$ anticommute, where the S_X's are related to the definite projections R_X by $S_X = R_X - (I - R_X)$; the state is chosen to make the impossibility of a joint distribution with the desired properties particularly easy to demonstrate. The details of the example are, however, unimportant; as long as the timelike separated pairs fail to commute, there will be *some* states that yield correlations between the spacelike-separated pairs that violate a Bell Inequality.

Restricting the set of definite observables to ones whose time-evolutes commute does not seem to be a viable option, if the interpretation is to achieve the goal of providing a sufficiently rich set of definite observables to count as a realist resolution of the measurement problem. This requires, at minimum, definiteness of certain observables—pointer observables, and the like—represented by quantum operators that undergo non-trivial evolution and hence are represented at different times by noncommuting operators. A straightforward extension of a Bub-style modal interpretation to a relativistic context is, therefore, not a possibility.

How do modal interpretations that use the spectral decomposition of a system's density operator to pick out the set of definite properties fare? Although, in the relativistic context, a density operator representing the state may not be available, Clifton's prescription can be substituted for the spectral decomposition rule. This reduces to the familiar prescription when the state can be represented by a density operator. The no-go theorems for density-operator modal interpretations of Dickson and Clifton ([1998]) and Arntzenius ([1998]), and the extension in Myrvold ([2002]) of the argument to such interpretations, are thus readily adapted to interpretations that employ the Clifton rule. As Earman and Ruetsche point out (566–567), it will not do, in formulating a no-go theorem for such interpretations, to restrict one's attention to situations in which the systems in question are isolated (so that their Schrödinger-picture state evolution is unitary), as, for such evolutions, the set of definite observables simply follows the evolution of the density operator's eigenprojections, and neither a Bell inequality nor a Kochen-Specker obstruction will be forthcoming. Translated into the relativistic context: for such evolutions, the Clifton prescription picks out the same observable as definite in regions A and C. Accordingly, the cited no-go theorems for density-operator interpretations involve, as they must, not isolated evolutions, but situations in which the systems in question are coupled to other systems (thought of as measuring devices, though this is of course inessential). The coupling forces an evolution of the particles' definite properties, as picked out by their reduced density operators; given appropriate choice of initial state and coupling, it is not difficult to obtain sets of definite properties such that no joint distribution can yield the Born-rule correlations for the spacelike-separated pairs. Though the set of observables picked out as definite in a region X depends only on the restriction of the global state to the local algebra $\mathcal{A}(X)$, and the prescription for picking out definite values selects, for each of these regions, a set of observables that can be consistently assigned definite values, the values assigned to these observables are also responsible to the global state, if the Born-rule correlations are to be recovered for spacelike-separated pairs of observables. The Clifton rule does not guarantee that the observables picked out as definite in timelike-related regions commute, nor that the global state's correlations for all spacelike-separated pairs of local definite observables be recoverable from a mixture of joint assignments of definite values to these observables.

The question arises whether states and evolutions of the sort require for these proofs, that is, states and evolutions giving rise to sets of local observables that admit of no joint probabilites satisfying the relativistic Born rule, will arise in physically realistic situations; perhaps relativity (or something else) precludes states and evolutions of this sort.[3] The evolutions involved are local evolutions, imposed separately on spacelike separated systems; the relativistic causal structure does not forbid us from approximating such evolutions under suitable laboratory conditions. The states employed in the proofs, however, are states defined on the limited set of observables explicitly considered, not on the full set of degrees of freedom treated by a field theory, and here there is room for worry that the theorems might not apply to field theories. However, even if our most fundamental theory is a field theory, in concrete applications to experiments we typically treat all but finitely many degrees of freedom as negligible, and we take it that the restriction of the state to these degrees of freedom can serve as a stand-in for the full field-theoretical state. We want our interpretation to yield the correct correlations in Bell-type experiments, involving measurements made at spacelike separation on entangled systems. In analysis of such experiments we are led naturally to consideration of theories in which relativistic considerations other than the relativity of simultaneity are disregarded. That is, we are led to consideration of chimerical theories, aptly characterized by Earman and Ruetsche as "ordinary QM, relieved of a privileged foliation of time slices, and subject to the relativistic Born rule" (560).

The no-go theorems for the density-operator interpretations can be thought of as applying, in the first instance, to such chimerical theories. They are, however, of relevance to the viability of a modal interpretation of a relativistic quantum field theory. Suppose we construct a chimerical theory by starting with a quantum field theory and restricting our attention to a limited set χ of the full theory's observables, say, by an energy cut-off beyond which we regard everything as effectively negligible in the situation at hand. Suppose that the modal interpretation of the full-blown theory picks out, for a state ρ of a local algebra $\mathcal{A}$, a definite subalgebra $\mathcal{D}\rho(\mathcal{A})$. Now consider a subalgebra $\mathcal{A}_\chi$, and let ρ_χ be the restriction of ρ to $\mathcal{A}_\chi$, and consider the definite algebra $\mathcal{D}_{\rho_\chi}(\mathcal{A}_\chi)$ that is picked out by a modal interpretation of the reduced theory. If we can construct a no-go theorem for a modal interpretation, restricted in this way to subalgebras $\mathcal{A}_\chi$ of the local algebras, then the would-be modal interpreter of the full-blown theory is faced with a dilemma: either the definite observables in $\mathcal{D}_{\rho_\chi}(\mathcal{A}_\chi)$, or some approximation to them, are among the definite observables in $\mathcal{D}\rho(\mathcal{A})$, or not. If not, the interpretation runs the risk of having too few observables to count as a realist solution to the measurement problem. If there are, among the full theory's definite observables, some that approximate those of the chimerical theory, the modal interpretation of the full theory is in the same boat as the modal interpretation of the chimera, unable to recover the correlations between spacelike separated pairs of definite observables as mixtures of value assignments to these observables.

The Clifton-Earman-Ruetsche trivialization results suggest that a modal interpretation employing the Clifton rule, applied to a quantum field theory, will, for typical states, be caught on the "too few" horn of the dilemma, picking out no non-trivial definite observables at all— the extreme case of too few definite observables. This horn might be escaped by, *e.g.* making a move to a theory in which only finitely many degrees of freedom are associated with bounded regions of spacetime. This will be of no avail in escaping the second horn, however.

Note that we have employed very little in the way of assumptions about the spacetime structure. The same considerations would, in fact, apply to a modal interpretation of nonrelativistic quantum mechanics that sought to recover expectation values from assignments of definite values defined, not only on equal-time sets of Heisenberg observables, but on sets including timelike-related Heisenberg observables. Suppose that A, B are open bounded subsets of a spacelike hyperplane in Galilean spacetime, and C and D open bounded subsets of a later hyperplane. The causal structure of Galilean spacetime does not require R_A and R_D to commute, but if these are observables corresponding to quantum systems that do not interact with each other, they will commute anyway, and similarly for R_B and R_C. Then a Heisenberg-picture modal interpretation of nonrelativistic quantum mechanics that attempted to recover expectation values from two-valued homomorphisms over a set of observables including all of $\{R_A, R_B, R_C, R_D\}$ and closed under commuting products would run into the same troubles. An option that is available to the modal interpreter in Galilean spacetime, that is not available to the modal interpreter in a relativistic spacetime (that is, any spacetime without a causally distinguished foliation), is the option of picking out, for each time, a set of observables definite at that time, with the closure and probability conditions restricted to equal-time sets of observables.

What options are left for the would-be relativistic modal interpreter? One option might be to try to pare down the sets of definite observables, so that histories formed out of them are consistent histories, as is suggested by Dieks ([2000]). On such an approach, the set of definite observables in a local algebra $\mathcal{A}(X)$ would not be determined by the local reduced state ρ_X alone; some further restriction would have to be invoked to ensure the consistency of histories composed of them. Whether this can be done in a systematic manner while respecting relativistic causality remains a possible avenue for further research. One could also deny the assumption that there are local definite observables intrinsic to bounded regions of spacetime, perhaps by adopting a 'radical perspectivalism' (Dieks [2005]) on which the definite observables are relativized to hypersurfaces of simultaneity. This is the approach adopted by Berkovitz and Hemmo ([2005a, b]). Discussion of the viability of such an approach is beyond the scope of this paper.

4 Evolving states in a relativistic context

We have seen that the no-go theorems need not make use of evolving states associated with spacelike hyperplanes. Although relativistic quantum theories are typically cast in terms of the Heisenberg picture, which is a natural setting for a relativistic theory, it can be useful, for some purposes, to reintroduce time-evolving states, in analogy with the Schrödinger picture states of nonrelativistic quantum mechanics.

In nonrelativistic quantum mechanics, the state associated with a spacelike hypersurface yields expectation values for experiments performed to the future of the hypersurface, conditional on events (including any state collapses) to the past. This suggests that, if we want, in the context of an algebraic relativistic quantum theory, to set up an association between three-dimensional spacelike hypersurfaces and states we first associate with a spacelike hypersurface σ the algebra

$\mathcal{R}(\sigma) = \mathcal{A}(D^+(\sigma) \setminus \sigma)$, where $D^+(\sigma)$ is the future domain of dependence of σ.[4] If σ is open as a subset of a 3D spacelike hypersurface, then $\mathcal{R}(\sigma)$ is an open region of spacetime, and if σ is bounded, then for well-behaved spacetimes, so is $\mathcal{R}(\sigma)$.

In an algebraic relativistic quantum theory, we associated with an open bounded spacetime region $\mathcal{O}$ an algebra $\mathcal{A}(\mathcal{O})$. The self-adjoint elements of $\mathcal{A}(\mathcal{O})$ correspond to observables measured in experiments carried out in $\mathcal{O}$. We will have available to us a representation α_Λ of the Lorentz group, such that $\alpha_\Lambda(\mathcal{A}(\mathcal{O})) = \mathcal{A}(\mathcal{O}_\Lambda)$. If we want to mimic the Schrödinger picture and associate the same operator with a given experimental procedure, regardless of when it is performed, we can do so as follows.

Let $\mathcal{K}_\mathcal{A}$ be the dual space of the algebra $\mathcal{A}$, that is, the space of bounded linear functionals on $\mathcal{A}$. Define, for an automorphism $\alpha : \mathcal{A} \to \mathcal{A}$, the adjoint map $\alpha^\dagger : \mathcal{K}_\mathcal{A} \to \mathcal{K}_\mathcal{A}$ by

$$\alpha^\dagger(\omega) := \omega \circ \alpha.$$

Let $\{\sigma_t\}$ be a foliation of spacetime into spacelike hypersurfaces, indexed by a time parameter t. Assume, in addition a timelike congruence, or fibration of spacetime: a family F of disjoint inextendible timelike curves that cover spacetime, to be thought of as yielding a 'same place' relation on distinct hypersurfaces.[5] Let σ_0 be one member of the foliation, σ_T another, related to related to σ_0 by a timelike translation T along members of F. Let $\mathcal{O}$ be a bounded subset of $\mathcal{D}^+(\sigma_0)$, and let $\mathcal{O}_T = \alpha_T(\mathcal{O})$. Take $A \in \mathcal{A}(\mathcal{O})$, and let $A_T = \alpha_T(A)$ be the corresponding operator in $\mathcal{A}(\mathcal{O}_T)$. Then, to calculate the expectation value of these observables, we can either use the same state ρ and the time-evolved operator A_T, or else we can use the same operator A to represent the experiment, regardless of whether it is performed in $\mathcal{O}$ or $\mathcal{O}_T$, and use the time-evolved state $\rho_T = \alpha_T^\dagger(\rho)$ for the experiment performed in $\mathcal{O}_T$, since

$$\rho(A_T) = \rho_T(A).$$

In this way, given $\{\sigma_t\}$ and F, we get a corresponding family of states. We can also transform, in a straightforward way, between states associated with one such family, and states associated with another, corresponding to a different foliation and congruence.

Note that passing from a state associated with one spacelike hypersurface to another, whether they are members of a common foliation or not, involves dynamical evolution.[6] This is, of course, true in the classical context as well.

There is a value in reintroducing evolving states into a relativistic context that goes beyond concern for modal interpretations. We will want our relativistic quantum theories to have nonrelativistic quantum mechanics as a limiting case. One route from a relativistic theory to nonrelativistic Schrödinger-picture quantum mechanics would be to obtain nonrelativistic Heisenberg-picture quantum mechanics as a limiting case, and from there obtain Schrödinger-picture quantum mechanics. But the relation between the relativistic theory and Schrödinger-picture quantum mechanics could perhaps be seen more perspicuously if we can take a more direct route, without a Heisenberg-picture intermediary. On this route, there will be a different intermediary, which is what Bell ([1987]) calls "relative time translation invariant quantum mechanics."

Consider a one-dimensional Lorentz boost,

$$\begin{aligned} x' &= \frac{1}{\sqrt{1-\beta^2}}\,(x - \beta\, ct) \\ ct' &= \frac{1}{\sqrt{1-\beta^2}}\,(ct - \beta\, x), \end{aligned}$$

where $\beta = v/c$. To first order in β, this gives, not the Galilean transformation, but rather,

$$\begin{aligned} x' &= x - \beta\, ct \\ ct' &= ct - \beta\, x, \end{aligned}$$

Suppose we have two (or more) systems located in small regions a large distance apart, so that $\beta \Delta x$ is non-negligible even for $\beta << 1$. Then what we have is a relative time translation of the t-coordinates t_i at the locations x_i. When the systems are non-interacting, ordinary, nonrelativistic quantum mechanics is invariant under such relative time translations; a residue, as Bell puts it, of Lorentz invariance. A really relativistic theory, in a low energy regime, applied to systems localized (at least approximately) in regions that are small compared to the distances between them, should yield relative time-translation invariant quantum mechanics as an approximation.

More needs to be said about how this limit is to come about! As Clifton ([2000], 175) remarks, "since the algebras in the Galilean case are invariably type I, this limit is bound to be mathematically singular, and its *physical* characterization needs to be dealt with carefully. But this is a problem for *any* would-be interpreter of relativistic quantum field theory, not just modal interpreters."

Having reintroduced evolving states into the relativistic context, we obtain the no-go theorems in the form they originally appeared; these theorems can be thought of as no-go theorems for modal interpretations of relative time translation invariant quantum mechanics.

As argued in §3, above, no-go theorems for modal interpretations of such theories are relevant to the viability of modal interpretations of a fully-blown relativistic quantum theory. The upshot of such considerations is: for would-be modal interpreters in a relativistic spacetime, the strait between the Scylla of too many definite observables, and the Charybdis of too few, is narrow indeed.

5 Acknowledgmens

I would like to thank Laura Ruetsche for helpful comments on an earlier draft of this paper, and three anonymous referees for *BJPS* for constructive criticisms.

Notes

[1]Earman and Ruetsche mention Dickson and Clifton ([1998]), Arntzenius ([1998]), and Myrvold ([2002]); Berndl *et al.* ([1996]) should also be mentioned as a theorem of the same ilk.

[2]See, *e.g.* Bombelli *et al.* ([1987]), Henson ([2006]), Rovelli ([2004]).

[3]I am grateful to two anonymous referees for pressing this point.

[4]This is a slight departure from the usual way to make the association, which is to take the diamond-shaped region $D^+(\sigma) \cup D^-(\sigma)$. This will make no difference in the case of unitary evolution. If, however, we want to extend our account to collapse theories, it is observations in the future domain of dependence of a hypersurface whose probabilities the state on the hypersurface will concern.

[5]In Minkowski spacetime, the most natural choice is to take F to be a family of parallel inertial trajectories, and $\{\sigma_t\}$ a family of spacelike hypersurfaces orthogonal to all F. But we need not restrict ourselves to such choices.

[6]This point has been emphasized by Gordon Fleming ([2002, 2003]). "[T]he transition from the physical state of affairs on any one hyperplane to any other, whether the hyperplanes intersect or are parallel, is always an instance of dynamical evolution between them" (Fleming [2003], 10).

References

Arntzenius, F. [1998]: 'Curiouser and Curiouser: A Personal Evaluation of Modal Interpretations', In Dieks and Vermaas ([1998]), pp. 337–377.

Bell, J. S. [1987]: 'Are There Quantum Jumps?', In *Speakable and unspeakable in quantum mechanics*Cambridge, Cambridge University Press, pp. 201–212.

Berkovitz, J. and Hemmo, M. [2005a]: 'Can Modal Interpretations of Quantum Mechanics Be Reconciled with Relativity?', *Philosophy of Science*, **72**, pp. 789–801.

Berkovitz, J. and Hemmo, M. [2005b]: 'Modal Interpretations of Quantum Mechanics and Relativity: A Reconsideration', *Foundations of Physics*, **35**, pp. 373–396.

Berndl, K., Dürr, D., Goldstein, S. and Zanghì, N. [1996]: 'Nonlocality, Lorentz invariance, and Bohmian quantum theory', *Physical Review A*, **53**, pp. 2062–2073.

Bombelli, L., Lee, J., Meyer, D. and Sorkin, R. D. [1987]: 'Spacetime as a Causal Set', *Physical Review Letters*, **59**, pp. 521–524.

Clifton, R. [2000]: 'The Modal Interpretation of Algebraic Quantum Field Theory', *Physics Letters A*, **271**, pp. 1–7. Reprinted in Jeremy Butterfield and Hans Halvorson, (*eds.*), *Quantum Entanglements: Selected Papers*, Oxford, Oxford University Press, 2004, pp.143–163.

Dickson, M. and Clifton, R. [1998]: 'Lorentz-Invariance in Modal Interpretations', In Dieks and Vermaas ([1998]), pp. 9–47.

Dieks, D. [2000]: 'Consistent histories and relativistic invariance in the modal interpretation of quantum mechanics', *Physics Letters A*, **265**, pp. 317–325.

Dieks, D. and Vermaas, P. E. (eds.) [1998]: *The Modal Interpretation of Quantum Mechanics*, Dordrecht, Kluwer Academic Publishers.

Earman, J. and Ruetsche, L. [2005]: 'Relativistic Invariance and Modal Interpretations', *Philosophy of Science*, **72**, pp. 557–583.

Fleming, G. N. [2002]: 'The Dependence of Lorentz Boost Generators on the Presence and Nature of Interactions', <philsci-archive.pitt.edu/archive/00000663/>.

Fleming, G. N. [2003]: 'Observations on Hyperplanes: I. State Reduction and Unitary Evolution', <philsci-archive.pitt.edu/archive/00001533/>.

Henson, J. [2006]: 'The Causal Set Approach to Quantum Gravity', <gr-qc/0601121>.

Myrvold, W. C. [2002]: 'Modal Interpretations and Relativity', *Foundations of Physics*, **32**, pp. 1773–1784.

Rovelli, C. [2004]: *Quantum Gravity*, Cambridge, Cambridge University Press.