Observational Equivalence of Deterministic and Indeterministic Descriptions and the Role of Different Observations

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1 Introduction

Recently some results have been presented which show that certain kinds of deterministic descriptions and indeterministic descriptions are observationally equivalent (Werndl 2009*a*, Werndl 2010). These results prompt interesting philosophical questions, such as what exactly these results show or whether the deterministic or indeterministic description is preferable when there is observational equivalence. There is hardly any philosophical discussion about these questions, and this paper contributes to filling this gap.

More specifically, first, I will discuss the philosophical comments made by mathematicians about observational equivalence, in particular Ornstein & Weiss (1991). Their comments are vague, and I will argue that, according to a reasonable interpretation, they are misguided. Second, the results on observational equivalence raise the question of whether the deterministic or indeterministic description is preferable relative to all evidence. If none of them is preferable, this would amount to underdetermination. I will criticize Winnie's (1998) argument that, by appealing to different observations, one finds that the deterministic description is preferable. In particular, I will clarify a confusion in this argument. Furthermore, I will argue that if the concern is a strong form of underdetermination, the argument delivers the desired conclusion but this conclusion is trivial; and for the other kind of underdetermination of interest the argument fails.

This paper proceeds as follows. In section 2 I will introduce deterministic and indeterministic descriptions. In section 3 I will present the relevant results on observational equivalence. These results are highly technical, and I will keep the discussion at an intuitive level. In section 4 I will discuss the mathematicians' claims about observational equivalence. Section 5 will be about Winnie's argument on the role of different observations. Finally, in section 6 I will summarise the results.

2 Deterministic and Indeterministic Descriptions

I will introduce the relevant deterministic and indeterministic descriptions informally; for the technical details the reader is referred to Werndl (2010). Deterministic and indeterministic descriptions are of two kinds: either the time increases in discrete steps or the time-parameter varies continually. Because the latter case is more important in the sciences, this paper focuses on descriptions involving a *continuous time parameter*.[1]

Deterministic Systems

We will be concerned with measure-theoretic deterministic descriptions, in short deterministic systems. A *deterministic system* is a triple (X, T_t, p); the set X, called the *phase space*, represents all possible states of the system; $T_t(x)$, $t \in \mathbb{R}$, are functions, called the

[1] For more on descriptions where the time varies discretely see Werndl (2009*a*).

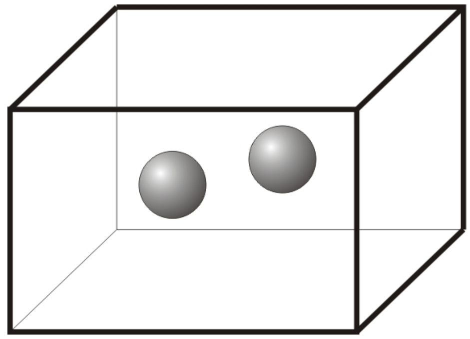

Figure 1: Two hard balls in a box

evolution functions, which tell one that the system in state x evolves to $T_t(x)$ after t time steps; and p assigns a probability to regions of X. It is obvious that these systems are deterministic according to the standard philosophical definition, namely that determinism means that two solutions which agree at one time agree at all future times (Butterfield 2005). A *solution* is a possible evolution of the system in the course of time, i.e., a function $s_x(t) : \mathbb{R} \rightarrow X$, $s_x(t) = T_t(x)$ for an arbitrary $x \in X$. Deterministic systems thus defined are among the most important descriptions in science. For instance, all deterministic descriptions in Newtonian and statistical mechanics are of this kind.

Example 1. Two hard balls in a box. This is a system in Newtonian mechanics where two balls which interact by elastic collisions and which have a finite radius but no rotational motion move in a three-dimensional box (cf. Simányi 1999). Figure 1 shows the hard ball system in a specific state. Mathematically, this system is represented as follows. X is the set of all possible states, i.e., the set of all vectors consisting of the position and velocity coordinates of the two balls. This means that the specific state of the system shown in Figure 1 is represented by exactly one $x \in X$. The evolution functions tell us that the hard ball system in state x will evolve to $T_t(x)$ after t time steps. For an arbitrary region A in phase space, p assigns the probability $p(A)$ to the event that the two hard balls are in one of the states represented by A. And a solution is a possible evolution of the hard ball system in the course of time.

Finally, when a deterministic system in state x is observed, a value $\Phi(x)$ is observed which is dependent on x (but may be different from it). Thus observations are modeled as *observation functions*, i.e., functions $\Phi(x) : X \rightarrow X_O$ where X_O is the set of all possible observed values.

Stochastic Processes

The indeterministic descriptions which I will be concerned with are stochastic processes, which are processes that are governed by probabilistic laws. A *stochastic process* $\{Z_t\}$ consists of a family of functions $Z_t : \Omega \rightarrow E$, $t \in \mathbb{R}$. The set E, called the *outcome space*,

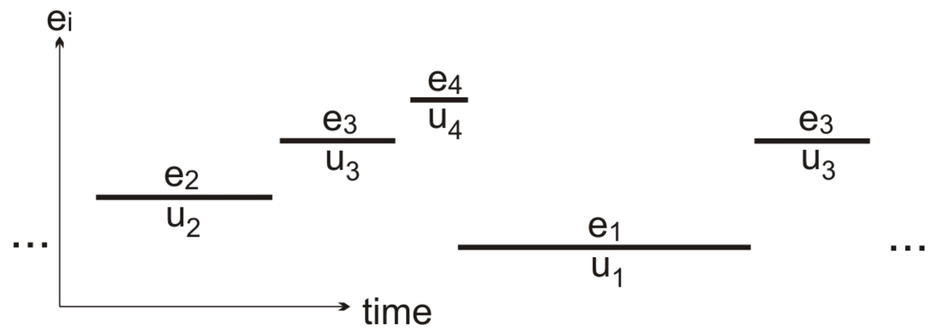

Figure 2: A realisation of a semi-Markov process

represents all possible outcomes of the process, and $Z_t(\omega)$ represents the outcome of the process at time t. Furthermore, probability distributions $P(Z_t \in A)$ tells us the probability that the outcome of the process is in A at time t for any region A of E and any $t \in \mathbb{R}$, and probability distributions $P(Z_s \in A$ given that $Z_t \in B)$ tell us the probability that the process is in A at time s given that it is in B at time t for arbitrary regions A, B of E and any $t, s \in \mathbb{R}$. A *realisation* of the stochastic process is a possible evolution of the process in the course of time, i.e., a function $r_\omega(t) : \mathbb{R} \to E$, $s_\omega(t) = Z_t(\omega)$ for an arbitrary $\omega \in \Omega$. (Here one sees that, intuitively, ω encodes the evolution of the stochastic process.) Stochastic processe are usually indeterministic. If the process takes a specific outcome, there are many outcomes that might follow, and a probability distribution measures the likelihood of them. Stochastic processes are the main indeterministic descriptions in the sciences.

Example 2: semi-Markov processes. A semi-Markov process is a process with finitely many possible outcomes $e_1, \ldots, e_n$. The process takes the outcome e_i for a time u_i, and which outcome follows e_i depends only on e_i. Figure 2 shows a realisation of a semi-Markov process with four possible outcomes e_1, e_2, e_3, e_4. The probability distributions of the semi-Markov process tell us, for instance, the probability that the process takes a specific outcome at time t, such as $P(Z_t = e_3)$, or the probability that an outcome at time t is followed by another outcome at s, such as $P(Z_s = e_3$ given that $Z_t = e_4)$ for $t, s \in \mathbb{R}$ (cf. Ornstein & Weiss 1991, Werndl 2010). Semi-Markov processes are widespread in the sciences (cf. Janssen & Limnios 1999)

3 Observational Equivalence of Deterministic and Indeterministic Descriptions

Observational equivalence as understood here means that *the deterministic description, when observed with an observation function, and the stochastic process give the same predictions.* Let me explain what "give the same predictions" means. The predictions derived from a stochastic process are the probability distributions over its realisations. Because there is a probability measure p defined on a deterministic system, when applying an observation function, the predictions obtained are the probability distributions over the solutions of the system coarse-grained by the observation function. Consequently, I say that a stochastic process $\{Z_t\}$ and a deterministic system (X, T_t, p) observed with an observation function Φ give the same predictions iff: (i) the outcome space E of $\{Z_t\}$ and the set of possible values of Φ are identical, and (ii) the probability distributions over the solutions of the deterministic system coarse-grained by Φ and the probability distributions over the realisations are the same.

Suppose that a deterministic system (X, T_t, p) is observed with an observation function Φ. *Then* $\{\Phi(T_t)\}$ *is a stochastic process which is observationally equivalent to* (X, T_t, p) *as observed with* Φ. Let me explain this with an example. Consider the system of two hard balls moving in a box (Example 1) and an observation function of this system with four possible values e_1, e_2, e_3, e_4. Because a probability measure p is defined on the phase space X, one obtains probabilities such as $p(\Phi(T_t) = e_1)$ and $p(\Phi(T_s) = e_2$ given that $\Phi(T_t) = e_4)$ for all $t, s \in \mathbb{R}$. Now $\{\Phi(T_t)\}$ is exactly the stochastic process with outcomes e_1, e_2, e_3, e_4 and the probability distributions determined by observing the hard ball system with Φ. Hence the possible outcomes of $\{\Phi(T_t)\}$ are the possible observed values of the hard ball system; and the realisations of $\{\Phi(T_t)\}$ and the solutions of the hard ball system coarse-grained by Φ have the same probability distributions. That is, $\{\Phi(T_t)\}$ and the hard ball system observed with Φ are observationally equivalent.

The question which immediately arises is whether the stochastic process $\{\Phi(T_t)\}$ is nontrivial. To highlight the issue: if Φ is the identity function, $\{\Phi(T_t)\} = \{T_t\}$; hence this stochastic process has only trivial probabilities (0 and 1) and is really the original deterministic system. *It turns out that* $\{\Phi(T_t)\}$ *is often nontrivial.* Let me state a result proven in Werndl (2010).

Theorem 1 *If for the deterministic system* (X, T_t, p) *there does not exist an* $n \in \mathbb{R}^+$ *and a* $C \subseteq X$, $0 < p(C) < 1$, *such that* $T_n(C) = C$, *then for any arbitrary nontrivial finite-valued observation function* Φ, $\{Z_t\} = \{\Phi(T_t)\}$ *is nontrivial in the following sense: for every* $k \in \mathbb{R}^+$ *there are* $e_i, e_j \in E$ *such that* $0 < P(Z_{t+k} = e_i$ *given that* $Z_t = e_j) < 1$.

This result is strong because one obtains nontrivial stochastic processes regardless of which finite-valued observation function is applied. *Theorem 1 applies to several systems of importance in science*, e.g., to hard ball systems which are important in statistical mechanics; in particular, to two hard balls moving in a box (Example 1) and to almost all systems of a finite number of hard balls moving on a torus of arbitrary dimension (Simányi 1999, Simányi 2003); to geodesic flows of negative curvature, i.e., frictionless motion of a particle moving with unit speed on a compact manifold with everywhere negative curvature (Ornstein & Weiss 1991); to many types of billiard systems (Chernov & Markarian 2006); and also to dissipative systems such as the Lorenz system which has been used to model weather dynamics and waterwheels (Luzzatto, Melbourne & Paccaut 2005).

The discussion so far was about how, given deterministic systems, one finds observationally equivalent stochastic processes. There are also results about how, given stochastic processes, one finds observationally equivalent deterministic systems. First of all, given *any* arbitrary stochastic process, one can construct a deterministic system, called the *deterministic representation*, such that the following holds: observed with a specific observation function Φ_0 the deterministic representation is observationally equivalent to the stochastic process. Yet the phase space of the deterministic representation is defined as the set consisting of all possible realisations of the stochastic process and thus this construction involves a cheat (see Werndl 2010). Apart from the deterministic representation, there are results which show how, given certain kinds of stochastic processes, one finds observationally equivalent deterministic systems (cf. Ornstein & Weiss 1991). Let me present two results in this direction.

Theorem 1 tells us that deterministic systems in science and stochastic processes can be observationally equivalent. Yet it is silent about the nature of these processes. So one might wonder whether deterministic systems in science can be observationally equivalent to stochastic processes in science (where systems and processes in science are those systems and processes that are derived with help of scientific theories). The following theorem shows that this is indeed the case.

Theorem 2 *If the deterministic system (X, T_t, p) is a continuous Bernoulli system, then there are observation functions Φ such that $\{\Phi(T_t)\}$ is a semi-Markov process (Ornstein 1970).*

Several deterministic systems in science are continuous Bernoulli systems (e.g., all the systems listed after Theorem 1). *Hence several deterministic systems in science yield stochastic processes in science* (namely semi-Markov processes (Example 2)).

One can go even further and ask: can deterministic systems in science only yield stochastic processes in science for some specific observation functions? Or can deterministic systems in science yield stochastic processes in science regardless at which observation level they are observed? It turns out that the latter is true.

Theorem 3 *If the deterministic system (X, T_t, p) is a continuous Bernoulli system, then for every $\alpha > 0$, (X, T_t, p) is α-congruent to a semi-Markov process (Ornstein & Weiss 1991).*

Intuitively speaking, being α-congruent means to be observationally equivalent at observation level α (see Werndl 2010). As just mentioned, several deterministic systems in science are continuous Bernoulli systems. *Thus Theorem 3 shows that several deterministic systems in science are observationally equivalent at every observation level to stochastic processes in science* (namely semi-Markov processes (see Example 2)).

Now that the results on observational equivalence have been presented, let me turn to the mathematicians' comments about them.

4 Mathematicians' Comments on Observational Equivalence

Hardly any mathematicians comment on the philosophical significance or implications of the results on observational equivalence. The main exception is the following:

> Our theorem [Theorem 3] also tells us that certain semi-Markov systems could be thought of as being produced by Newton's laws (billiards seen through a deterministic viewer) or by coin-flipping. This may mean that there is no philosophical distinction between processes governed by roulette wheels and processes governed by Newton's laws. {The popular literature emphasises the distinction between "deterministic chaos" and "real randomness".} In this connection we should note that our model for a stationary process (§ 1.2) [the deterministic representation] means that random processes have a deterministic model. This model, however, is abstract, and there is no reason to believe that it can be endowed with any special additional structure. Our point is that we are comparing, in a strong sense, Newton's laws and coin flipping.[2] (Ornstein and Weiss 1991, 39–40)

Let me first focus on the claim that there may be no "philosophical distinction between processes governed by roulette wheels and processes governed by Newton's laws". The most direct reading of the claim is that there may be no conceptual distinction between deterministic and stochastic descriptions. This seems wrong. *This conceptual distinction will always remain, regardless of any results on observational equivalence.*

In the above quote Ornstein & Weiss (1991) also comment on the meaning Theorem 3. On the most plausible reading, they claim that it expresses that deterministic systems in science, when observed with specific observation functions (called "viewers"), can be observationally equivalent to stochastic processes in science (namely semi-Markov processes). This also illuminates why Ornstein & Weiss mention the deterministic representation, namely, to highlight that this is a case of observational equivalence different from the deterministic representation, which is not a system is science. *However, this claim is puzzling.* As discussed in the previous section, already Theorem 2 shows that deterministic systems in science can be observationally equivalent to semi-Markov processes; and Theorem 2 was known before Theorem 3 was proven and is weaker than Theorem 3. Still, this is the most plausible reading (cf. Werndl 2010). In the previous section I argued that Theorem 3 shows that deterministic systems in science are observationally equivalent at every observation level to stochastic processes in science. So one expected Ornstein & Weiss to claim this. But this seems not to be the case because, first, they do not refer to all possible observation levels, and second, if they claimed this, there would be no reason to mention the deterministic representation.

[2] The text in braces is in a footnote.

Suppes (1993, 254) claims that Ornstein & Weiss (1991) prove the following (referring to Theorem 3): "There are processes which can equally well be analysed as deterministic systems of classical mechanics or as indeterministic semi-Markov processes, no matter how many observations are made." Clearly, Theorem 3 only proves some results about observational equivalence and not that processes can be equally well analysed as deterministic and indeterministic descriptions. It is not clear that the latter follows from the former. Indeed, as I will argue in the next section, the results on observational equivalence do not imply that the phenomena can be equally well analysed as deterministic and indeterministic descriptions.

To my knowledge, Winnie (1998, 310) is the only philosopher who discusses the above quote by Ornstein & Weiss. He takes the claim that there may be no "philosophical distinction between processes governed by roulette wheels and processes governed by Newton's laws" to mean what Suppes (1993) claims, namely the following: the phenomena can be equally well analysed as deterministic and indeterministic descriptions. Because it is not clear that the absence of the philosophical distinction means the same as that the phenomena are equally analysable, it is unclear whether Ornstein & Weiss (1991) want to say this. But if they do, their claim will not be generally true. As just mentioned, I will argue in the next section that the results on observational equivalence do not imply that the phenomena can be equally well analysed as deterministic and indeterministic descriptions.

5 Winnie on the Role of Different Observations

Choice and Underdetermination

We have seen that, in certain cases, deterministic and stochastic descriptions are observationally equivalent. Hence there is a choice between a deterministic description and a stochastic description obtained by applying an observation function Φ to the deterministic description, and the question arises of which description is preferable. I will assume that the deterministic and the stochastic description are about the same level of reality, e.g., they both describe the motion of two hard balls in a box.[3] And I will focus on the question of which description is preferable *relative to all evidence*.

It is important to note that there is *no* underdetermination between a deterministic system and a stochastic process obtained by applying Φ relative to all *in principle possible observations which show whether there are more states than the ones given by the observation function* Φ. Clearly, if in principle possible observations show whether there are more states than the ones given by the observation function Φ, then only the deterministic system or the stochastic process is in agreement with the possible observations.

[3]If the descriptions are about different levels of reality, the situation is quite different. For instance, in certain cases one might argue that at one level of reality the deterministic description is preferable and at another level of reality the stochastic description is preferable.

However, other kinds of underdetermination are possible, namely all kinds of underdetermination which are relative to observations which do not show whether there are more states than the ones given by the observation function Φ. In particular, suppose that an observation function Φ is given and that current technology does not allow one to find out whether there are more states then the ones given by Φ (from the deterministic perspective this means that Φ is, or is finer than, the finest possible observation function). This implies that the predictions of the deterministic system and of the stochastic process which results from applying Φ to the deterministic system agree at all currently possible observation levels. If the possible evidence does not favour a description, this represents a case of underdetermination *relative to all currently possible observations* (cf. Laudan & Leplin 1991). I take it that Suppes' (1993) claim that the phenomenon in question is equally well analysable by deterministic and stochastic descriptions implies that there is underdetermination.

In what follows I will concentrate on an argument against underdetermination put forward by Winnie (1998). I will criticize this argument and, in particular, I will clarify a confusion in it.

Trivial Transition Probabilities to Coarser Observations

Winnie (1998) starts with the following thought. For a deterministic system (X, T_t, p) consider an observation function Ψ and an observation function Φ which is *coarser* than Ψ, i.e., there is at least one value of Φ such that two or more values of Ψ corresponds to one value of Φ. Even if $\{\Phi(T_t)\}$ and $\{\Psi(T_t)\}$ are nontrivial stochastic processes, the following can hold for a time period t: for every value o_Ψ of Ψ and every value o_Φ of Φ the probability that o_Ψ will lead to o_Φ after t time steps is 0 or 1. *Thus there are trivial transition probabilities from the observation* Ψ *to the coarser observation* Φ, where the *transition probabilities* are the probabilities that any arbitrary value follows another arbitrary value.[4] Winnie (1998, 314–315) comments on this:

> Thus, the fact that a chaotic deterministic system [...] has *some* partitioning that yields a set of random or stochastic observations in no way undermines the distinction between deterministic and stochastic behaviour for such systems. [...]

[4]Here is a mathematical example. On the unit square $X = [0,1] \times [0,1]$ consider

$$T((x,y)) = (2x, \frac{y}{2}) \text{ if } 0 \leq x < \frac{1}{2};\ (2x-1, \frac{y+1}{2}) \text{ if } \frac{1}{2} \leq x \leq 1. \quad (1)$$

For the Lebesgue probability measure p one obtains the discrete-time deterministic system (X, T, p), called the *baker's transformation.* Consider $\Phi((x,y)) = o_1\chi_{\alpha_1}((x,y)) + o_2\chi_{\alpha_2}((x,y))$ where $\alpha_1 = [0,1] \times [0,1/2], \alpha_2 = [0,1] \times (1/2,1]$ and $\Psi((x,y)) = \sum_{i=1}^{4} q_i\chi_{\beta_i}((x,y))$ where $\beta_1 = [0,1/2] \times [0,1/2], \beta_2 = (1/2,1] \times [0,1/2], \beta_3 = [0,1/2] \times (1/2,1], \beta_4 = (1/2,1] \times (1/2,1]$ ($\chi_A(x)$ is the characteristic function of A, i.e., $\chi_A(x) = 1$ for $x \in A$ and 0 otherwise). It is clear that if one observes q_1 (with Ψ), the probability that one next observes o_1 (with Φ) is 1; if one observes q_2, the probability that one next observes o_2 is 1; if one observes q_3, the probability that one next observes o_1 is 1; and if one observes q_4, the probability that one next observes o_2 is 1.

> As successive partitionings are exemplified [...] the determinism underlying the preceding, coarser observations emerges. To be sure, at any state of the above process, the system may be modeled stochastically, but the successive stages of that modeling process provide ample—inductive—reason for believing that the deterministic model is correct [original emphasis].

In order to understand this quote, note the following. From the fact that there are trivial transition probabilities from an observation (Ψ) to a coarser observation (Φ) after t time steps, it does not follow that the observed phenomenon is deterministic and Winnie also does not claim this. It may well be that $\{\Psi(T_t)\}$, or any stochastic process at a smaller scale, really governs the phenomenon under consideration.

The argument Winnie seems to make is the following. Consider the observation functions which one can apply, some of them finer than others. Suppose that for some observation functions there are trivial transition probabilities from finer to coarser observation functions after t time steps for some fixed time t. Now consider all possible observation functions such that there are trivial transition probabilities from finer to coarser observation functions, and suppose that one finds that for finer observation functions one observes stochastic processes at a smaller scale (i.e., processes where there is at least one outcome of the stochastic process at a larger scale such that two or more outcomes of the stochastic process at a smaller scale correspond to one outcome of the process at a larger scale). Then the deterministic description is preferable relative to evidence.

My first criticism is that it is unclear *why it is required in this argument that there are trivial transition probabilities from finer to coarser observations*. The force of the argument seems only that for finer observation functions what one observes are stochastic processes at a smaller scale and that the stochastic processes at a smaller scale explain how the probabilities of the stochastic process at the larger scale arise. Simple examples show that there are a set of observation functions such that finer observation functions yield stochastic processes at a smaller scale, but the transition probabilities from finer to coarser observation functions are not always (or even never) trivial.[5] The force of the argument also seems to apply to these examples. From the text it is not entirely clear whether Winnie thought that trivial transition probabilities to coarser observations are decisive for the argument that the deterministic description is preferable. If yes, as just argued, this is puzzling because the force of the argument does not seem to hinge on this. If not, it is confusing that trivial transition probabilities to coarser observations are highlighted in his argument for the deterministic description.

I will now criticize Winnie's argument and my criticism will apply regardless of whether or not one requires that the observation functions are such that trivial transition probabilities

[5]Here is a mathematical example. Consider the baker's transformation (X, T, p). Let $\Psi((x,y)) = \sum_{i=1}^{4} q_i \chi_{\beta_i}((x,y))$ be as in the previous footnote, and consider $\Phi((x,y)) = o_1\chi_{\gamma_1}((x,y)) + o_2\chi_{\gamma_2}((x,y))$ where $\gamma_1 = [0,1/2] \times [0,1], \gamma_2 = (1/2,1] \times [0,1]$. Clearly, for all i, $1 \leq i \leq 4$, and all j, $1 \leq i \leq 2$, the probability that q_i is followed by o_j is $1/2$. Still Φ is coarser that Ψ, and for the observation Ψ at the finer level one obtains a stochastic process at a smaller scale.

are observed from finer to coarser observations.

Criticism of Winnie's Argument About Finer Stochastic Processes

Winnie does not state which kind of underdetermination he is concerned with. Suppose that it is underdetermination relative to in principle possible observations which show whether there are more states than the ones given by the observation function Φ. As argued at the beginning of this section, here it is trivial that there is no underdetermination. Assume that the phenomenon of concern is deterministic. Then Winnie's argument indeed establishes that the deterministic description is preferable because the following holds only for the deterministic description: when considering all possible observation functions (and, maybe, select only those possible observation functions such that there are trivial transition probabilities from finer to coarser observations), then finer observation functions lead to stochastic processes at a smaller scale. This is so because there will be no finest observation function and hence only the deterministic descriptions will be in agreement with the observations. Still, *Winnie's argument seems unnecessary in the sense that the reason why there is no underdetermination is simply that only the deterministic description agrees with the in principle possible observations.*

Suppose Winnie is concerned with underdetermination relative to all possible observations given current technology. I will argue that in this case Winnie's argument fails. To criticise Winnie's argument, it will suffice to present a scenario that could happen in science (regardless of whether this is actually the case) where the premises of the argument are true but the conclusion is not. Let me outline such a scenario (for more details, see Werndl 2009*b*).

This scenario appeals to *indirect evidence*, which is generally regarded as an important kind of evidence (Laudan 1995, Laudan & Leplin 1991, Okasha 1998, Okasha 2002). Let me given an example of indirect evidence (cf. Laudan 1995). Galileo's law of free fall is only about the motion of bodies near the Earth's surface, while Kepler's law is only about the motion of planets. Galileo's and Kelper's law are derivable from Newtonian theory (under certain assumptions). This means that predictions derived from Galileo's law which are in agreement with the observations can provide evidence for Kelper's law, even though this evidence is not derivable from Kepler's law (and hence constitutes indirect evidence for Kelper's law). As Laudan & Leplin (1991) point out, indirect evidence can be an argument against underdetermination. Suppose the same predictions are derivable from a hypothesis H than from Kepler's law but that H is not derivable from Newtonian theory. Because of the indirect evidence for Kepler's law, Kelper's law is preferable to H relative to evidence, and there is no underdetermination.

An analogous argument for our concern—descriptions and not hypotheses—can easily be found. Suppose that the stochastic description S which arises from applying Φ to the deterministic system derives from a well-confirmed theory W and the deterministic description D does not derive from any theory. Furthermore, suppose that current technology does not allow one to find out whether there are more states than the ones given by Φ. Then for the observation functions which one can apply (and where, maybe, only those possible observation functions are considered where there are trivial transition probabilities from finer to coarser observations) the following holds: for finer observations one observes stochastic processes at a smaller scale; hence the premises of Winnie's argument are true. Even though S and D are observationally equivalent, there are many descriptions which are not derivable from S or D but which support W. Now suppose that some of these descriptions provide indirect evidence for S. *Then the stochastic description S is preferable relative to evidence, and the conclusion of Winnie's argument is not true.* Furthermore, there is no underdetermination. Consequently, Suppes's (1993) claim that the phenomena are equally well analysable as deterministic and stochastic descriptions fails.

Regardless of what exactly indirect evidence amounts to, the above argument shows that Winnie's argument fails. Note that being derivable from the same hypothesis or statement cannot be sufficient for indirect evidence because this would lead to the paradox that any statement confirms any statement (Okasha 1998).[6] A promising account is that there is indirect confirmation when two statements are strongly coherent because of a unifying theory; in our examples Galileo's law and Kepler's law strongly cohere because of the unifying power of Newtonian theory, and S and the other descriptions strongly cohere because of the unifying power of W.

6 Conclusion

This paper started by presenting, in an intuitive manner, some recent results on the observational equivalence of deterministic and indeterministic descriptions. After that philosophical questions prompted by these results were examined.

First, I discussed the philosophical comments made by mathematicians about observational equivalence, namely Ornstein & Weiss (1991), and I argued that they are misguided. For instance, on a direct reading, Ornstein & Weiss claim that the results on observational equivalence may show that there is no conceptual distinction between deterministic and indeterministic descriptions. However, regardless of any results on observational equivalence, this distinction will always remain.

Second, if there is a choice between a deterministic and an indeterministic description, the question arises of which description is preferable. I investigated Winnie's (1998) argument that the deterministic description is preferable which goes as follows. Consider the possible observation functions which one can apply and which are such that there are trivial transition probabilities from finer to coarser observations after t time steps. Suppose that stochastic processes at a smaller scale are observed when finer observation functions are applied. Then the deterministic description is preferable relative to all evidence. I clarified

[6]Statement A confirms itself; A is derivable from $A \wedge B$ where B is any statement; B is derivable from $A \wedge B$. Hence, A confirms B.

a confusion in this argument: it unclear why trivial transition probabilities are required from finer to coarser observations because the force of the argument does not seem to hinge on this. Then I argued that, regardless of this confusion, if the concern is a strong form of underdetermination, the argument delivers the desired conclusion but this conclusion is trivial. And if the concern is underdetermination relative to the possible observations given current technology, the argument fails. The question of whether the deterministic or the stochastic process is preferable is an interesting one and, as my discussion has hopefully shown, it deserves further investigation.

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