SPONTANEOUS SYMMETRY BREAKING AND CHANCE

Abstract

In this paper I explore the nature of spontaneous symmetry breaking in connection with a cluster of interrelated concepts such as Curie's symmetry principle, chance, and stability.

1. A model for spontaneous symmetry breaking

Even though spontaneous symmetry breaking (SSB) as a technical concept has its origin in condensed matter physics and high-energy physics (cf. Coleman 1975), its domain of application is much broader, which includes some simple classical systems. In this section I give a general characterization of the notion of SSB (from a simple model). It will then become clear that the phenomena of SSB exhibit some unusual features a detailed examination of which will shed new light on our understanding of such notions as Curie's principle of symmetry, random perturbations, determinism, chance, and stability. A discussion of the latter will appear in later sections.

In another paper I have described in detail how SSB occurs in a simple mechanical model, which shares all the structural features of SSB that the ones in condensed matter physics and high-energy physics have (cf. Greenberger 1978 & Sivardiere 1983). Here I describe those features without going into the mathematical arguments.

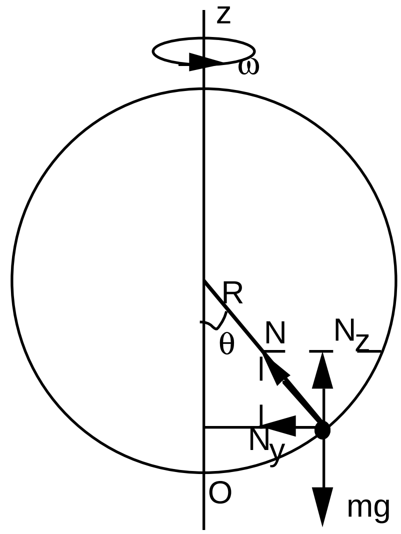

Figure 1. A bead with mass, $m \neq 0$, is free to move frictionlessly on a circular wire of radius, R, which rotates around the z-axis with variable ω.

The model comprises a metal ring vertically suspended and free to rotate (without friction) and a bead frictionlessly threaded on it (see Figure 1). Imagine that we set the ring into rotation and very slowly[1] increase its angular velocity ω. A complete story of the system's mechanical behavior can be told in classical mechanics. At first when the angular velocity is low, the ground state of the system -- the state at which the bead has the lowest energy -- is the one with the bead resting at the lowest point of the ring (i.e. at O or θ = 0). In other words, the mathematical argument concludes that the potential energy of the bead has its minimum when it is at rest at O. This situation remains until the angular velocity passes a certain value -- henceforth the 'critical value' -- when O stops being the ground state of the system. Instead, the ground state begins to 'split' and 'climb' the ring as the angular velocity continues to increase. From Figure 1 it means that the bead will be in its ground state at successive values of θ or $-\theta$, where $|\theta| \neq 0$, with corresponding angular velocities. The position of the bead's ground state will become higher and higher on the ring (i.e. the angle θ will become larger and larger as the angular velocity increases, and it approaches 90° (or -90°) when the angular velocity approaches infinity). Therefore, the bead, originally resting at O, will in seeking the new ground state depart from O and ascend one or the other side of the ring; and because this rotating ring-bead system initially possesses the reflectional symmetry between the two sides of the ring with respect to the z-axis, this symmetry is said to be broken by its ground states after the angular velocity passes its critical value (cf. Wigner 1979, pp. 3-50). The symmetry appears to be spontaneously broken because there are no apparent causes (asymmetries) in the model that are responsible for the breaking.

2. The nature of SSB

Are the symmetries truly broken *spontaneously* in SSB -- i.e. without any causes in the form of asymmetrical antecedents? No, for if one thinks that the bead will inevitably climb the ring from O in our simple model -- exactly as it is described -- then one is mistaken. If there were a physical system in reality that is exactly like our model, we would have no reason to believe that the bead would budge at all however fast the ring rotates. This is because even though the point O is no longer the ground state of the system -- the lowest energy state of the bead -- beyond the critical point, it is still an equilibrium point in the following sense: there is nothing in the mathematical argument which indicates that O is not a state in which the bead will remain forever unless bumped or disturbed, however slightly, toward another state. Therefore, it seems that the actual breaking of a symmetry is only attributable to some perturbations of the system (which may or may not be caused by small external disturbances). In our model the bead won't actually ascend the ring unless some perturbation (or fluctuation) at O causes it to do so (cf. Poincaré 1952, pp. 64-90 & Ismael 1997, pp. 179-180).

However, there is a sense of SSB in which perturbations (or fluctuations) are not relevant. In our model in which no asymmetrical elements are considered, the symmetrical ground state at O is 'broken up' -- i.e. rendered unstable -- when the angular velocity passes the critical value. The bifurcation of the ground state which provides the possibility for a breaking of the symmetry is present without any asymmetrical antecedents. This is connected to the following general feature of SSB. What the initial unique ground state breaks into after the parameter crosses its critical value is always a set of ground states -- in our model they are those at θ and $-\theta$ -- which are *degenerate*, meaning that they have the same value so that any transformation -- of the same symmetry they are supposed to have broken -- from one state to another within the set does not change the value. Hence one may say that the ground states together as a set still preserve the symmetry which each breaks.

3. Curie's principle and SSB

Although the actual breakings of a symmetry in SSB are not without causes (or asymmetrical antecedents) if the analysis given in the previous section is correct, they are none the less fundamentally different from most cases of symmetry breaking that are the results of either asymmetrical external influences, such as forces and impacts, or explicitly asymmetrical initial and/or boundary conditions. In both situations, the asymmetrical antecedents could, and should, be made explicitly in the model so that the broken symmetries in question are properly accounted for. This is not the case with SSB. Even though we assume that they are the results of perturbations, it is not possible either to include any precise information about individual perturbations in the model -- i.e. in its mathematical description -- so as to make the symmetry breaking one of the common type, or to determine by other means which perturbation causes the system in question to fall into which symmetry-breaking ground state.

This feature of SSB, a feature that is regarded as separating it from the common types of symmetry breaking (and in some sense justifies calling the symmetry-breaking 'spontaneous'), is of course the same feature I explained above, namely, the 'breaking' in a model of SSB always results in a set of degenerate ground states which together preserve the very symmetry(ies) each breaks.

Several issues now arise regarding the nature of SSB. One may first wonder whether Curie's principle of symmetry -- which in slogan form reads: 'asymmetry out only if asymmetry in' -- holds in SSB? I have argued elsewhere that the principle holds if we regard SSB as a deterministic phenomenon. To give a rough summary we may say that the asymmetries responsible for the actual breaking of the symmetry(ies) in question are provided by the perturbations. And if we disregard the perturbations, there won't be any *actual* breaking of the symmetry despite the shift of the solution from one symmetrical ground state to a set of asymmetrical ones.

But then one may ask whether one should treat SSB as a deterministic phenomenon. It is common to hear an expert say that in SSB a symmetry is broken by pure chance; or more precisely, whenever there is SSB in a system, which symmetry-breaking ground state the system will eventually end up is a matter of chance. I shall argue that Curie's principle no longer holds in the indeterministic contexts, although it does apply at some level there; so if SSB is an indeterministic phenomenon, it violates Curie's principle[2]. The conclusion of the argument is in short that the principle is violated by the truly chancy processes, while it can be seen to be preserved by the probabilistic regularities -- some of which are natural laws -- that prevail in those processes. In other words, no asymmetry seems present to be responsible for the obtaining of particular results of a truly stochastic process, while some asymmetry is expected to be present if any asymmetrical distribution of the results obtains.

The idea that Curie's principle does not hold in indeterministic processes is briefly explained in van Fraassen 1989 (pp. 239-243) and 1991 (pp. 23-24). Here is an argument in the spirit of van Fraassen's. Suppose that we have a system that radiates particles one at a time along a single spatial dimension -- in the positive or the negative direction with respect to the emitter -- and the chance of one particle in either direction is 1/2. And suppose that the state of the system before any emission is reflectionally symmetrical. The resulting system after any emission -- the emitter and the emitted -- would be a state that is not reflectionally symmetrical. Given the emissions are indeterministic, namely, there is no hidden mechanism in the system that determines which particle is emitted in which direction, the Curie's principle is violated by this model: no asymmetry in but some asymmetry out.

However, at the level of chance distribution, the system before and after any emission is always reflectionally symmetrical. Indeed, were the chances in the two directions not equal, we would be entirely justified to assume that some asymmetrical elements exist in the emitter which cause the unequal chances. I know of no instance of a scientific theory in which an asymmetrical probabilistic distribution is not accounted for by any asymmetrical antecedents or, failing that, is regarded as an entirely satisfactory result.

Here a detour is in order to prevent a probable misunderstanding. The legitimate request for asymmetrical antecedents for resulting asymmetrical chance distributions is different from the type of requests commonly known in the quantum mechanics literature as those for hidden variables. The former is at the level of determining chance distributions and the latter individual measurement results. In fact, at least in quantum mechanics, chance distributions evolve deterministically because the evolution of either state functions (in the Schrödinger representation) or observables (in the Heisenberg representation) which determines such distributions at any instant given an initial distribution is entirely deterministic.

The above should be sufficient to resolve an apparent conflict of view between van Fraassen and Ismael, where van Fraassen (ibid.) claims that Curie's principle fails, while Ismael (1997) claims that it holds, in the contexts of indeterminism. They ask different questions and, not surprisingly, get different answers. Van Fraassen's question is whether the principle holds in any indeterministic processes, and the answer, as one can see from the above, is clearly a 'no.' Ismael's question is 'does Curie's principle have any application where the laws in question are *in*deterministic...(p. 176)' and the answer, also in accord with what is just said, is a 'yes.' The application of the principle, as Ismael explains in detail, is indeed on the level of probabilistic distributions. Ismael also realized that the principle does not apply among the purely chancy processes, but she does not regard such as a violation of the principle in the indeterministic contexts[3].

4. Chance and SSB

Given that at least some SSB happen in deterministic systems, we now ask first whether and in what sense we are justified to call the actual breakings a matter of chance, where by chance I mean some property of physical systems whose values obey probability calculus. And secondly, why are the breakings in SSB equally probable?

It is of course possible, or even very likely, that at the most fundamental level all physical processes are indeterministic. However, that does not make SSB automatically an indeterministic phenomenon or the chance in it a chance of indeterminism. For systems such as the one represented by ring-bead model, even if every molecule or field in them is governed by classical deterministic laws, the SSB would still occur (as a theoretical result). Hence, the existence of SSB, very much like objective chance as we shall see shortly, is compatible with determinism.

I also want to mention before our in-depth analysis what might be the first case in which a connection between chance and SSB is made, although it was not recognized as such. To my surprise (and delight) the first example in Poincaré's now famous discussion of chance in (Poincaré 1952b) is a special case of SSB: a stationary cone balanced on its point on a flat surface. It will topple towards an unpredictable direction since rotational symmetry is already 'broken' in that the upright position of the cone in balance is an unstable equilibrium state and the stable one (i.e. the ground state) is one of the infinite number of states, [0, 2π), in which the cone lies on its side. The toppling of the cone towards any particular direction may be caused by a particular 'very slight trepidation, or a breath of air. (p. 67)'[4] Again, why is the toppling of the cone in any direction a matter of (equal) chance, even though it is determined by a specific perturbation?

Those who are familiar with the literature of the foundations of statistical mechanics (SM) (cf. Sklar 1993 & Guttmann 1999) may think that the answers to our questions are, details aside, simple and straightforward. To put it roughly, if the use of objective probabilities among observable (macro-)states (i.e. the coarse-grained states) is consistent with the assumed underlying determinism among dynamical (micro-)states (i.e. the fine-grained states) in SM, then one can simply say the same (again details aside) for the chance in SSB. This is however not true, even if we assume that the antecedent of the above conditional is unproblematic. The case of SSB is similar to, but not the same as, those in standard SM. I first point out some important differences (i.e. the one that are relevant to the treatment of chance in the deterministic contexts), and then I try a direct answer to our questions.

To begin with, SSB systems are not the ones in thermo-equilibrium; instead they are in transitions from one equilibrium state to another. Therefore, the appearance of chance in them cannot be justified in the same way as is the appearance of chance in systems of equilibrium. Given that ergodicity is what we now know justifies with rigor the consistency of chance in the observable states with determinism in the dynamical states in equilibrium SM, ergodicity would not be directly applicable to SSB systems.

That leaves us with the possibility of regarding SSB as non-equilibrium phenomena, as processes that eventually approach equilibrium. We may think of the new ground states (the states obtained when the parameter is beyond the critical value) as equilibrium states and the transfer from the unstable state to them as analogous to what one encounters in the transport phenomena in the kinetic theory. Again, things are not so simple as this.

The kinetic theory -- at whose core is the Boltzmann equation of the one-particle density function -- does not apply straightforwardly to SSB systems. First of all, it is one of the main challenges of the theory to show that for a thermo-system the equilibrium state at which the Boltzmann distribution -- the most probable distribution -- holds is indeed the state to which all systems in other (non-equilibrium) states will eventually approach and from which they, once there, will never permanently leave. The other challenge is to show that the approach to equilibrium is irreversible regarding observable states despite the fact that the laws that govern the dynamical states are completely time-reversible. None of these is a problem for the SSB systems. An SSB system when regarded as a thermo-system -- considering the ring-bead system and its immediate environment as a collection of molecules strictly obeying the dynamical laws of classical mechanics -- is *'attracted'* towards one of its new ground states, θ or $-\theta$, not because of its tendency of becoming 'one of the most probable state,' as in the kinetic theory case, but rather because -- in a simpler sense of attraction -- its tendency of moving to a lower energy state. This is true both at the observable and the dynamical level, because even if we look exclusively at the individual trajectories in the phase space of the ring-bead system, all of them should be 'deterministically' attracted towards the subspaces defined by θ or $-\theta$ and the corresponding potential energy.[5] In other words, the transition of the ring-bead system from the former ground state at the bottom of the ring to one of the latter ground state higher up the ring is not really a probabilistic phenomenon; it is a mechanical one. Essentially the same holds for the case of Poincaré's cone. It is not a matter of 'becoming more probable' that the upright cone topples, but rather it is mechanically determined to do so. Because of this feature, there is no good sense in which one may talk about the physical possibility that a large fluctuation, for instance, may return the system, the bead on the ring or the Poincaré cone, to its old ground state, while this would certainly be possible if the system is a case of the kinetic theory. In other words, while it is only an overwhelming probability that prevents the cream from suddenly separating on its own from the coffee once the two are well mixed, there is an energy gap -- a difference at the observable level -- that prevents the system in question to go back to the previous (symmetrical) ground state in SSB.

Therefore, we cannot directly borrow from SM to answer our questions, and yet a combination of some notions there, perhaps slightly modified, may just get us what we wanted. The rest of the section provides a conception of such an attempt. I shall first give the answer in intuitive but imprecise terms, and then try to precisify it by connecting it, wherever possible, to what is well established in the foundations of SM.

The actual breakings of a symmetry in a case of SSB are a matter of chance partly because they are caused by perturbations[6] that are randomly distributed in the neighborhood of the unstable equilibrium point, and partly because the stable equilibrium states are symmetrically distributed around (but not close to) it. In our ring-bead model, we may conceive of the system as composed of a large number of molecules and surrounded by some kind of gas. When the angular velocity goes beyond the critical value, the state at O (see Figure 1) becomes unstable, which when translated into the language of ensembles means the following. Of all the possible systems in the ensemble in the phase-space neighborhood of that state, only those with configurations that make them occupy exactly the position O will not depart permanently from O and make the transition to one of the new ground states, θ or $-\theta$. Any system whose configuration makes it deviate from O, either because of any slight disturbance from its immediate environment or from its own thermo-agitation, will, because of the energy gap, make the transition. Such perturbations appear to be so small and so numerous, we may as well regard them as randomly distributed; and the object, e.g. the bead, is as likely to depart from O toward one direction as toward another. And since the possible states that such departures will end up are all degenerate, meaning that they all have the same value for the macroscopic quantity, e.g. the energy gap, that does the 'attraction,' there is no reason why the system should not end up in any of them with equal probability.

Let us now make this answer more accurate. At the most general level, the answer has the following structure.

1. An SSB system acquires a chancy character when its symmetrical ground state becomes unstable. The values of the chance -- the probabilities -- are determined by how the perturbations, if present, are distributed before the state becomes unstable, i.e. when is it a stable equilibrium state.
2. The actual symmetry breakings -- the transitions to the lower-energy ground states -- are entirely determined by such individual perturbations by individual dynamical processes.
3. The symmetrical arrangement of the asymmetrical ground states and the degeneracy of the values of the SSB-causing property, such as the energy gap, ensure that the actual breakings have the same probability distribution as the distribution of the perturbations.

To fill out the structure, we begin with the question of how the perturbations are distributed. We can simply say as I did earlier that the perturbations are *randomly* distributed, and then go on to give a precise sense of randomness. If the latter turns out to be good enough to justify the equal-chance character of the actual symmetry breakings, then we can rest satisfied. It is beyond the scope of this discussion to even touch on the various approaches to, and controversies over, a precise definition of randomness (cf. Earman 1986, chs. 8-9; Emch & Liu, 2001, 188-215); it suffices to predict that no satisfactory answer is forthcoming if one seeks a direct connection between randomness and the equal chances of perturbations in all directions from O in the ring-bead model. Further, even if such a connection can be figured out accurately, we are still left with the more difficult question of how we know that the perturbations are indeed random.

Fortunately, there is another way of approaching the problem. We know that before the possibility of symmetry breaking opens, an SSB system is in a stable equilibrium state. If the system can be suitably modeled as a thermo-system -- systems with a large number of degrees of freedom -- there is a possibility that we can employ the notion of ergodicity, or comparable notions such as quasi-ergodicity, to justify the uniform distribution of the perturbations we intuitively suspect.

First, can we model SSB systems as thermo-systems? Some of them obvious can readily be so modeled, e.g. the systems in which such SSB as ferromagnetism, superconductivity, and superfluidity take place. The SSB systems in quantum field theories -- e.g. the gauge fields and gauge symmetry-breakings -- are certainly not thermo-systems in any straightforward sense. However, there is so much similarity between the SSB in quantum SM, such as in ferromagnetism, and the SSB in gauge field theory (cf. Aitchison 1984, ch.6), and from a foundational perspective quantum fields are many-particle systems of infinite degrees of freedom (cf. Strocchi 1985), I may venture a conjecture here that whatever accounts for the chance of SSB in the former case can, without any conceptual modifications, be applied to the latter case. It is beyond the scope of this paper to work this out explicitly, so I shall put aside the questions of whether or not the SSB in quantum fields are chancy and if they are, what their nature is.

More importantly, the question is whether or not such SSB systems as our ring-bead system or Poincaré's cone can be modeled as thermo-systems. *Prima facie*, they are modeled as rigid-body systems, which are not amenable to thermo-statistical laws. But that is only true when no 'very slight trepidation, or a breath of air' is considered. When we consider what I have explained earlier to be the causes of the actual symmetry breakings in these systems, we should see that they are indeed distributed as in a thermo-system. To put it another way, either the perturbations (together with the small disturbances that cause them) are excluded from these models so that they are not thermo-systems, but then there are no actual symmetry breakings, or the perturbations (together with the small disturbances that cause them) are included, and then they can and should be modeled as thermo-systems. Hence, even in the case of the ring-bead or the

Poincaré model we do have thermo-systems in equilibrium before the symmetrical breaking becomes possible.

But, second, how do we know that such systems are at least ergodic? For otherwise, to a system in thermo-equilibrium but not being ergodic we still do not have the justification for using either microcanonical (if the system is isolated) or canonical (if it is in equilibrium with a heat reservoir) ensemble to model it. In other words, we still do not know why the distribution of the actual symmetry breakings is given by the theory of SM (in this case, of classical SM).

For those who know the story of the struggle with ergodicity, it should be obvious that it is practically impossible to give a rigorous proof here that the systems, with respect to SSB, are or are not ergodic. Theorists only know how to construct such a proof for physical systems that do not even look like thermo-systems, and yet many simple thermo-systems seem obviously ergodic from an intuitive point of view (cf. Emch & Liu 2001, chs. 8-9, especially pp. 317ff; Toda et al 1995, ch. 5). At the intuitive level what would prevent a system from being ergodic is for a possible trajectory of it to wonder into a region of its phase space and stay there (practically) forever. The reason for this is quite simple. For ergodicity to hold for a system, the time average of any function of its variables (as $t \to \infty$) must equal its ensemble average, so that it justifies the idea that the probability that the system will be found in a certain region of its phase space is proportional to the phase area of that region (in, say, Lebesgue measure). But for this to be true, it is obviously necessary that the system's trajectory should not stay in a certain region for a disproportionally long time. Such islands in which trajectories may wonder in and never leave are called KAM tori in the literature, and to be an ergodic system is not to have KAM tori in the system's phase space. It is usually very difficult to prove the absence of KAM tori even if it is rather obvious from an intuitive inspection.

Our models when in the symmetrical state of stable equilibrium seem to be ergodic from an intuitive level in the same way that many simple thermo-systems are. There do not seem to be islands or subspaces in the neighborhood of the state (except the state itself) into which any phase trajectory of the system -- either the ring-bead or the cone -- would wonder and stay forever. Because of the energy constraint, any trajectory starting away from (or leading out of) the lowest energy state would eventually, if not quickly, come (return) to that state. Given that the systems can be modeled as thermo-systems and they are ergodic, we are able to conclude that the probability of having a particular perturbation occur in a certain region of the phase-space neighborhood of the unstable equilibrium state should be proportional to the area of that region. This implies that perturbations are uniformly distributed around that state, namely, a small deviation from it in one direction is equally likely as one in another direction. (To put it more accurately, think of a phase-space ring around the unstable equilibrium state. Given that the system is ergodic, the probability that a perturbation occurs in one finite section of the ring must be the same as the probability of its occurring in any other section of the same size. The result is the same even if we let the size of the section approach zero. Hence a perturbation from the equilibrium state is equally likely in any directions.)

The second element (#2 above) in our answer is the transitions from the unstable equilibrium state to the stable ones, which are caused by the perturbations. When the systems concerned reach a state in which their SSB-parameter is beyond the critical value, the state becomes unstable -- which means that the systems will not return to it when they acquire however small a perturbation from it -- and yet the distribution of the perturbations for these systems should not be affected. This means that when symmetry breaking becomes possible for a system, the system will be disposed to make a transition from one state to another with the presence of any perturbation. But since the distribution of the perturbations is the same, the probability of the system departing in one direction should be the same as its departing in any other direction.

However, how the symmetry in question are actually broken also depends on what the ground states (the new stable equilibrium states) are and how they are situated (#3 above).

If they do not form a set of degenerated states which together has the same symmetry -- i.e. one being transformable into any other by the same symmetry group, then the probability of actual breakings may not be the same as that in which the system is *disposed* to break its symmetry. For example, if in our ring-bead system the perturbations from O are equally distributed between the left and the right direction and yet only one of the ground states, say, θ (but not $-\theta$), is allowed (never mind how this can actually be the case, but it is certainly physically possible), then the probability of the bead to deviate from O would be 0.5 in either direction while the probability of it settling into the ground state is unity; and therefore, the two probabilities do not match. (Here I assume that every perturbation deterministically leads to a transition of the system to its (or one of its) ground state(s).)[7]

As a final point, I would like to entertain the following question about chance and SSB. Does SSB have anything to do with the very possibility of chance in a deterministic world? Here the question is not just whether the existence of chance is consistent with such a world but what that world has to be in order for chancy processes to arise. There are more than one ways to answer this question. Ergodicity theory appears to be the dominating approach, while other approaches include, Jaynes's (Jaynes 1983), Khinchin's (Khinchin 1949), and most recently, Albert's

(Albert 2000). However, the minimal requirement for the existence of chances in thermo-systems would be the possibility of coarse-graining and the existence of instability (cf. also, Clark 1987). I want to emphasize here how in our universe instability depends on coarse-graining and how the latter is relative and at best inter-subjective in the sense that the degrees to which a phase space is coarse-grained is in principle arbitrary and in practice determined partly by the limits of the size and capacity of the investigative agents and partly by the agents' investigative interests. To see this point, let us remind ourselves of what it means to have instability in nature[8]. If a partition of a phase space is given by the coarse-graining, then it is up to nature whether or not any system is stable. If there are trajectories which begin at time t in the same cell of the coarse-grained phase space and end up at t'(>t) in different cells, then the systems having these trajectories are considered unstable at t; and otherwise, no systems are. On the other hand, given the complexity of our universe, for any set of trajectories, it is possible to coarse-grain the phase space in such a way that they belong to the same cell at t but different cells at t'(>t). The *complexity* of the 'universe' -- in the precise sense of *not* having all the trajectories in a phase space keeping their distance (in some given measure) invariant through time -- is important, because it is certainly possible that a universe is so 'simple' that no coarse-graining -- indeed no 'any'-

graining -- of the phase space is able to produce instability. (In other words, there are mechanical models that are absolutely stable.) And in such deterministic models, chance really cannot have a place, for however one partitions the phase space in question, one gets the result that any set of trajectories belonging to a single cell at one time belongs to a single cell at any other time. That means that the probability of any of those trajectories ending up in one cell at time t' given it comes from another cell at time t ($<t'$) is either 1 or 0.

Therefore, complexity is necessary for ensuring the possibility of instability in a model. There are certainly many different ways for a system (or a universe) to become complex (or more complex); but they all have the general feature of bringing about stable (or more stable) observable states that 'cover'[9] more and more dynamical states. One of the simplest examples of such a process would be the free expansion of a gas from one volume to another bigger volume. Imagine a gas first confined in one of the two halves of a container, which is separated by a dividing door in the middle. When the door is suddenly withdrawn, the possible positions for the gas molecules are by this process doubled and the number of possible momenta increased. When the gas finally permeates the whole container and reaches the equilibrium, the system has become more complex in exactly the sense I gave above.

Now, SSB are not unlike the free expansion of gas. The crossing of the critical value of the parameter is analogous to the withdrawing of the dividing door, for when either happens, the space of possible observably distinct states is widened (in the gas case, having the gas in the other half of the container and in the ring-bead model, having the bead at rest at positions other than O); and with the parts of the system in question occupying the possible states, the system becomes more complex. Of course, the free-expansion case of gas is not a case of SSB, so SSB are only one of the ways by which the above mentioned take place. One may say that an SSB is the process by which new observable states result, which have distinct symmetries, each being different from the one of the initial observable state. Although the examples I considered in this paper happen to involve spatial symmetries, there are other kinds of symmetries, such as the gauge symmetries, that are not spatio-temporal.

5. Conclusion

In this essay I first described a simple model in which the structural features of SSB can be plainly seen. Then I showed that there are really two different meanings for SSB, one, as given in our model, specifies the conditions under which the *possibility* of SSB is present; and the other, as given by the model plus perturbations, which are the real causes of actual breakings, describes (in not in detail) conditions for the *actual breakings* of the symmetry. And then I argued that Curie's principle of symmetry is indeed violated by actual processes of SSB, even though the results of such processes, at the level of their distribution, still make the principle applicable. Furthermore, the justification of the use of (equal) chance in SSB turns out to have a three-element structure: (1) the uniform distribution of the perturbations that holds even when the ground state becomes unstable; (2) the deterministic transition from the unstable state to the stable states; and (3) the symmetrical (of the same symmetry) arrangement of the symmetry-breaking states. Each of these can be justified if the systems can be models as ergodic thermo-systems. SSB systems indeed can be so modeled, or so I have argued. Lastly, I tried to explain how SSB is a type of instability which is responsible for producing diverse chancy processes in our universe, which contributes to its complexity.

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[1] As a technical term, 'very slowly' means that the ring-bead system is at equilibrium at every value of the angular velocity.

[2] Even though most of the known types of SSB are regarded as happening in deterministic systems, there may be indeterministic cases of SSB. In fact, one of the most interesting and difficult questions about SSB is whether the processes of quantum measurement -- the heart of indeterminism -- are of SSB. However, one must note that the SSB cases in condensed matter physics and in high-energy physics are not examples of indeterministic SSB. The symmetries are spontaneously broken in those cases at the level of probabilistic distributions rather than at the level of purely chancy events.

[3] Ismael's interpretation of Curie's principle is in fact such that the purely chancy processes in the indeterministic contexts are not in the proper domain of the principle's application.

[4] One should note that the difference between our ring-bead system and Poincaré's cone with respect to SSB is almost trivial. The cone case can be easily modified so that it also has a one-parameter controlled process of SSB. For instance, one can think of a rotationally symmetrical system whose lower part continuously change its shape (which is the parameter) from spherical to an inverted cone shape.

[5] One must note that in this paper, and especially in the rest of it following this note, there are two kinds of spaces: the 3-dimensional space and the 6N-dimensional phase space (N being the number of particles). In principle one should always make it clear which space one is talking about when one describes a situation and avoid mixing them in such descriptions. But there are occasions where a mixed used is convenient and free of the risk of confusion. So, I will occasionally say things such as 'the phase states -- or states -- of the ring-bead system around O,' which simply means 'the phase states ... around the phase point that corresponds to the bead's being at rest in the 3d space at O.'

[6] From here on I will use the word 'perturbation' in a narrower sense, namely, only to mean a small deviation of the configuration of a system from a certain understood state -- the unperturbed state -- which may be caused from an equally small external cause or is due to a small internal fluctuation. This is the sense commonly used in physics as a technical term, such as in 'the perturbation of a celestial orbit' or 'the perturbational expansion' of some equation.

[7] If one finds this case difficult to imagine (because switching directions inevitably requires the passing of the point O), then think of the case of Poincaré's cone, where all directions of perturbations from the upright position are possible and yet not all directions along which the cone lies on its side are allowed; and the set of these disallowed directions is not of measure zero. The same argument, *mutatis mutandis*, goes through.

[8] It does not seem meaninful to assume any mathematical rigor in the following discussion, given the level of rigor adopted in general for the paper. For a rigorous treatment of the related notions -- stability, orbit distances, and sensitive dependences -- for non-specialists, see Earman (1986, ch. 9) and Smith (1998, 15, 102-105, 167ff). The following can be taken as an application of those notions in the contexts of coarse-grained phase spaces.

[9] Whenever I say that an observable or macroscopic state covers or contains a set of dynamical or microscopic states, I mean no more than that no permutations among the members of the latter can change the value of the former.