Naive Quantum Gravity

Steven Weinstein

Abstract

In this paper we consider a naive conception of what a quantum theory of gravity might entail: a quantum-mechanically fluctuating gravitational field at each spacetime point. We argue that this idea is problematic both conceptually and technically.

1 Introduction

The world of classical, relativistic physics is a world in which the interactions between material bodies are mediated by fields. The "black body catastrophe" provided the first indication that these fields (in particular the electromagnetic field) should be "quantized."[1] Modern field theory contains quantum field-theoretic descriptions of three of the four known interactions (forces)—all except gravity. It is characteristic of the theories of these three forces that the values of the fields carrying the forces are subject to the Heisenberg uncertainty relations, such that not all the field strengths at any given point can be specified with arbitrary precision.

Gravity, however, has resisted quantization. There exist several current research programs in this area, including superstring theory and canonical quantum gravity.[2] One often comes across the claim that the gravitational field must be quantized, and that quantization will give rise to a similar local

[1] See the first chapter of Bohm's textbook [1] for a concise history of the origins of quantum theory.

[2] See [12] for a recent review.

uncertainty in the gravitational field. Here we will examine this claim, and see how the very things that make general relativity such an unusual "field" theory not only make the quantization of the theory so technically difficult, but make the very idea of a "fluctuating gravitational field" so problematic.

2 What is a field?

Maxwell's theory of electromagnetism describes the interaction of electrically charged matter (consisting of "charges") and the electromagnetic field.[3] Charges act as "sources" for the field, and the field in turn exerts a force on the charges, causing them to accelerate. The field is specified against a background of space and time, assigning values for the various components E_i, B_j, etc., of the electric and magnetic fields to each point in space at a given time.[4] The acceleration of a charged object at a given point is then given by the Lorentz law

$$\ddot{\vec{x}} = \frac{\vec{F}}{m} = \frac{q_E(\vec{E} + \dot{\vec{x}} \times \vec{B})}{m} \quad (1)$$

where m is the mass of the object, q_E is its electrical charge, and $\dot{\vec{x}}$ the velocity of the object (in appropriate units). In short, the acceleration of a given object in a given field is directly proportional to the charge, and inversely proportional to the mass.

Maxwell theory is the paradigmatic field theory, yet there are three other "interactions" known in nature, associated with different sorts of charge.[5]

[3] Here we understand "field" to mean an assignment of properties (the "values" of the field) to each point in space or spacetime. "Spacetime" will be represented, for our purposes, by a 4-dimensional differentiable manifold equipped with a Lorentz metric satisfying Einstein's equations. "Space" then refers to a spacelike hypersurface in some spacetime.

[4] For simplicity of presentation, we use the canonical picture for electromagnetism and thus make an arbitrary split of spacetime into space and time.

[5] Quantum-mechanically, these fields are usually identified with various symmetry groups: $U(1)$ for the electromagnetic interaction (Maxwell theory), $SU(2)\times U(1)$ for the electroweak interaction (combined theory of the electromagnetic and weak interaction), and $SU(3)$ for the strong interaction.

As noted in the introduction, the theories of the strong and weak nuclear interactions are also field theories, specifying the fields associated with their respective charges, and the resulting forces on the charges.

The remaining interaction is gravity. As in the theories of other interactions, objects carry a "charge", the charge acts as a source for something like a field, and the theory quantifies how the properties of this field affect the behavior of the object (and vice-versa). What is uniquely characteristic of gravity is that the gravitational "charge" q_G of an object is identical to its mass (in Newtonian theory) or mass-energy (in general relativity). This has far-reaching consequences. One, it means that gravity is universal, since *all* objects have a mass (respectively, mass-energy). Two, it means that all objects behave the same in a gravitational field (because the ratio of the charge to the mass $q_G/m = m/m = 1$). This equivalence of gravitational charge and inertial mass is what we shall refer to as the "principle of equivalence" or "equivalence principle."[6]

If gravity were universal, yet objects reacted differently to gravitational effects, then there would be no particular reason to associate the gravitational field with spacetime geometry. It is the fact that objects behave the same in a gravitational field that leads to describing gravity as a property of spacetime itself.[7] The reason for this is that "behave the same" means "follow the same spacetime trajectory." Einstein noticed that if these trajectories were construed as characteristic features of a curved spacetime geometry, then gravity could be represented geometrically. They can—the

[6] There are many different versions of the equivalence principle in the literature—the version here is what Ciufolini and Wheeler [2] call the "weak equivalence principle." The review article by Norton [10] contains an excellent taxonomy of the various senses of "equivalence principle."

[7] We are actually considering an idealized limit in which we ignore the contribution of the object itself to the gravitational field. It is only in this limit that it makes sense to talk about two different massive objects moving in the same field, for the objects themselves change the field in proportion to their energy and momentum.

special trajectories are “geodesics”.[8]

An alternative way to conceive of gravity would of course be to follow the lead of other theories, and regard the gravitational field as simply a distribution of properties (the field strengths) in *flat* spacetime.[9] What ultimately makes this unattractive is that the distinctive properties of this spacetime would be completely unobservable, because all matter and fields gravitate. In particular, light rays would not lie on the “light cone” in a flat spacetime, once one incorporated the influence of gravity. It was ultimately the unobservability of the inertial structure of Minkowski spacetime that led Einstein to eliminate it from his theory of gravitation and embrace the geometric approach.

Nonetheless, we shall see that this attribution of gravity to the curvature of spacetime leads to great conceptual and technical difficulties, essentially because it makes it difficult, if not impossible, to treat gravity within the conceptual and mathematical framework of other field theories. Thus it is worth asking whether it is at all *possible* to construe gravitation as a universal interaction that nonetheless propagates in flat, Minkowski spacetime. The idea might be to still construe the field geometrically (retaining part of Einstein’s insight into the significance of the equivalence principle), but to construe the geometrical aspect as “bumps” on a special, flat background.

The short answer is, “No”, for three reasons. First, the “invisibility” of the flat spacetime means that there is no privileged way to decompose a given curved spacetime into a flat background and a curved perturbation about that background. Though this nonuniqueness is not particularly problematic for the classical theory, it is quite problematic for the quantum theory, because different ways of decomposing the geometry (and thus retrieving a flat background geometry) yield different quantum theories.[10] Second, not

[8] The ambiguities that arise in the geometry when the equivalence principle is not respected are discussed in Weinstein [21].

[9] An interesting philosophical analysis of this line of thinking may be found in Reichenbach [11].

[10] However, the decomposition of a curved spacetime into a flat part and a curved

all topologies admit a flat metric, and therefore spacetimes formulated on such topologies do not admit a decomposition into flat metric and curved perturbation.[11] Third, it is not clear *a priori* that, in seeking to make a decomposition into background and perturbations about the background, that the background should be *flat*. For example, why not use a background of constant curvature?

The upshot is that, for general spacetimes, the gravitational field can only be *locally* decomposed into a flat Minkowski background and a curved foreground, and even then there is no unique way to do it. Thus we are stuck with a theory in which the gravitational field seems irrevocably tied to a fully geometric description, which in particular means that the field, such as it is, defines its own background—it is both "stage" and "actor".

3 The uncertainty of quantization

Quantum theory applies to all sorts of systems. In a quantum theory, the determinate properties of classical mechanics are replaced by indeterminate properties, represented by self-adjoint operators on a Hilbert space. For example, objects such as low-energy particles have indeterminate position $\vec{x} = (x_i, x_j, x_k)$ and momentum $\vec{p} = (p^i, p^j, p^k)$. These quantities (the components of the vectors) are represented by self-adjoint operators $\hat{x}_i$ and $\hat{p}_j$ satisfying commutation relations

$$[\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij} \ . \tag{2}$$

As conventionally understood, the commutation relations imply that the position and momentum of the particle cannot be specified with arbitrary accuracy at a given time.

---

perturbation is useful in many classical (i.e., non-quantum) applications. See chapter 11 of Thorne [18] for a popular exposition.

[11] For example, S^4 does not admit a flat metric. See the classic paper by Geroch and Horowitz [5] for further discussion of this and related topological issues.

Quantum fields, such as the quantum electromagnetic field, are similarly represented. The relevant observable properties of the electromagnetic field are the various components of the electric and magnetic field at each point in space (at some time—we are working in the canonical framework), and these are formally represented by the six operators $\widehat{E}_i(\vec{x})$ and $\widehat{B}_i(\vec{x}')$ $(i = 1, 2, 3)$.[12] For a scalar field, we have simply $\widehat{\phi}(x)$ and its conjugate $\widehat{\pi}(x)$. These operators all satisfy canonical commutation relations.[13] For example,

$$[\widehat{\phi}(\vec{x}), \widehat{\pi}(\vec{x})] = i\hbar\delta^3(\vec{x} - \vec{x}') \ . \tag{3}$$

One might well think that the gravitational field should also be quantized, and that analogous commutation relations should hold for the operators representing *its* properties. This line of thinking is implicit in the writings of many physicists. But in fact it is not at all obvious what it even *means* for the gravitational "field" to be subject to uncertainty relations. The two obstacles are:

1. The uncertainty relations apply to physical, observable quantities, such as the position and the momentum of a particle, or the values of the magnetic and electric fields at each point. Such observable quantities correspond to the canonical degrees of freedom of the theory. But no one has succeeded in isolating such quantities for the gravitational field.[14]

2. We use classical matter and fields to physically identify points of space-time. If all fields except for the gravitational field are treated quantum-mechanically, we can still use the gravitational field. But what does it

[12]Technically, these objects correspond to operator-valued distributions, which must be "smeared" with test functions in order to yield well-defined operators. Chapter 3 of Fulling [4] contains a lucid discussion.

[13]The canonical commutation relations for the electromagnetic field are rather messy, due to the presence of the constraints $\vec{\nabla} \cdot \vec{B} = 0$ and $\vec{\nabla} \cdot \vec{E} = 0$ (in the vacuum case). The commutation relations are $[\widehat{E}_i(\vec{x}), \widehat{B}_j(\vec{x}')] = i\hbar\left(\delta_{ij} - \frac{\partial_j\partial_i}{\nabla^2}\right)\delta^3(\vec{x} - \vec{x}')$.

[14]Technically, the point here is that we lack an explicit characterization of the reduced phase space of general relativity.

even mean to talk about the values of the gravitational field *at a point* (or commutation relations *between* points) if the field itself is subject to quantum "fluctuations"?[15]

Regarding the first obstacle, even though it is relatively well-known that gravity has not been reduced to a true canonical system, the relevance of this to the lack of local observables seems to be quite underappreciated. Perhaps the reason for this is that, insofar as one understands the gravitational field to be represented by the Riemann tensor $R^{\alpha}{}_{\beta\gamma\delta}$ (itself composed of first- and second-derivatives of the metric $g_{\alpha\beta}$), and insofar as this tensor has a value at every point, it is thought that the gravitational field is well-defined at every point. In the second half of the next section we will discuss the utility of this characterization of the field.

The second point is more straightforward. The significance of the "diffeomorphism invariance" of general relativity is that one needs some sort of classical structure like the metric or other physically meaningful tensorial objects (such as the Maxwell tensor $F_{\alpha\beta}$ corresponding to the electromagnetic field) in order to give physical meaning to "spacetime points." Thus if we quantize the metric and other fields, it is difficult to see how to talk meaningfully about the relation between the quantum fluctuations of a field at, and between, points.[16] We shall explore this idea further in the subsequent section.

[15] This point is taken up at greater length in [24].

[16] We are speaking loosely here. There are only four canonical degrees of freedom per spacetime point, whereas the metric has ten components. Thus if we are attempting to quantize general relativity as we would quantize an ordinary field theory, only four of the ten components should be subject to "quantum fluctuations."

4 Quantifying the effects of gravity—local field strength

4.1 Absolute acceleration

Traditionally, the physical significance of the values of a field at a given point is to determine the motion of a charge at that point. More specifically, the strength and direction of the field at the point determines the acceleration of the charge. So for instance in Maxwell theory, the acceleration of an electric charge at a given point is directly proportional to the strength and direction of the electric and magnetic fields at that point (see (1) above).

Note that the energy density of the field is calculated from these field strengths. For Maxwell theory, the energy H in an infinitesimal spatial volume dV is $dH = (\left|\vec{E}\right|^2 + \left|\vec{B}\right|^2)\ dV$. This is significant in that the fact that the function giving the total energy $H = \int_\Sigma dH$ over a region of space Σ is the Hamiltonian, and the Hamiltonian is the "generator" of time-evolution in the canonical formalism. Thus the fact that the energy of the field is well-defined corresponds to the fact that the time-evolution is well-defined. As we shall see, in general relativity the energy is not well-defined in general, and the time-evolution is ambiguous.[17]

Implicit in the definition of field strength here is the use of inertial frames as canonical reference frames. The acceleration of the charged particle is defined with respect to inertial frames—acceleration is the deviation from inertial motion. But as we saw above, the presence of gravity means that there *are* no inertial frames. On the face of it, this presents a problem for the definition of field strength in Maxwell theory in the presence of gravity. But one can recover a useful analog of the previous definition by utilizing the nearest approximation to inertial motion in curved spacetime, which is motion along a geodesic. The field strengths in curved spacetime then just

[17] This ambiguity is behind the notion of "many-fingered time" that one finds in texts such as [9].

give the acceleration with respect to a given geodesic, i.e., the deviation from geodesic motion. (Technically, they assign a "four-acceleration" to a charge at each point, the value of which determines the extent to which the charge will deviate from geodesic motion.)

It now follows that, according to this definition of field strength, the gravitational field strength at a point is always zero, no matter what the value of the Riemann tensor is at that point! If a freely falling observer (i.e., one following a geodesic) releases a gravitational "test charge" (any massive object, i.e., any object at all), then the test charge will not accelerate relative to the observer.[18] Rather, it will remain stationary with respect to the observer. In short, if one conceives of field strength as deviation from geodesic motion, then the gravitational field strength must be zero everywhere. Similarly, the energy density must be zero everywhere, since the magnitude of the velocity of a test particle never changes.

This claim, that the gravitational field strength is zero at each point, must be taken with a grain of salt. The argument is really that *if* one carries over to gravity the traditional notion of field strength, then one finds that the gravitational field strength is zero. Though it will turn out that there is no fully adequate local characterization of the gravitational field, we can do a bit better, and it is instructive to see how.

4.2 Relative acceleration and the Riemann tensor

Of course, we can and do observe the effects of gravitation. But as we have seen, what we observe is neither the acceleration of test objects relative to inertial observers (for there are no inertial observers) nor with respect to

[18] To be precise, the test charge will not accelerate relative to the observer as long as its center-of-mass and the observer's center-of-mass coincide at the time of release. If the observer holds the test object out to one side and lets it go, then the difference in the gravitational field at the point where the centers-of-mass of the two objects are located will result in a relative acceleration if the Riemann tensor is non-zero at those points. (This is known as a "tidal effect.")

their nearest gravitational analogues, geodesics in curved spacetime. Typically, what we observe are *tidal* effects, which involve the way in which bits of matter (or observers) distributed in space accelerate toward or away from each other. This relative acceleration is encoded in the Riemann curvature tensor $R^{\alpha}{}_{\beta\gamma\delta}$.

As with any tensor, the Riemann tensor is defined at every spacetime point, and thus it might seem that it offers a way of characterizing the local properties of the gravitational field. Given an observer at a point, we can find the relative acceleration a^{α} of nearby matter (which will follow nearby geodesics) by the geodesic deviation equation

$$a^{\alpha} = R^{\alpha}{}_{\beta\gamma\delta} u^{\beta} v^{\gamma} u^{\delta} \,, \tag{4}$$

where u^{α} is the tangent vector to the observer's worldline (representing his velocity) and v^{α} is a "geodesic selector", the purpose of which is to select a particular neighboring geodesic to compare with the geodesic traced out by the observer (i.e., her worldline). The quantity a^{α} then represents the relative acceleration of the two geodesics.

The fact that the Riemann curvature tensor seems to encode the effects of gravity in the neighborhood of any given point might suggest that it, like the Maxwell tensor in electromagnetism, fully characterizes the gravitational field. If this were the case, then one might expect that knowing the Riemann tensor at a given time would determine the Riemann tensor at future times, just as knowing the Maxwell tensor at a given time (the $\vec{E}$ and $\vec{B}$ field at a given time) determines the Maxwell tensor in the future.[19]

Looked at in a certain light, this construal of the Riemann tensor has a certain plausibility. After all, just as one can form the Maxwell tensor $F_{\alpha\beta}$ from the derivatives of a vector potential A_{α} via the equation $F_{\alpha\beta} = \nabla_{[\alpha} A_{\beta]}$, one can form the Riemann tensor $R^{\alpha}{}_{\beta\gamma\delta}$ from derivatives of the metric $g_{\alpha\beta}$.

[19]Here we are supposing that the Cauchy problem is well-posed, i.e., that the spacetime is spatially closed or that appropriate boundary conditions have been specified.

In the electromagnetic case, the quantities of physical significance are captured in the Maxwell tensor, and transformations $A_\alpha \longrightarrow A'_\alpha$ of the vector potential that leave the Maxwell tensor unchanged are thus regarded as non-physical "gauge" transformations. One might be tempted to guess, then, that since the Riemann tensor may be formed from derivatives of the metric, that transformations of the metric which leave the Riemann tensor invariant are physically meaningless "gauge" transformations. However, this is not the case.

To understand this, let us consider a situation in which one is given a manifold (thus a topology) and the Riemann tensor on the manifold. Suppose we have the manifold $S^1 \times \mathbb{R}$, and we are told that the Riemann tensor vanishes everywhere. This means that the metric is flat, and therefore that we are considering a cylinder. Does this information determine the metric on the cylinder? No, it does not. If it *did*, it would tell us the circumference of the cylinder, hence its radius. But since all cylinders have the same (vanishing) curvature, the curvature underdetermines the metric.

To what extent does the curvature underdetermine the metric? In cases of high symmetry, e.g., the hypersurfaces of constant curvature in typical idealized cosmological models, it underdetermines it by quite a bit. In the general case, where the Riemann tensor varies from point to point, one can often determine the metric up to a conformal factor. But this is insufficient to extract any unambiguous physical information from the Riemann tensor.

For example, suppose one is given the Riemann tensor at a point, and one wants to know the way in which particles in the neighborhood of the point will accelerate toward or away from a given observer at the point. An observer is characterized by a worldline in spacetime, and an observer at a given point is characterized by the tangent vector u^α to the worldline at that point. Therefore, one could construct a tensor

$$Z^\alpha_{\ \gamma} = R^\alpha_{\ \beta\gamma\delta} u^\beta u^\delta \tag{5}$$

$Z^\alpha_{\ \gamma} = R^\alpha_{\ \beta\gamma\delta} u^\beta u^\delta$ which represents the acceleration of nearby matter relative to an observer moving along a worldline with tangent vector u^α. However, there is something wrong with this picture, and it has to do with how we choose the tangent vector. A tangent vector is constrained to be of unit length, but we cannot tell how long a vector is without the metric. Therefore, to each of the conformally-related metrics $g_{\alpha\beta}$ associated with a given Riemann tensor, there is associated a different set of candidate tangent vectors u^α. In the absence of a specific metric, one cannot even form the tensor $Z^\alpha{}_\gamma$, because one has no way of normalizing candidate tangent vectors u^α. In short, the fact that the Riemann tensor by itself contains no physical information suggests that it is a mistake to regard it as fully characterizing the gravitational field, in any conventional sense.

5 Causal structure

In the previous section, we examined one of the difficulties in applying the uncertainty principle to the gravitational field, the difficulty that the values of the gravitational field at a point are not even well-defined in general relativity. In this section and the next, we will address another difficulty, having to do with the status of commutation relations in a theory in which the spacetime geometry itself is quantized.

In a conventional classical field theory in a flat background spacetime, the causal structure tells us the "domain of dependence" of the field values at a point. In other words, we know that the values of a field at spacetime point x are related to the values of the field at points in its forward and backward lightcones. In the corresponding quantum field theory, this is reflected in the fact that the (covariant) field operators $\widehat{\phi}_i(x)$ at spacelike separated points x and y commute:

$$[\widehat{\phi}_i(x), \widehat{\phi}_j(y)] = 0 \; . \tag{6}$$

The intuitive physical picture behind this is that measurements of the field at point x do not reveal anything about the field at point y, because they are not in causal contact with each other.

Of course, when one incorporates the gravitational effects of a classical field, one formulates it in a curved spacetime, where the curvature respects the stress-energy properties of the matter in accord with Einstein's equation. If one wants to treat the fields quantum-mechanically, there are two choices: one can attempt to leave the spacetime classical and use that structure in the quantization ("semi-classical" gravity), or one can attempt to quantize gravity. The difficulty with the former is that one wants a determinate spacetime structure despite the indeterminate (because quantum) stress-energy of the field. One can pursue this by using the "expectation value" (the average value) of the fields in a given state to determine the spacetime curvature—this is the approach taken by those working in the field of "quantum field theory in curved spacetime."[20] In such a theory, one can make sense of commutation relations like (6), because one can determine whether or not two points are spacelike separated. But such a theory can make no claim to being fundamental [3][7].

Suppose, then, that we opt for the second alternative and allow some of the components of the gravitational field to "fluctuate", so that, for example, the curvature at each point is subject to quantum fluctuations. In that case, we would expect that the metric itself is subject to quantum fluctuations (since the curvature is built from derivatives of the metric). But if the metric is indefinite, then it is by no means clear that it will be meaningful to talk about whether x and y are spacelike separated, unless the metric fluctuations somehow leave the causal (i.e., conformal) structure alone.

Assuming that the metric fluctuations do affect the causal structure, one would expect that the commutation relations themselves should reflect this by also undergoing quantum fluctuations of some sort. However, it is not at all clear what this means, or how it might be represented. And in particular, it should be noted that such a commutator would have no

[20] See Fulling [4] or Wald [20] for a modern introduction.

apparent counterpart in the classical theory. In allowing metric fluctuations to affect causal structure, one is clearly at some remove from ordinary field-theoretic quantization schemes.

6 What's the point?

We began by taking a close look at how one might characterize the gravitational field at a given point, and we then went on to examine the consequences of turning whatever local quantities we might find into operators, in a quantum theory of gravity. Both of these are problems peculiar to gravitational physics, in that they arise as a result of the principle of equivalence, the equivalence of gravitational and inertial mass that practically compels us to regard classical gravity as a theory of spacetime geometry.

The final point, however, has less to do with gravity *per se* than the fact that any theoretical framework incorporating gravity must seemingly be diffeomorphism-invariant. Up to this point we have adopted the polite fiction that, for example, the Riemann tensor at a point x is a physically meaningful quantity. In practice, however, we need to know how to locate x in order to extract such information. Classically, this is not a problem, as long as reference objects or observers are part of the model. Thus we can make sense of the value of the Riemann tensor at x, if x means something like "in the southeast corner of the lab at 5 o'clock" or "where Jim will be standing in 10 minutes." But it is entirely unclear how to carry this sort of thing over to the case in which all matter (including the lab and Jim) and all fields including gravity, are quantized. If we treat the lab quantum mechanically, then the location of the southeast corner of the lab at 5 o'clock will not designate a particular point at all.

This is true in ordinary quantum field theory as well. In order to even give any physical content to a field operator defined at a spacetime point x, we need a physical object that we can identify the point x. In practice, this means that we need objects which are very *massive*, so that, barring macroscopic Schrödinger-cat states, they track a definite spacetime trajectory. However, this will not do for a quantum theory of gravity, for although increasing the mass of an object localizes it, it also amounts to increasing its gravitational "charge". This means that the more accurate (with respect to a classical background) one's reference system might be expected to be, the more it actually interacts with the quantum-gravitational background one is trying to measure. Ultimately, of course, this is why gravitational observables are diffemorphism-invariant—one cannot isolate a system gravitationally, and all matter, including the reference objects, must be included in the description. But this raises havoc for a quantum-field-theoretic treatment of gravity.[21]

7 Conclusion

We began by looking at the idea of a gravitational field subject to quantum fluctuations at each point of spacetime, a naive yet popular conception of what a quantum theory of gravity might entail. Upon examination, it turned out that the only way in which to quantify the effects of gravity at a point makes use of relational properties, which fail to capture all observable gravitational phenomena. Furthermore, because any fluctuations in the field would mean fluctuations in the spacetime structure itself, one is left with no way of individuating the points that lends itself to the structure of quantum theory.

In the real world of quantum gravity research, one finds these problems cropping up, albeit in sometimes oblique ways. In canonical quantum gravity, the most obvious counterpart of the first problem is the extreme difficulty of finding any observables [19].[22] Should one find them, one would

[21] See [24] for a more extensive discussion of the difficulties of diffeomorphism-invariant quantum theory.

[22] Rovelli and Smolin [13] claim that the area and volume operators in loop quantum gravity are observables, but this relies on a somewhat contentious procedure in which

expect that they would not be local observables, but some sort of nonlocal or perhaps global (i.e., over all of space) observables. It is worth noting in this connection that one of the great ironies of quantum gravity is that it is a theory which is generally supposed to be applicable only at an incredibly small scale (the Planck length is 10^{-33}cm), yet any candidate gravitational observables would have to be highly nonlocal.

Another counterpart to the first problem is the notorious "problem of time" in quantum gravity. As we saw, the lack of any complete specification of local field strength for gravity implies that there is no adequate definition of local energy density. For the important case of spatially-closed spacetimes, this raises great difficulty for a global characterization of energy. In conventional physics, the function that characterizes the energy (the "Hamiltonian") is the function that mathematically generates time translation, and the ill-definedness of energy in general relativity corresponds to our inability to isolate a Hamiltonian for the theory. In this light, it is not surprising that time-evolution is inherently ambiguous, and that consequently there are great difficulties in even formally constructing a quantum theory.[23]

The counterpart of the second problem, identifying the causal structure, is skirted in canonical quantum gravity by positing a split of spacetime into space and time at the outset. This is not without consequences, however. Among the most serious is the fact that the diffeomorphism group (the invariance group of the full theory of general relativity) is represented in a distorted way in the canonical theory, so that it is unclear that one is actually quantizing general relativity at all. Furthermore, it is characteristic of the classical theory that hypersurfaces which begin as spacelike can evolve into null surfaces, thus killing the evolution. One should expect an analogue of this problem in the canonical quantum theory, though how this would arise depends on how the problem of time is resolved. All this suggests that a

matter, treated classically, is used to "gauge fix" the theory.

[23]Excellent reviews of the problem of time are Isham [6] and Kuchař [8]. See also Weinstein [23] [22].

theory that truly unifies quantum theory and gravity will be one in which the idea of local fluctuations in a field plays no role, and so a theory which is radically different from any quantum field theory with which we are familiar at present.[24]

References

[1] D. Bohm. *Quantum Theory.* Prentice-Hall, Englewood Cliffs, 1951.

[2] I. Ciufolini and J.A. Wheeler. *Gravitation and Inertia.* Princeton University Press, Princeton, 1995.

[3] M.J. Duff. Inconsistency of quantum field theory in a curved spacetime. In C.J. Isham, R. Penrose, and D. Sciama, editors, *Quantum Gravity 2: A Second Oxford Symposium*, pages 81–105. Clarendon Press, Oxford, 1981.

[4] S.A. Fulling. *Aspects of Quantum Field Theory in Curved Space-Time.* Cambridge University Press, Cambridge, 1989.

[5] R. Geroch and G. Horowitz. Global structure of spacetimes. In S.W. Hawking and W. Israel, editors, *General Relativity: An Einstein Centenary Survey*, pages 212–293. Cambridge University Press, Cambridge, 1979.

[6] Chris J. Isham. Canonical quantum gravity and the problem of time. In *Integrable Systems, Quantum Groups, and Quantum Field Theories*, pages 157–288. Kluwer Academic Publishers, London, 1993.

[7] T.W.B. Kibble. Is a semi-classical theory of gravity viable? In C.J. Isham, R. Penrose, and D. Sciama, editors, *Quantum Gravity 2: A Second Oxford Symposium*, pages 63–80. Clarendon Press, Oxford, 1981.

[24] For examples of some of the more radical speculations, see [14][15][16][17].

[8] Karel Kuchař. Time and interpretations of quantum gravity. In *Proceedings of the 4th Canadian Conference on General Relativity and Relativistic Astrophysics*, pages 211–314. World Scientific, Singapore, 1992.

[9] C.W. Misner, K.S. Thorne, and J.A. Wheeler. *Gravitation.* W.H. Freeman and Company, New York, 1973.

[10] J. Norton. General covariance and the foundations of general relativity: eight decades of dispute. *Reports on Progress in Physics*, 56:791–858, 1993.

[11] H. Reichenbach. *The Philosophy of Space and Time (English translation).* Dover Publications, New York, 1958. First published 1927.

[12] C. Rovelli. Strings, loops and others: A critical survey of the present approaches to quantum gravity. 1998. Forthcoming in the proceedings of the GR15 conference, Pune, India, 1997.

[13] C. Rovelli and L. Smolin. Discreteness of area and volume in quantum gravity. *Nuclear Physics*, B442:593–622, 1995. Errata: B456:753, 1995.

[14] L. Smolin. The Bekenstein bound, topological quantum field theory and pluralistic quantum cosmology. 1995. gr-qc/9508064.

[15] L. Smolin. Linking topological quantum field theory and nonperturbative quantum gravity. 1995. gr-qc/9505028.

[16] L. Susskind. The world as a hologram. *Journal of Mathematical Physics*, 36:6377–6396, 1994.

[17] G. 't Hooft. Distinguishing causal time from Minkowski time and a model for the black hole quantum eigenstates. 1997. gr-qc/9711053.

[18] K.S. Thorne. *Black Holes and Time Warps: Einstein's Outrageous Legacy.* W.W. Norton, New York, 1994.

[19] Charles G. Torre. The problems of time and observables: some recent mathematical results. 1994. gr-qc/9404029.

[20] R.M. Wald. *Quantum Field Theory in Curved Spacetime and Black Hole Thermodynamics*. University of Chicago Press, Chicago, 1994.

[21] S. Weinstein. Strange couplings and space-time structure. *Philosophy of Science*, 63:S63–S70, 1996.

[22] Steven Weinstein. *Conceptual and Foundational Issues in the Quantization of Gravity*. PhD dissertation, Northwestern University, 1998.

[23] Steven Weinstein. Time, gauge, and the superposition principle in quantum gravity. In *Proceedings of the Eighth Marcel Grossman Meeting*, Singapore, 1998. World Scientific. gr-qc/9711056.

[24] Steven Weinstein. Absolute quantum mechanics. 1999. Forthcoming in *British Journal for the Philosophy of Science*.