A locus for "now"

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Abstract

We investigate the concepts of past, present, and future that build upon a modal distinction between the settled past and the open future. The concepts are defined in terms of a pre-causal ordering and of qualitative differences between alternative histories. Finally, we look what an event's past, present, and future look like in the so-called Minkowskian Branching Structures, in which histories are isomorphic to Minkowski spacetime.

"What is the present?"[1] We typically understand this question as being relative to events, that is, we fix our attention on some event (frequently, an event of our utterance) and query what the present of this event is. The question has many facets, two of which we have set apart for the purposes of this paper. First, we may be concerned with when the "now" is, or what a locus for "now" is. The aim is to indicate a part of our world, or a region of spacetime, as a locus for the present of an arbitrary event. In other words, the aim is to define the set of events co-present (contemporaneous) with a given one, or the set of locations of such events. Secondly, one may wonder what differentiates the present of an event from its past and its future. This second question quickly leads us to considerable metaphysical queries: "Is becoming real?", or "Is the distinction between tenses objective or mind-dependent?". Clearly, the second question is much harder; moreover, a positive answer to it presupposes some answer to the first question. So we put it aside, and focus upon what a locus for "now" is.

The problem is that starting with the papers of Rietdijk (1966) and Putnam (1967), there have been arguments showing that special relativity (SR) is inimical to any intuitive notion of the present, where "intuitive" here means that it is based on co-presence that is transitive and neither the identity nor the universal relation on Minkowski spacetime.[2] Although these arguments deserve a separate analysis, to keep the length of this paper short, we will limit ourselves to this not-so–rigorous formulation of the result:

R The following set of premises is logically incoherent:
(1) the relations used to define co-presence and co-presence itself are invariant with respect to automorphisms of Minkowski space-time,
(2) co-presence is a transitive relation on Minkowski spacetime,
(3) of two co-present events, one cannot be causally before (or after) the other, and
(4) co-presence is neither the identity nor the universal relation on Minkowski spacetime.

Attempts to blunt the impact of this result boil down to arguing that a failure of one of premises (1) - (4) is not as bad as it looks.[3]

In this paper, motivated by an intuition that associates the future with contingency, we construct a spatiotemporally extended and frame-independent notion of the present. The construction does not contradict result **R** (how could it?): to accommodate contingency, we will distinguish a special set of points of Minkowski spacetime, called splitting points, and thought of as locations of chancy events. To define the present, we will use relations like "x is a splitting point and y lies within the future-light cone of x", which clearly is not invariant with respect to automorphisms of Minkowski spacetime. Moreover, we will end up with a notion of tenses that will be separate from causal notions defined in terms of light-cones. The approach is intended to be conciliatory: on the one hand, we invite the reader to modify her notion of the present. On the other, we take it that neither special relativity nor general relativity are our ultimate truths, so perhaps one day there will be a theory of both spacetime and chanciness. Needless to say, we bet on our world turning out to be chancy.[4]

1 Main intuition

There is a strand in philosophy that associates the future with open possibilities, the past with settled facts, and the present with a region of passage from possibility to settledness. The view had a strong proponent in Aristotle.[5] In recent times, the idea was defended by Whitrow (1961, pp. 295–296):

> Strict causality would mean that the consequences pre-exist in the premises. But, if the future history of the universe pre-exists logically in the present, why it is not already in the present? If, for the strict determinist, the future is merely "the hidden present", whence comes the illusion of temporal succession? The fact of transition and 'becoming' compels us to recognize the existence of an element of indeterminism and irreducible contingency in the

> universe. The future is hidden from us—not in the present, but in the future. Time is the mediator between the possible and the actual.

Similar elaborations on this view can be found in Eddington (1949) and (1953). The doctrine that the objectivity of the distinction between the past, the present, and the future requires indeterminism (or some aspect of contingency, or a failure of the universal causation) has been vigorously opposed.[6] But, strangely enough, no friends or foes of the doctrine have belaboured the underlying association between future and contingency to a point of stating it with a rigor that would make the association amenable to formal treatment. It is exactly this task to which we now turn.

How then is the future different from the past and the present? Supposedly, in contrast to the latter, the future has some aspect of contingency. Yet what is this aspect, exactly? Note that once we decide on how to respond to this question, we will get a grip on a concept of the future, from which a characterization of the past and the present would fall in naturally. We will define events in the past of event e as those events from which perspective e was in the future. Having had the notions of "events in the past of e" and "events in the future of e", we will declare that events co-present with e are exactly those events that are neither in the past nor in the future of e.

In the above elucidation of what the past is we used tenses ("was in the future"); similarly we will invariably use words like "after" or "before" in our final definition of the past, the present, and the future. This might bring in an objection that our definition is circular. Clarifying this possible confusion, we assume here a pre-causal ordering of the totality of possible point events, and that this ordering is partial. The ordering is similar to the SR ordering in terms of light cones, but generalized to modal contexts. We will read the ordering $e \leqslant e'$ as "e can causally influence e'", or "e' belongs to a possible continuation of e". The 'after" and "before" will refer to this ordering. In a similar vein, the tense operators will be standardly defined in terms of pre-causal ordering. As a consequence of this approach, we will get a certain separation between causal notions (including the tense operators) and the notions of past, present, and future.[7]

Turning to belabouring on a future–contingency link, let us begin with the question: why does my toast at the New Year's Eve 2012 belong to the future of my present utterance? As a first approximation, take the answer

"It belongs to the future only if it might fail to occur."

Evidently, this answer is too strong, as it relegates from the future of e any event that occurs after e in every possible continuation of e. In other words, an event deterministic from the perspective of e cannot belong to the future of e on this construal. As an improvement consider this:

> "My toast at the New Year's Eve 2012 belongs to the future of my present utterance only if the way it will occur is not settled yet".

On this proposal, the toast in question belongs to the future of my present utterance since, for instance, it is not yet settled *where* I will have it. This answer is again too strong, for exactly the same reason as the previous one. What seems to me a minimal link between future and contingency is the following formulation:

> "My toast at the New Year's Eve 2012 belongs to the future of my present utterance iff the toast is consistent with the utterance and before the toast there is an event and some aspect of it that is not settled yet."[8]

Here "before" is understood weakly, as "before or identical to". The requirement of consistency excludes from the future of our utterance those possible events that do not occur in a history to which our utterance belongs. To illustrate this analysis, although it is inevitable that my old-fashioned mechanical wall-clock will strike in 52 minutes, this event belongs to the future of my present utterance, because there are some events before it that are in some respect contingent from the present perspective.

The New Year's Eve examples suggest that the future-contingency link should be minimal, which strongly favors our third analysis. There seems to be, however, an opposite intuition as well, which takes the event of our clock striking in 52 minutes as not really belonging to the future, since (given our assumptions) it is already settled that the clock will strike in 52 minutes. The feeling is that the clock mechanism is somewhat "isolated" from its surrounding, and especially from chancy events in its past. No matter what, it will strike. On reflection, the truth of the "already settled" sentence above means that, even if there are many histories to which my utterance belongs, in every such a history there is our clock striking in 52 minutes. There is thus a disjunctive event of our clock striking that is contained in many histories. Lewis calls such events "non-fragile" since, even if our clock stroke a bit differently, we would call it "the same event as the actual striking of our clock". There is however another concept of events, fragile or non-disjunctive events. On this concept of events, if our clock's striking were minimally different from the actual one, even by merely having a minimally different past, this event would not count as identical with the actual striking. The feeling of a mechanism isolated from a neighboring chancy event stems from our concentration on settled truth and the underlying disjunctive events. At the level of non-disjunctive events, the phenomenon is absent: a slight chancy event brings in a non-erasable difference for the future.[9]

To further elaborate on our third analysis, we will put it down in words as below:

Condition 1 *f is in the future of e iff e is consistent with f and there is some event e' before or identical to f and a subject matter A such that at e it is contingent that A at the space-time location of e'.*

To put rigour into our intuition, we need to combine spacetime with modality. The only rigorous framework for this task is the theory of branching space-times (BST) of Belnap (1992), which in turn is a development of an earlier theory of branching time (BT).[10] The development consists of the fact that BST is able to account for spatial and relativistic aspects in addition to modal and temporal aspects analyzed in BT.

Both branching theories can be seen as addressing two problems. One is an ontological question: what does the indeterministic world look like? The other problem is semantic, namely, how to formally model a language with tenses, modal operators and indexicals? It is BST's capacity to handle the second (semantic) problem that we need in the present paper. The basic insight of branching theorists, owed to Prior, is that sentences are evaluated as true or false at the event-history pairs, which leads to giving more structure to evaluation points. Designating evaluation point by e/h, we will have, for a sentence A unsettled in the future of e:

$$e/h_1 \models Will\colon A \text{ but } e/h_2 \not\models Will\colon A.$$

1.1 Models of BST

A model of BST, $\langle W, \leqslant \rangle$, is a non-empty partially ordered set of possible point events ordered by a pre-causal relation, subject to some postulates.[11] Histories in $\langle W, \leqslant \rangle$ are identified with particular (upward directed) subsets of W.

A BST model $\langle W, \leqslant \rangle$ can serve as a basis of a semantic model $\langle \langle W, \leqslant \rangle, \mathcal{I} \rangle$ for a propositional language with tenses and modal operators, and the indexical "here-and-now". Above $\mathcal{I}$ is an interpretation function $\mathcal{I}\colon Atoms \Rightarrow \mathcal{P}(W)$, where *Atoms* is the set of atomic formulas. It is understood that atomic formulas of this language have the form: "Here-and-now there is property A". Turning to truth-conditions, here are a few examples: (For more information on BST semantical models, cf. Belnap (2007), Müller (2002), and Placek and Müller (2007).) To avoid lengthy notation, we abbreviate the point of evaluation $\langle \langle \langle W, \leqslant \rangle, \mathcal{I} \rangle, e/h \rangle$ to e/h.

$e/h \models A$ iff $e \in \mathcal{I}(A)$ for A an atomic formula;
$e/h \models \neg\varphi$ iff it is not the case that $\langle\langle W, \leqslant\rangle, \mathcal{I}\rangle, e/h \models \varphi$;
$e/h \models \mathit{Will}\colon \varphi$ iff $\exists e' > e\colon e'/h \models \varphi$;
$e/h \models \mathit{Was}\colon \varphi$ iff $\exists e' < e\colon e'/h \models \varphi$;
$e/h \models \mathit{Poss}\colon \varphi$ iff $\exists h'\colon e \in h' \wedge e/h' \models \varphi$.

Note that in the last clause, since we quantify over histories on its right-hand side, the reference to history on the left-hand side is redundant. We will thus write $e \models \mathit{Poss}\colon A$ instead of $e/h \models \mathit{Poss}\colon A$.

Some (but not all) BST models allow for more structure, as one can define spacetime locations (st-locations for short) on them. A set *Loc* of st-locations for BST model $\langle W, \leqslant\rangle$ is a partition of W that is conservative with respect to ordering $\leqslant$—cf. Müller (2005). St-location is a relativistic counterpart of our everyday thinking of what would happen at the time or in the location of a given event, if things went differently at some junction in the past. Note that we have thus arrived at the distinction between event (i.e., an element of W) and st-location of an event (an element of a particular partition *Loc* of W). To denote the st-location of event e, we will write $loc(e)$.

In what follows, we need to consider sentences of the form "At st-location x it is φ", like "The value of electromagnetic field at t, x, y, z is such-and-such." The truth conditions for such sentences can only be formulated with respect to a BST model with set *Loc* of st-locations:

$$\langle\langle W, \leqslant, \mathit{Loc}\rangle, \mathcal{I}\rangle, e/h \models \mathit{At}_x\colon \varphi \text{ iff } \exists e'\colon e' \in h \cap x \wedge e'/h \models \varphi, \text{ where } x \in \mathit{Loc}.$$

We are now able to formulate the intuition of Condition 1 within the language of BST:

Definition 2 *An event f belongs to the future of event e, $f \in \mathit{Future}(e)$, iff there is event e' and an atomic formula A such that*

1. *there is history h such that $e, f \in h$ and*
2. *$e' \leqslant f$ and*
3. *$e \models \mathit{Poss} : \mathit{At}_{loc(e')} \colon A$ and*
4. *$e \models \mathit{Poss} : \mathit{At}_{loc(e')} \colon \neg A$.*

Event p belongs to the past of event e, $p \in \mathit{Past}(e)$, iff event e belongs to the future of p.
Event e' belongs to the present of event e, $e' \in \mathit{Present}(e)$, iff there is a history h such that $e, e' \in h$ and e' belongs neither to the past nor to the future of e.

The future, present and past as defined above are global, that is, whether an event belongs to the past / present / future of event e depends on possibilities open in a history to which e belongs. Technically speaking, clauses (3) and (4) of the definition of the future of e require quantification over *all* histories comprising e. L. Wroński suggested to me (in a private communication) that for some purposes relativised notions of the past / present / future are more adequate. Typically we do not know about all possibilities available from a given history. We might thus want to relativise the investigated notions to some set of possibilities, those we know or those that are available in our vicinity. Technically, this proposal amounts to relativising the operator $Poss$ to a set H of histories:
for $H \subseteq \{h \in Hist \mid e \in h\}$, $e \models Poss_H\varphi$ iff $\exists h\ h \in H \wedge e/h \models \varphi$.
As a result of replacing $Poss$ by the relativised operator $Poss_H$ in Definition 2, typically the future of e as well as the past of e would become smaller, making the present of e larger.

1.2 Minkowskian Branching Structures (MBS)

Although the above definition adequately (we believe) captures our informal statement of Condition 1, it does not permit us to "see" what the future, and hence the past and the present, of an event are. This is a consequence of the generality of BST, which leaves it open what structure BST histories have, as long as they are maximal upward directed subsets of a base set. Thus, to address the "see" question, we need to make it relative to a specific concept of spacetime, and then consider such BST models, in which histories are isomorphic to the spacetime in question. We will investigate the problem for Minkowski spacetime.

A particular class of BST models, in which every history is isomorphic to Minkowski spacetime has been investigated by Müller (2002), Wroński and Placek (2009), and Belnap and Placek (2010). To begin with an informal notion (to be proved identical to BST histories), a possible scenario can be thought of as Minkowski spacetime plus physical content. The content can be represented by an attribution of "point properties" (typically, strengths of physical fields), i.e., a function from $\Re^4$ to $\mathcal{P}(P)$, where P is the set of point properties. To get a modal aspect[12], we need a system of such "physical contents". A system of this sort is represented by a property attribution $F : \Re^4 \times \Sigma \to \mathcal{P}(P)$, where Σ is the set of labels for scenarios.

Since we haven't (yet) imposed any restrictions on property attribution functions, we should expect that they will produce strange property attributions, or at least, ones incapable of obtaining a BST reading. Thus, in an attempt to arrive at BST models, we single out the class of "proper" property attributions.

We shall put our requirement informally first: for F to be a proper property attribution, we require that every two scenarios $\sigma, \eta \in \Sigma$ are qualitatively different somewhere and if they are different at some point, there is a special point $c \in \Re^4$ below it (called *splitting point* for σ and η). Its special character consists in that (1) σ and η agree at and below c, and that (2) for a point x above c, no matter how close x is to c, there is always an even closer point above c at which σ and η disagree in content.[13] Note that while postulating a complete qualitative agreement at and below c, we do not require a complete disagreement above c; we readily permit that over large regions above c the scenarios are qualitatively the same– as long as they are different at locations arbitrarily close to c and above c. The locutions "above" and "below" refer here to the so-called Minkowskian ordering $\leqslant_M$ of $\Re^4$:

$$x \leqslant_M y \text{ iff } \sum_{i=1}^{3}(x^i - y^i)^2 \leqslant (x^0 - y^0)^2 \text{ and } x^0 \leqslant y^0, \tag{1}$$

with a resulting strict ordering $<_M$ defined in a usual way. The relation of being space-like related (SLR) is also typically defined: two points are SLR iff they are incomparable by $\leqslant_M$. Putting the above informal explanation in symbols, we have this :

Definition 3 *A property attribution* $F : \Re^4 \times \Sigma \to \mathcal{P}(P)$ *is* proper *iff for every* $\sigma, \eta \in \Sigma$ $(\sigma \neq \eta)$ *there is* $x \in \Re^4$ *such that*

$$F(x, \sigma) \neq F(x, \eta), \textit{ and} \tag{2}$$

(for every $x \in \Re^4$*) if* $F(x, \sigma) \neq F(x, \eta)$*, then there is* $c \in \Re^4$ *such that* $c <_M x$ *and*

$$\forall z \in \Re^4 (z \leqslant_M c \to F(z, \sigma) = F(z, \eta)) \textit{ and} \tag{3}$$

$$\forall x' \in \Re^4 (c <_M x' \to \exists y \in \Re^4 (c <_M y <_M x' \ \wedge \ F(y, \sigma) \neq F(y, \eta))). \tag{4}$$

Points of $\Re^4$ that satisfy conditions 3–4 constitute what we call the set $S_{\sigma\eta}$ of splitting points for σ and η. From this definition of proper property attribution some desired properties of sets of splitting points are deducible.[14] To state them, it is useful to distinguish special subsets of $\Re^4$, thought of as regions of no qualitative difference of histories, and defined as

$$R_{\sigma\eta} := \{x \in \Re^4 \mid \neg\exists c \ (c <_M x \wedge c \in S_{\sigma\eta}) \text{ for } \sigma, \eta \in \Sigma.$$

Fact 4 *Assume that* $F : \Re^4 \times \Sigma \to \mathcal{P}(P)$ *is a proper property attribution. Then:*

1. $\sigma \neq \eta \rightarrow S_{\sigma\eta} \neq \emptyset$;
2. $S_{\sigma\eta} = S_{\eta\sigma}$;
3. $\forall c, c' \in S_{\sigma\eta}\ (c \neq c' \rightarrow c\ SLR\ c')$;
4. $x \in R_{\sigma\eta} \rightarrow F(x, \sigma) = F(x, \eta)$; *and*
5. $\forall \sigma, \eta, \gamma \in \Sigma\ \ R_{\sigma\eta} \cap R_{\eta\gamma} \subseteq R_{\sigma\gamma}$.

Clearly, Σ is not a set of BST histories, and $\leqslant_M$ is not a BST ordering. To produce a BST model, we need to construct these latter notions, showing that they satisfy BST postulates. In this task, we follow Müller's (2002) construction, to which the reader should turn to for more information. First, we define relation $\equiv$ on $\Re^4 \times \Sigma$:[15]

$$x\sigma \equiv y\eta \text{ iff } x = y \text{ and } x \in R_{\sigma\eta}.$$

Provably $\equiv$ is an equivalence relation on $\Re^4 \times \Sigma$. Next, we define a BST event as an equivalence class with respect to $\equiv$, that is

$$\{y\eta \mid y\eta \equiv x\sigma\} := [x\sigma]$$

A BST ordering is defined as follows:

$$[x\sigma] \leqslant [y\eta] \text{ iff } [x\sigma] = [x\eta] \wedge x \leqslant_M y.$$

Importantly, it turns out that Σ is indeed a set of labels for histories, as every BST history is of the form: $\{[x\sigma] \mid x \in \Re^4\}$ for $\sigma \in \Sigma$. Moreover, given that a property attribution is proper and an additional postulate is satisfied,[16] $[x\sigma]$ is a maximal element in the overlap of two histories $\{[x\sigma] \mid x \in \Re^4\}$ and $\{[x\eta] \mid x \in \Re^4\}$ iff x is a splitting point for these histories, i.e., $x \in S_{\sigma\eta}$. The construction should finish with proofs that the resulting structure is indeed a BST model.[17]

Figure 1 illustrates two Minkowskian Branching Structures, first with two histories and one splitting point, and the second —with four histories and two splitting points. The shaded area indicates where a given history overlaps with the first history.

Formally speaking, an MBS is a triple $\langle \Sigma, P, F \rangle$, where Σ is a set of labels for scenarios, P is a set of point properties, and F is a proper property attribution. A merit of this construction is that $\langle \Sigma, P, F \rangle$ provides a natural semantic model for a propositional language with tense operators and modal operators, and whose atomic sentences have the form:

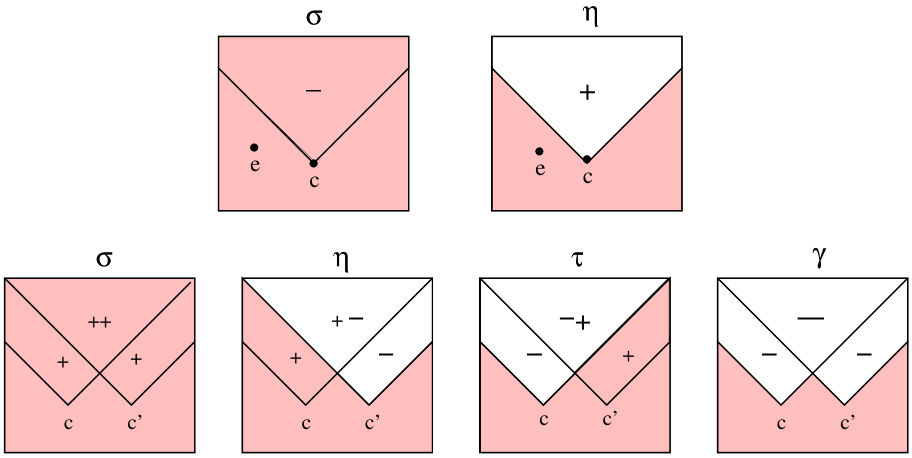

Figure 1: Top: an MBS with one splitting point and two histories. Bottom: an MBS with two splitting points and four histories. Shadowed regions indicate the intersection of a given history with a reference history σ.

It is ψ here-and-now,

where $\psi \in P$. Furthermore, the proper property attribution F determines interpretation function $\mathcal{I}$ in the following manner:

$[x\sigma] \in \mathcal{I}(A)$ iff $\psi \in F(x, \sigma)$, where $A =$"It is ψ here-and-now".

The BT/BST truth conditions for tense and modal operators can be readily reformulated in the MBS framework. For a point of evaluation we take $\langle\langle \Sigma, P, F\rangle, [x\sigma]/\sigma\rangle$, which we abbreviate as $[x\sigma]/\sigma$. As an example, here are the truth conditions for *Poss* and At_y ($y \in \mathbb{R}^4$):

$$\begin{array}{l} [x\sigma]/\sigma \models Poss\!:\!B \text{ iff there is } \eta \in \Sigma \text{ such that } [x\sigma] = [x\eta] \text{ and } [x\sigma]/\eta \models B \\ [x\sigma]/\sigma \models At_y B \text{ iff } [y\sigma]/\sigma \models B, \text{ where } y \in \mathbb{R}^4. \end{array} \tag{5}$$

Since in the clause for *Poss* the reference to label σ after the stroke is redundant, we will write $[x\sigma] \models Poss\!: B$ for $[x\sigma]/\sigma \models Poss\!: B$.

2 What do the presents look like?

We will now apply Definition 2 to some selected MBS's in order to get a grasp on what the future, the present and the past of a given event are.

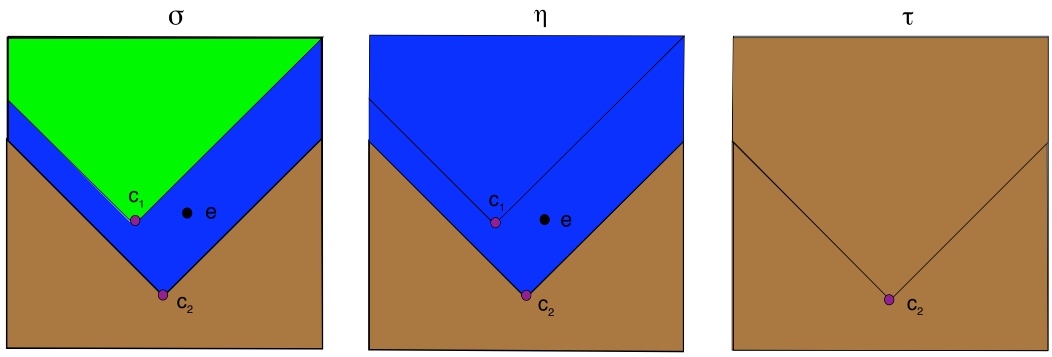

Figure 2: The past, the present, and the future of $e = [x\sigma]$.

Two splitting points, time-like. Consider first an MBS with three histories, i.e. $\Sigma = \{\sigma, \eta, \tau\}$, in which the proper property attribution yields two splitting points $c_1, c_2 \in \mathbb{R}^4$ such that $c_2 <_M c_1$ and $S_{\sigma\eta} = \{c_1\}$ and $S_{\sigma\tau} = S_{\eta\tau} = \{c_2\}$ (See Figure 2.). Pick now an event $e := [x\sigma]$ that is "between" c_2 and c_1 in the sense that $c_2 <_M x \not>_M c_1$ and ask: (1) What is the future of e? (2) What is its past? (3) And what is its present?

We claim now: For x such that $c_2 <_M x \not>_M c_1$,

1. The future of $e = [x\sigma]$ is the set of events that are strictly above $[c_1\sigma]$: $Future([x\sigma]) = \{[z\gamma] \mid c_1 <_M z \wedge \gamma \in \{\sigma, \eta\}\}$.
2. The past of $e = [x\sigma]$ is the set of events that are in history σ and not strictly above $[c_2\sigma]$: $Past([x\sigma]) = \{[z\sigma] \mid c_2 \not<_M z\}$.
3. The present of $e = [x\sigma]$ is the set of events in history σ and "between" c_2 and c_1 in the sense: $Present([x\sigma]) = \{[z\sigma] \mid c_2 <_M z \wedge c_1 \not<_M z\}$.

PROOF:
Ad. 1 $\Rightarrow$ Let $[z\gamma] \in Future([x\sigma])$. There is then $[z'\gamma']$ such that (†) $[z'\gamma'] < [z\gamma]$ and (‡) $[x\sigma] \models Poss\colon At_{z'}A$ and $[x\sigma] \models Poss\colon At_{z'}\neg A$ for some atomic formula A. It follows that for some $\beta, \beta' \in \Sigma$: ($\star$) $[x\sigma] = [x\beta] = [x\beta']$, and $[z'\beta]/\beta \models A$ and $[z'\beta']/\beta' \models \neg A$. The latter entails ($\diamond$) $F(z'\beta) \neq F(z'\beta')$, and hence $\beta \neq \beta'$. Given the location of x, it follows from ($\star$) that $\beta = \sigma$ and $\beta' = \eta$ (or vice versa), so ($\diamond$) implies that $c_1 <_M z'$. (†) implies $z' <_M z$, and hence $c_1 <_M z$. The consistency clause requires $\gamma = \eta$ or $\gamma = \sigma$.
$\Leftarrow$ Let $c_1 <_M z$. Since $c_1 \in S_{\sigma\eta}$ for some z' such that $c_1 <_M z' <_M z$: $F(z'\sigma) \neq F(z'\eta)$. Hence for some atomic A: $[z'\sigma]/\sigma \models A$ and $[z'\eta]/\eta \models \neg A$

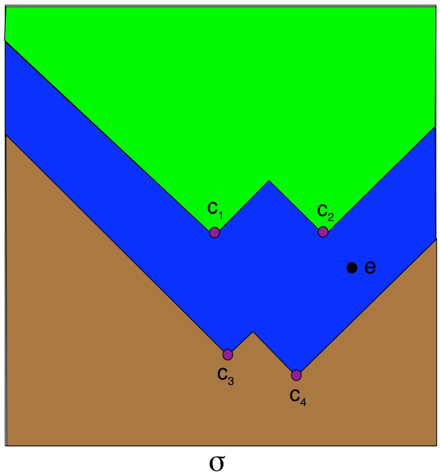

Figure 3: History $\{[x\sigma] \mid x \in \Re^4\}$ with four splitting points $c_1, \ldots, c_4$. The present of event $[x\sigma]$ is indicated by the shaded area.

(or vice versa). By the location of x, $[x\sigma] = [x\eta]$, and hence $[x\sigma] \models Poss: At_{z'}A$ and $[x\sigma] \models Poss: At_{z'}\neg A$. Further each $[z\sigma]$ and $[z\eta]$ is consistent with $[x\sigma]$ ($= [x\eta]$) and $[z'\sigma] < [z\sigma]$ and $[z'\eta] < [z\eta]$.

Ad. 2 By an argument analogous to the one given above, for every z such that $c_2 \not<_M z$, $[x\sigma] \in Future([z\sigma])$, from which the sought-for result follows.

Ad. 3 Immediate from (2) and (3) above. □

Note that the present of $e = [x\sigma]$ turns out to be a spatially extended and temporally thick collection of events. Its temporal thickness depends on the Lorentz interval of the (time-like) vector c_1c_2.

Four splitting points, layered in two SLR pairs. Consider an MBS with five histories, i.e., $\Sigma = \{\sigma, \eta, \tau, \nu, \gamma\}$, with $S_{\sigma\eta} = \{c_1\}$, $S_{\sigma\tau} = \{c_2\}$, $S_{\sigma\nu} = \{c_3\}$, and $S_{\sigma\gamma} = \{c_4\}$. Pick an event $[x\sigma]$, with x located "between" two pairs of splitting points, $\langle c_1, c_2\rangle$ and $\langle c_3, c_4\rangle$, each pair being space-like related and each element of the top pair lying above each element of the bottom pair—see Figure 3. That is, $\forall_{i=1,2}\forall_{k=3,4}\ c_k <_M c_i$ and ($c_3 <_M e$ or $c_4 <_M e$) and ($c_1 \not<_M x$ and $c_2 \not<_M x$). Applying our Definition 2, we get this result:

$$\text{Present}([x\sigma]) = \{[y\sigma] \mid (y >_M c_3 \vee y >_M c_4) \wedge (y \not>_M c_1 \wedge y \not>_M c_2)\}.$$

Thus, the present of $[x\sigma]$ turns out to have a shape of a thick letter W.

Extreme cases: no point / every point is a splitting point. Consider an MBS with history $\{[x\sigma] \mid x \in \Re^4\}$ in which no point is a splitting point, i.e., $\forall x \in \Re^4\ \forall \rho \in \Sigma\ x \notin S_{\sigma\rho}$. By Definition 3 of proper property attribution, the MBS considered consists of exactly one history, that is $\Sigma = \{\sigma\}$. This is global determinism. Then for every event, its past as well as its future are empty, from which it follows that for every event, its present is the entire history. We thus have a block universe, indeed.

At the other extreme, if a history splits at every point with some other history, that is, if $\forall x \in \Re^4\ \exists\ \eta \in \Sigma\ x \in S_{\sigma\eta}$ for some $\sigma \in \Sigma$, then for every event $[x\sigma]$, its present consists merely of the event itself.

3 Discussion

Taking as a guide an (alleged) link between tenses and modalities, we defined a frame-independent notion of "the present of an event". The presents of events can be extended as well as point-like. Importantly, the underlying relation of co-presence is transitive. The shape of the present of event e occurring in history h depends on the splitting points of h, which are locations of chancy events in h. These depend in turn on the localization of qualitative differences between h and other histories. The definition allows for a non-extended present as well as the global cosmic present, that, the entire universe.

On this construal, the present of e is a set of events, that is, it is an event-like concept. It is not a location-like concept; consequently, one cannot ask in this framework what the present of a given spatiotemporal location is? Observe that two different events sharing the same location must have different presents, since they must belong to alternative histories. Further these two presents, that is, different sets of events, may have different locations, as chancy events in these alternative histories may have different spatiotemporal locations.

Finally, according to our definition, there is a full separation of tenses and causal notions. It might happen that e is in the causal past of f (i.e., within the past light-cone of f), but belong to the present of f. In the other direction, f might lie outside the causal future of e (i.e., e SLR f), but nevertheless belong to the future of e. This is the price to be paid for not requiring in Definition 2 that the future of e is (causally) after e. The definition also allows that for some two events, each belongs to the future of the other, which further entails that for each event of this kind, its future overlaps with its past.

There is a straightforward remedy that prohibits this controversial consequence and removes the separation of tenses and causal notions in one direction: strengthen the clauses 1 and 2 of Definition 2, with the following result:

Definition 5 *An event f belongs to the future of event e, $f \in Future(e)$, iff there is event e' and an atomic formula A such that*

1. $e < e' \leqslant f$ *and*
2. $e \models Poss : At_{loc(e')} : A$ *and*
3. $e \models Poss : At_{loc(e')} : \neg A$.

Event p belongs to the past of event e, $p \in Past(e)$, iff event e belongs to the future of p.
Event e' belongs to the present of event e, $e' \in Present(e)$, iff there is a history h such that $e, e' \in h$ and e' belongs neither to the past nor to the future of e.

As a result, every event in the future of e will be the causally after e, and every event in the past of e will be causally before e. In general, this change of Definition 2 will result in smaller futures and smaller pasts, but larger presents. In particular, in a Minkowskian Branching Structure, if there are splitting points arbitrarily close below and arbitrarily close above a given event e, the future of e is the union of alternative possible future light-cones of e, its past—the past light-cone of e, and its present in a given history containing e—the set of events that belong to this history and are space-like related to e.

Despite the altered definition, some separation of tenses and causal notions remains. To use Aristotle's sea battle, suppose that the two admirals have already brought their hostile fleets near our harbor and have decided to have a battle tomorrow.[18] Suppose further that with their decision, and all the circumstances, no matter what, the battle must happen. And not only this: assume as well that there is not a single trace of chanciness between our present thinking and the battle. No quantum decay, no agent's differing to do this rather than that. If these conditions are satisfied, the battle is *now*; it belongs to the present of your reading these words now. But obviously the battle is *tomorrow*, that is, in one day: by the definition of tense operators (as well as by common sense) we say: "There *will* be the sea battle tomorrow"and yet it is also now, presenting us with a clear paradox.

Let us finally reflect on where we arrived. We elaborated on an Aristotelian tradition of associating future with contingency, choosing for our analysis what seems to be a very weak link between these notions. We then used a rigorous framework of BST and MBS's to write down our definitions, and to see what the past, the present, and the future look like according to these definitions, if the underlying spacetime is Minkowski.[19] But then, nicely, we hit upon a paradox. Can we tame it or explain it away? Or is this paradox a *reductio* of the idea of associating future with contingency? We leave it for the reader to decide.

Notes

[1] I would like to thank the audience at the ESF workshop "Physical and Philosophical Perspectives on Probability, Explanation and Time" and the audience of my lunch talk at the Center for Philosophy of Science of the University of Pittsburgh in February 2010. The paper also owes much to the discussions I had with Jacek Wawer and Leszek Wroński. The MNiSW research grant 668/N-RNP-ESF/2010/0 is gratefully acknowledged.

[2] Cf. Malament (1977), Dieks (1988), Stein (1991), van Benthem (1991), or Rakić (1997). For a present assessment of Putnam's argument, cf. Dorato (2008).

[3] For an example, see Savitt (2000) or Dieks (2006).

[4] The construction developed here is in some aspects similar to that of Müller's (2006) and to a model I gave at *Logica* 2002 (unpublished). Some ideas presented here were born in discussions I had with T. Müller in the years 2002-2004. I am very grateful to him for sharing his insights with me. It seems to me that Fred Muller once held similar views on loci for the past, present, and future.

[5] Cf. *Cael* I.12: "No capacity relates to being in the past, but always to being in the present or future."

[6] See e.g., Gale (1963), as it is an attempt to rebut Whitrow's and Eddington's arguments.

[7] I am indebted to D. Dieks, K. Kishida, and J. Wawer for clarifying the distinction between a causal ordering and past, present, and future, and for the perception that tense operators ($Will$, Was) are defined in terms of the causal ordering, and separated from notions of past, present, and future, as here analyzed.

[8] The word 'iff' abbreviates 'if and only if'.

[9] This problem was brought to my attention by Bryan Roberts.

[10] BT theory was suggested in S. Kripke's letter to A. N. Prior (dated September 3, 1958, unpublished), discussed then briefly in Prior (1967) and worked out in Thomason (1970).

[11] For the postulates, and more information on BST models, see Belnap (1992).

[12] As exemplified for instance in saying "It is φ at $x \in \Re^4$, but it could be ψ there".

[13] The background of the requirement is the density of $\leqslant$, which is a BST postulate.

[14] We follow here the construction of Belnap and Placek (2010), which is more "physical" than the others, since it derives BST structures from property attributions. Apart from the proper property attributions, these authors assume a topological postulate and a condition on chains of splitting points.

[15] To avoid eyestrain, we write $x\sigma$ rather than $\langle x, \sigma \rangle$.

[16] The postulate is: every convergent sequence in a set $S_{\sigma\eta}$ is convergent to an element of $S_{\sigma\eta}$—cf. Belnap and Placek (2010).

[17] For the proofs we refer the reader to Belnap and Placek (2010), or Müller (2002) and Wroński and Placek (2009).

[18] I owe to J. Bogen the perception of how acute the separation between $Will$ and the future is.

[19] As pointed out by J. Norton, physics neither exhibits branching structures similar to those of BST, nor splitting points, nor particular patterns of branching scenarios, the single exception being perhaps quantum measurement, but (ironically) this we hardly understand. Accordingly, BST might be not adequate for analyzing time in our physical world—the objection goes. But, in the present approach we *derive* the axioms of BST from the requirement that the attribution of properties to spatiotemporal points be *proper*, which is a week and intuitive requirement. (For the details, of the derivation, cf. Belnap and Placek (2010).) Perhaps physics has not yet grasped our everyday modal notion of indeterminism, which means that an event may happen, but not necessarily. (This notion is different from Laplacian indeterminism, elaborated by Montague (1962) and Lewis (1983)—for more on this, see our paper cited above.) As we said in the introduction, we bet that our world is indeterministic in the modal sense, and that physics will come to terms with it. A similar worry is that our analysis cannot be extended to general relativity, since BST axioms are incompatible with some solutions of this theory. Although we do not know how resolve this problem generally, some initial results in this direction are reported in Placek (2009).

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