The Kantian Framework of Complementarity*[†]

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In the literature on the Copenhagen interpretation of quantum mechanics, not enough attention has been directed to the similarities between Bohr's views on quantum mechanics and Kant's theoretical philosophy. Too often, the connection is either ignored, downplayed, or denied outright. This has, as far as a proper understanding of Bohr's views is concerned, been detrimental, for it has contributed to the common misconception of Bohr as either a positivist or a pragmatist thinker.[1] In recent years, however, there has

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[1] Baggot (2004), for example, does not mention Kant at all in relation to Bohr. Folse

been a growing number of commentators attentive enough to note the important affinities in the views of these two thinkers (for instance, Honner 1982, MacKinnon 1982, Shimony 1983, Kaiser 1992, Chevalley 1994, Faye 2008). All of these commentators are, I believe, correct; however the picture they present to us of the connections between Bohr and Kant is one that is painted in broad strokes. It is open to the criticism that these affinities are merely superficial.

The contribution that I intend to make in this essay, therefore, is to provide a closer, structural, analysis of both Bohr's and Kant's views that makes these connections more explicit. In particular, I will demonstrate the similarities between Bohr's argument, on the one hand, that neither the wave nor the particle description of atomic phenomena pick out an object in the ordinary sense of the word, and Kant's requirement, on the other hand, that both 'mathematical' (having to do with magnitude) and 'dynamical' (having to do with an object's interaction with other objects) principles must be applicable to appearances in order for us to determine them as objects of experience. I will argue that Bohr's 'Complementarity interpretation' of quantum mechanics, which views atomic objects as idealizations, and which licenses the repeal of the principle of causality for the domain of atomic physics, is perfectly compatible with, and indeed follows naturally from a broadly Kantian epistemological framework.

There are exegetical difficulties with respect to both Bohr and Kant. Their writings are dense and are considered to be obscure by many. Interpreting Kant has become something of an industry in philosophy. As for Bohr, J.S. Bell writes of him: "While imagining that I understand the position of Einstein ... I have very little understanding of the position of his principal opponent, Bohr." (2004 [1981], p. 155). Abner Shimony writes: "I must confess that after 25 years of attentive—and even reverent—reading of Bohr, I have not found a consistent and comprehensive framework for the interpretation of quantum mechanics." (1985, p. 109). I do not pretend to have succeeded, where these and other eminent physicists and philosophers have failed, in resolving all of the problems that go along with giving a comprehensive and consistent interpretation of Bohr's philosophical position. Bohr is known to have thought highly of the Pragmatist philosophy of

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(1985), on the other hand, flatly denies any similarities whatsoever that are not merely superficial. Baggot and Folse both view Bohr as a pragmatist. For examples of positivist construals of Bohr, see: Popper (1982), Bunge (1955a,b).

William James, and Bohr's philosophy represents, in all likelihood, a combination of Jamesian and Kantian strands (although even this is likely an oversimplification). In this essay it is the Kantian aspects of Bohr's views that I will focus on; I do not, however, believe this is the whole story.[2]

Understanding the Kantian aspects of Bohr's thought is important because, although Bohr's and Kant's philosophies do diverge ultimately, they nevertheless share (as I will argue) a common epistemological framework. Any interpretation of Bohr should, therefore, *start* with Kant. Further, comparing Kant and Bohr is also invaluable for our interpretation of Kant. By asking the question 'how can a Kantian make sense of quantum mechanics?', one gains valuable insight into the implications of the principles of quantum mechanics for Kantian philosophy—in particular, what the uncertainty relations, if accepted, entail for the applicability of Kant's principle of cause and effect.

The essay is structured as follows: section 1 is devoted to a discussion of Kant's characterization of objective cognition. In section 2, I review the history of quantum theory up to Heisenberg's development of the Uncertainty Principle. In section 3, I give an analysis of Bohr's arguments for Complementarity, and discuss the Kantian aspects of Complementarity in section 4. Finally, in section 5, I deal with some possible objections to my interpretation of Bohr. For those already familiar with Kantian philosophy and/or the early history of quantum mechanics, sections 1 and 2 may be skimmed over (or skipped) without disastrous results, and referred to as needed for clarification of the exposition that follows in sections 3 and 4.

1 The Kantian Framework

For Kant, there are two aspects to experience: on the one hand, there are *intuitions*. We subsume these intuitions, on the other hand, under *concepts*. Intuitions are mediated by *sensibility*: our mind's capacity to be affected by objects (CPR, A19/B33). The effect, on our sensibility, of some object is the *sensation* of that object, and *empirical intuition* is that aspect of an intuition that is associated with this sensation.

An *appearance* is "The undetermined object of an empirical intuition" (CPR, A20/B34) (for example, consider a person in a dark room who sees a shape against the far wall, but only after some scrutiny determines that

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[2] For more on Bohr and William James, see, e.g, Folse (1985, p. 49-51).

shape to be a chair. Before determining it to be a chair, the person is puzzled as to what it is: we can say that the person views it merely as the appearance of something indeterminate).[3] There are two aspects to an appearance. First, there is its *matter*: what we sense. Second, there is its *form*. This is how the matter is *related*, both to itself and to the subject. The two forms of appearances are: space, for outer appearances, and time, for both inner and outer appearances. As forms of appearances, they are the *formal conditions* for appearances; they are a priori (in a logical, not a temporal, sense), i.e., they are the necessary relations according to which sensations must be ordered in our mind (CPR, A20/B34).

So much for intuition. *Concepts of the understanding*, now, correspond to rules for synthesizing the manifold of intuition. For example, an empirical concept (e.g., a horse) corresponds to a rule according to which this bushy tail, that long nose, that mane, and those hoofs can be associated in one representation. When we synthesize, i.e., combine, some particular manifold of intuition according to the particular rule for a concept, we say that this manifold of intuition has been subsumed under the concept. Now a *pure* concept of the understanding (a 'category') is one of a set of meta-concepts that all empirical concepts necessarily presuppose. Like the pure forms of intuition, these categories are a priori.[4]

Associated with the categories are formal principles for their application to possible experience. Among these, Kant distinguishes between *mathematical* and *dynamical* principles for the possibility of experience (CPR, B198-B294).[5] The former are *constitutive for appearances*. They are necessary principles for the possibility of presenting an appearance to ourselves as *existing*. These say that in order for anything to appear to us, it must be apprehended as having, determinately, both an extensive (length, breadth, etc.) and an intensive magnitude (i.e., a degree). But that something appears to us as existing is, by itself, not enough to determine this something as an *object*. To determine this appearance as an object, we must apply the dynamical principles to it. The dynamical principles are not constitutive

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[3] For a more thorough discussion of this point, see Harper (1984, p. 110-111).

[4] For a list, CPR, A80/B106.

[5] The mathematical principles are the *Axioms of Intuition* and *Anticipations of Perception*; the dynamical principles are the *Analogies of experience* and the *Postulates of empirical thought as such*. As the Postulates do not have a direct bearing on our discussion, I will leave them aside here and focus exclusively on the Analogies.

but *regulative*.[6] They are principles, not for the apprehension, but for the *connection* of appearances in time; they presuppose that an appearance has already been apprehended in accordance with the mathematical principles. These dynamical principles state, first, that all change presupposes something permanent; second, that all change must occur according to the law of cause and effect; third, that all substances that are perceived as simultaneous are in mutual interaction. To determine an appearance as an object of a possible experience, therefore, we require that at a determinate instant in time, it has a determinate position in space (determined by the mathematical principles) and that there is a law (subject to the dynamical principles) by which it dynamically interacts with other objects.

In particular, the principle of causality tells us, according to Kant, that in order to cognize change in some object, there must be a rule by which we objectively associate our perceptions of the object through time; i.e., some objective ordering of our perceptions such that the state of the object at some moment in time is presented as being in a determinate relation with the states of the object at other times. To illustrate: suppose I lean against a fence at the bank of a river, and watch a log as it is carried downstream by the current.[7] At time t_1, I watch as it comes into view from around the bend in the river some yards upstream. I then daydream for a while. Eventually, I notice (t_2) that the log has travelled some distance from the place where I first spotted it. At t_3, I recall to myself the motion of the log down the river that I half-consciously observed while daydreaming, after which I continue to watch the log as it disappears into the forest (t_4). Later that afternoon, I recall that what aroused me from my daydream was a sparrow alighting on the log (t_5). If we list these representations in the order in which they are actually perceived, then this is a *subjective ordering*:

$$t_1, t_2, t_3, t_4, t_5$$

I can also give these perceptions an *objective ordering*, however, according to which the motion of the log must have actually proceeded in time:

$$t_1, t_3, t_5, t_2, t_4$$

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[6] This sense of regulative should not be confused with the sense that Kant uses with respect to the 'ideas of reason'. There the distinction is between that which is constitutive or regulative with respect to experience as a whole. Here, he uses regulative not in the context of experience in general, but in the context of particular objects of experience.

[7] This is a variation on Kant's example of the ship (*Cf.* CPR, B236-238).

To determine this objective ordering, I must take into account the position of the log in the river during each of my perceptions, as well as anything else that is relevant to the motion of the log. The particular rule of succession for the change of state of the log is something that can only be discovered empirically (e.g., by determining which way the river is flowing). However, *that there is* some rule to be discovered is what the principle of causality tells us. This is *a priori*, according to Kant.

Now, for Kant, the presentations of time and space are continuous, infinitely divisible, quantities (CPR, B211). No matter how small, every 'piece' of space or time always presupposes a possible further intuition of space or time within its boundaries. The principle of cause and effect, now, tells us that every series of perceptions has some objective ordering according to which it progresses in time. But since time is infinitely divisible, so is the progression of perceptions (CPR, B255). All *change* associated with a possible experience is continuous, therefore, and we can know this a priori, according to Kant.

Kant's views were highly influential among both scientists and philosophers, especially in the latter half of the 19^{th} century. In the 20^{th} century, however, the development of Special and General Relativity, and the development of Quantum Mechanics undermined Kant's views in the eyes of many. Relativity theory called into question the a priori status of space and time; Quantum Mechanics called into question the a priori status of both space and time and the principle of causality. It is to the latter theory that we now turn.

2 The Genesis of Quantum Mechanics

When certain substances are subjected to extremely high temperatures, they absorb energy and emit light (picture a blacksmith hammering metal in a forge; if the temperature is high enough, the metal becomes 'red hot' or 'white hot'). To study this phenomenon, 19^{th} century physicists conceived of the notion of a 'black body': a completely non-reflecting object capable of emitting radiation at an intensity directly proportional to the amount of energy it absorbs. In 1859-60, Kirchoff was able to show that the ratio of emitted to absorbed energy in these materials depended solely on frequency and temperature (and not on, e.g., the body's shape).

At the turn of the century, Planck hypothesized that the total energy of the system was distributed over a large collection of indistinguishable energy elements. The result was the now famous formula: $\varepsilon = h\nu$. That is, the energy, ε, in each element is equal to the constant of proportionality, h (Planck's constant), times the frequency, ν. His results implied that ε must be given in *integer multiples* of $h\nu$. This was unprecedented (in classical physics, physical quantities change continuously with time). Heisenberg speculates:

> ... he must soon have found that his formula looked as if the oscillator could only contain discrete quanta of energy—a result that was so different from anything known in classical physics that he certainly must have refused to believe it in the beginning. ... Planck must have realized at this time that his formula had touched the foundations of our description of nature ... (Heisenberg, 1959, p. 35).

The discovery of Planck's constant (the 'quantum of action') was key to the resolution of some other outstanding problems in physics at the time. In 1905, Einstein described light in terms of energy quanta. He later did the same for atoms and ions. In 1913, Bohr's theory of the atom was published, according to which electrons are confined to fixed orbits that depend on 'principal quantum numbers'. In his PhD thesis of 1924, de Broglie demonstrated the wave-particle duality of matter by relating Planck's relation $\varepsilon = h\nu$ with Einstein's mass-energy equivalence relation, $\varepsilon = mc^2$.

Schrödinger developed his famous wave function for the evolution of a quantum mechanical system in 1925. Its interpretation was the subject of some debate. On Schrödinger's view, the wave function represented a real disturbance in the electromagnetic field; elementary 'particles' were really just electromagnetic waves of different amplitudes, phases, and frequencies, which when superimposed, resulted in large, localized, waves (called 'packets') which gave the impression of particles moving through space and time. As Lorentz pointed out, however, wave packets persist only when they are large compared with their wavelength. When confined to small regions of space, wave packets, unlike elementary particles, disperse rapidly (*Cf.* Moore, 1992, pp. 214-217).

Heisenberg wrote his famous uncertainty paper in 1927, in which he argued for the in principle impossibility of precisely determining *both* the position and momentum of an elementary particle at any one time. Presupposing a particle interpretation of elementary objects, he presented the following thought experiment: imagine we wish to determine the position and momentum of an electron as it travels under a microscope. To determine position, we use high-frequency γ-rays, since the resolving power of the microscope is directly proportional to the frequency of the beam. Frequency, however, is directly proportional to energy. Now when a high energy photon (a light particle) collides with an electron, the electron is knocked off its path (the 'Compton effect'). But this means that making an accurate determination of the electron's position renders us incapable of accurately determining its momentum: to determine its momentum we require the position of the electron at two points along its path, but since the path has been altered by the first position determination, we cannot determine where the electron would have been had we not interfered with it.

We can avoid the Compton effect by using low frequency γ-rays. But recall that the resolving power of the microscope is directly proportional to the frequency of the beam. If we use low frequency photons to measure the *momentum* of the electron, then we lose the ability to measure its *position* accurately. Heisenberg showed, mathematically, that an *exact* determination of the position of the electron resulted in an *infinite* uncertainty in its velocity (and hence momentum), and vice versa (he also demonstrated an analogous uncertainty relationship between energy and time). In other words, the more certain we are of one parameter, the less certain we are of the other. In the limit, i.e., as the uncertainty in the determination of one parameter approaches 0, the uncertainty in the determination of the other parameter approaches infinity.

3 Complementarity

Bohr accepted the validity of the uncertainty relations, but disagreed with Heisenberg over their significance. For (the young) Heisenberg, the uncertainty relations represent an epistemic limitation on what we can know of some object; we presuppose, however, that in spite of this limitation, the object is perfectly determinate in itself—that it is a particle, in fact. For Bohr the significance of the uncertainty relations is deeper; it is not epistemic in this sense but rather *conceptual.* For Bohr the uncertainty relations express the fact that the fundamental 'classical concepts' which both the particle and wave description of elementary objects presuppose (spatiotemporal concepts, on the one hand, and dynamical concepts, on the other) are inapplicable in the atomic domain, and that therefore a definition of the object in terms of these parameters is precluded. Let us work our way towards this conclusion. In his Como paper, Bohr writes:

> ... [quantum theory's] essence may be expressed in the so-called quantum postulate, which attributes to any atomic process an essential discontinuity, or rather individuality, completely foreign to the classical theories and symbolised by Planck's quantum of action. (1928, p. 580).

Contrasted with the classical theories, here, is the irreducibly 'discrete' nature of atomic processes; the fact that, according to quantum theory, the observed state of an elementary object changes discontinuously with time. What this implies, Bohr goes on to say, is that in our observations of the results of experiments, the interaction with the 'agency of observation' (i.e., the experimental apparatus) is an ineliminable part of our description of phenomena (in his later writings, Bohr calls this an "essential wholeness." *Cf.* 1958b, p. 72).

That last step may seem like an inferential leap, but it is comprehensible in light of our earlier discussion of Kant. We, saw, with Kant, how the infinite divisibility of time implies that all change must be continuous. Bohr's argument tacitly makes use of this assumption. Thus, on the classical conception of nature, change is continuous. Yet the state transitions of elementary objects are irreducibly discontinuous. It follows from this that, from a classical point of view, something is 'missing' from our description. What is 'missing', according to Bohr, is a clean distinction between the experimental apparatus and the object of our investigations; the 'agency of observation' is, in some sense, *a part* of what we observe.

Is it the case, then, that quantum mechanical descriptions of phenomena are not objective? No, quantum mechanical descriptions of phenomena, like classical descriptions, are objective. However what is different is that for the classical (but not for the quantum) case it is always possible to determinately describe (and correct for) the *interaction* between apparatus and object. Suppose I wish to describe a fish swimming in the water below my motorboat. There are three components involved in my description: first, there is the apparatus (my eyes); second, there is the object (the fish); third, there is the interaction between my eyes and the fish. We describe this last component by means of light rays that reflect off the fish and travel through water, then air, and finally into my eyes. Now when I look at the fish, it appears displaced from its actual position in the water due to the refraction of the ray. However, in my description of the fish, I am able to describe the interaction between my eyes and the fish and I am able to compensate for this interaction in my description; I am, at least vaguely, aware of the laws for the refraction of light, and taking these into account, I am able to determine the actual position of the fish, as well as its movements, with reasonable certainty; I am able to distinguish the fish 'as it really is' (the object) from the fish as it appears (the phenomenon). But this is *not* possible for atomic phenomena. Although we must make some 'subject-object' distinction—some 'cut' in what we observe—it is an *arbitrary* cut—one in which the interaction between apparatus and object cannot be disentangled from our description of the object.

One might object that there is some arbitrariness to the cut we make in the classical realm as well; a geologist and an archaeologist, for instance, will have distinct objects of inquiry even though both observe the same physical stone. What is different is that in the classical case, as we correct for the interaction with the apparatus in our description of the object, we are constrained by the (according to Bohr) criteria for its independent reality: a precise location in space-time and a precise account of its interaction with other objects. In the quantum case, however, we cannot account for the interaction with the apparatus in a way that leaves the object with definite position/time and momentum/energy parameters. Our language nevertheless requires some distinction, so we arbitrarily impose one.

Bohr expresses all of the foregoing in the following concise paragraph:

> Now the quantum postulate implies that any observation of atomic phenomena will involve an interaction with the agency of observation not to be neglected. Accordingly, an independent reality in the ordinary physical sense can neither be ascribed to the phenomena nor to the agencies of observation. After all, the concept of observation is in so far arbitrary as it depends upon which objects are included in the system to be observed. Ultimately every observation can of course be reduced to our sense perceptions. The circumstance, however, that in interpreting observations use has always to be made of theoretical notions, entails that for every particular case it is a question of convenience at what point

> the concept of observation involving the quantum postulate with its inherent 'irrationality' is brought in. This situation has far-reaching consequences. On one hand, the definition of the state of a physical system, as ordinarily understood, claims the elimination of all external disturbances. But in that case, according to the quantum postulate, any observation will be impossible, and above all, the concepts of space and time lose their immediate sense. On the other hand, if in order to make observation possible we permit certain interactions with suitable agencies of measurement, not belonging to the system, an unambiguous definition of the state of the system is naturally no longer possible, ... (1928, p. 580).

Bohr must still explain exactly why the classical concepts are not applicable to elementary objects. He writes:

> The fundamental contrast between the quantum of action and the classical concepts is immediately apparent from the simple formulae which form the common foundation of the theory of light quanta and of the wave theory of material particles. If Planck's constant be denoted by h, as is well known,
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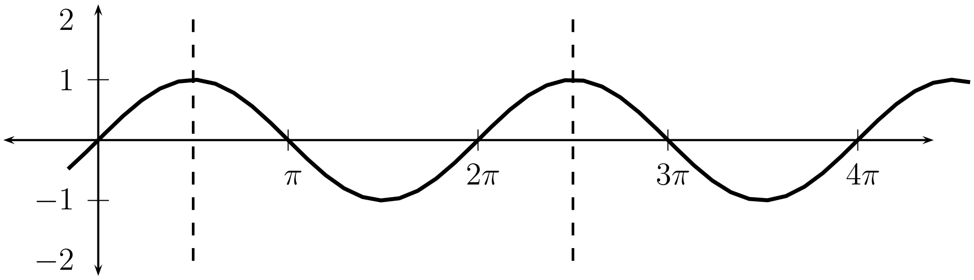
> $$E\tau = I\lambda = h, \quad . \quad . \quad . \quad (1)$$
>
> where E and I are energy and momentum respectively, τ and λ the corresponding period of vibration and wave-length. In these formulae the two notions of light and also of matter enter in sharp contrast. While energy and momentum are associated with the concept of particles, and hence may be characterised according to the classical point of view by definite space-time co-ordinates, the period of vibration and wave-length refer to a plane harmonic wave train of unlimited extent in space and time. (1928, p. 581).

In other words, in each case (i.e., for light and matter), Planck's constant relates two incompatible quantities. In the first relation, E (energy) is associated with the concept of a particle given with definite spatiotemporal coordinates, while τ (period of vibration) is associated with a wave-train 'of unlimited extent', not conceptualizable with respect to definite space-time coordinates. The case is the same for I and λ.



To illustrate the concept of a 'wave-train', consider the sine function. An individual wave (e.g., the one delimited by the two dashed lines in the figure) is a section of this function that stretches from one crest to the one immediately following it. The wave-train is made up of all the individual waves, extending infinitely in both directions along the x-axis. Clearly, it is not located *at* a particular point in space. Bohr's point is that it does not make sense to picture an object to ourselves that is, as the above relations express, *both* given at some definite spatiotemporal location *and* of unlimited extent in space and time. Nevertheless, physical theory does provide us with the resources we need to get around this difficulty, whether we assume a wave or a particle description of the object. The problem, as we shall see, is that neither description is precise.

For the case of the wave description, we can do this by using the superposition principle. Bohr writes:

> Only with the aid of the superposition principle does it become possible to obtain a connexion with the ordinary mode of description. Indeed, a limitation of the extent of the wave-fields in space and time can always be regarded as resulting from the interference of a group of elementary harmonic waves. (1928, p. 581).

A boat sailing over a smooth lake creates a wave disturbance behind it. A second boat, travelling alongside, also creates a disturbance. When the waves meet they intersect and constructively interfere. The result is one large, combined, wave. This is called superposition, and the combined wave is called a wave group. When enough waves are superimposed in just the right way, the resultant wave group can be very localized, spatiotemporally;

if it is so localized, then we call the group a wave *packet*, and we represent the velocity of the wave packet by its group velocity. This is what is behind Schrödinger's picture of a wave, manifesting particle-like properties, moving through space and time. However, although the superposition principle enables us to construct a description of an object in this way, it necessarily involves an element of indeterminacy with regard to that object.

> Rigorously speaking, a limited wave-field can only be obtained by the superposition of a manifold of elementary waves corresponding to all the values of ν and σ_x, σ_y, σ_z. But the order of magnitude of the mean difference between these values for two elementary waves in the group is given in the most favourable case by the condition
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> $$\Delta t \Delta \nu = \Delta x \Delta \sigma_x = \Delta y \Delta \sigma_y = \Delta z \Delta \sigma_z = 1 \qquad [1a]$$
>
> where $\Delta t, \Delta x, \Delta y, \Delta z$ denote the extension of the wave-field in time and in the direction of space corresponding to the co-ordinate axes. (Bohr, 1928, p. 581).

Here, ν refers to the frequency, and $\sigma_x, \sigma_y, \sigma_z$ refer to the wavenumbers for the elementary waves in the directions of the coordinate axes. Exactly how the waves constructively (or destructively) interfere depends, in part, on the wavenumbers/frequencies associated with the individual waves in the wave group. All else equal, the broader the range of wavenumbers in the group, the more spatially localized the resultant packet will be, and vice versa. This is what the expression (1a) is telling us; i.e.,

$$\begin{aligned} & \Delta x \Delta \sigma_x = 1 \\ \textit{implies}: \quad & \Delta \sigma_x = \tfrac{1}{\Delta x} \end{aligned}$$

It is the same for frequency and time. Now, according to the de Broglie relations, $E = \hbar\nu$, $I = \hbar\sigma$, where E and I are energy and momentum respectively, and $\hbar = \frac{h}{2\pi}$ is the reduced Planck's constant. If we multiply equation (1a) by $\hbar$, this gives us the uncertainty relations:

$$\Delta t \Delta E = \Delta x \Delta I_x = \Delta y \Delta I_y = \Delta z \Delta I_z = \hbar \qquad (2)$$

which give the upper bound on the accuracy of momentum/position determinations with respect to the wave-field.

Thus, as the wave-field associated with the object gets smaller —as we 'zoom in', so to speak, on its position and time coordinates—the possibility of precisely defining the energy and momentum associated with the object decreases in proportion. And the opposite is also true: in order to determine the object's momentum (or energy), we require a larger wave-field—we need to 'zoom out'—but this foregoes a precise determination of the object's position. 'Zooming in' and 'zooming out', however, are associated with different experimental arrangements. For the case of the γ-ray microscope, they are associated with the finite size of the microscope's aperture; the uncertainty in the position and momentum of the electron arises, not because of the interaction between two determinate entities (a photon and an electron), but rather because certain experimental arrangements, well-suited for precisely determining momentum, preclude *the definition* of the object in terms of continuously changing spatiotemporal coordinates, and vice versa.

> Indeed, a discontinuous change of energy and momentum during observation could not prevent us from ascribing accurate values to the space-time co-ordinates, as well as to the momentum-energy components before and after the process. The reciprocal uncertainty which always affects the values of these quantities is, as will be clear from the preceding analysis, essentially an outcome of the limited accuracy with which changes in energy and momentum can be defined, when the wave-fields used for the determination of the space-time co-ordinates of the particle are sufficiently small. (Bohr, 1928, p. 583).

Thus no one experimental setup allows for an *exact* definition of the object in terms of both quantities. One experiment can, at most, give us a picture of "unsharply defined individuals within finite space-time regions." (Bohr, 1928, p. 582).

Now let us see if we will have better luck if we begin, instead, with a particle description of the elementary object. Here, let us consider two different experimental arrangements, both variations of a 'one-slit' experiment, where we direct a photon at a thin diaphragm (a metal plate) into which an opening, or 'slit', has been made. On the other side of the diaphragm is a photographic plate which registers the light pattern that results. In one version of the experiment, designed to detect the particle's momentum, the diaphragm is not rigidly attached to the experimental apparatus, i.e., upon collision with the particle, the diaphragm will recoil slightly. When we direct the photon at the diaphragm, as it passes through the slit it will exchange momentum with the apparatus, which we can measure by the amount of recoil we observe in the diaphragm. However the recoil of the diaphragm gives rise to a corresponding uncertainty with regard to the position of the particle as it passes through the slit (the recoil of the diaphragm makes it impossible to precisely determine the location of the slit, and hence the particle, at the moment of impact):

> ... we lose, on account of the uncontrollable displacement of the diaphragm during each collision process with the test bodies, the knowledge of its position when the particle passed through the slit. (Bohr, 1935, p. 698).

On the other hand, suppose the diaphragm is rigidly fixed to the rest of the apparatus. In this case, as the photon passes through the slit, whatever momentum it exchanges with the diaphragm is completely absorbed by the apparatus—we thus lose the ability to make use of this momentum value in order to predict the location of the particle's impact on the photographic plate. Like the case for the wave picture, then, we have on our hands two experimental arrangements, one of which is compatible with a precise position determination; the other compatible with a precise momentum determination; however each of these *excludes* the other.

4 A Kantian View of Complementarity

Let us stop and reflect; consider the result of some experiment, say the mark on a photographic plate. The mark itself is a classical object. It has definite spatiotemporal coordinates, and it causally interacts in a definite way with its surroundings. However, *this* description of the phenomenon—of the mark *as a mark* on a photographic plate and nothing more—includes the photographic plate. To go further and describe the mark as a mark that has been left by some *independently existing object* that has interacted with the plate is what we desire to do, for this allows us to unify the marks resulting from different experiments as being different manifestations of the same independently existing object. Our goal is to 'get at' reality—the thing behind the phenomena—as it exists independently of the conditions of our experiments. We do this by eliminating the interaction between apparatus and object from our description of the latter.

Now from a Kantian perspective, in order to describe the object behind the phenomena as some independently existing[8] object of possible experience, we must ascribe, to the object, first: a determinate position, constrained by Kant's mathematical principles: the object must have a *definite* spatial extent and degree; second, a determinate momentum or 'quantity of motion', constrained by Kant's dynamical principles, telling us how the object interacts with its surroundings—in particular, how it *changes* through time.

To visualise the object, we make use, say, of the superposition principle. But by this means it is impossible to obtain *both* an exact position and an exact momentum determination (likewise for energy and time). It is possible to obtain an exact position determination, but in that case we completely forego a determination of the particle's momentum, and vice versa. We can, however, get something like a 'complete' object (i.e., one in which both causal and spatiotemporal parameters are present) by making our position and momentum determinations *inexact*—"unsharply defined". But in that case, although our description is objective, it is no longer the description of an object of possible experience (i.e., something physically real), for Kant—for in order for it to be physically real, we must assign determinate values to both parameters. Instead, the object is what Kant calls a noumenon, or abstract object.

To clarify: according to Kant, a concept of the understanding must be understood both in terms of its form and in terms of the content to which it can be applied. We can think of the form of a concept as analogous to a mathematical function, e.g., $f(x) = 2x + 4$. Now a determinate result can be obtained for this function only if something is filled in for x. By itself, the function only represents a form for the determination of a variable. Likewise for a concept: without *determinate* content, a concept gives us no *determinate* cognition. "Without [an object] it has no sense, and is entirely empty of content, even though it may still contain the logical function for making a concept out of whatever sort of *data* there are." (CPR, B298).

The concept of a noumenon is the concept of something *indeterminate*—analogous

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[8] I.e., in the sense of its being the same object in different experimental contexts. We can never abstract, on Kant's view, *completely* from the subjective conditions of observation (space and time), of course.

to x in the mathematical equation. The function above *cannot* be applied to x itself, but only to a value that has been filled in for x. Similarly for concepts: cognition of an object of possible experience requires that a concept be applied to a determinate, not indeterminate, intuition. A concept of some causal mechanism corresponds to a rule for the progression of perceptions *in time*, and the concepts of the understanding, in general, correspond to rules that must be applied to *our* sensible forms of intuition, space and time, which are always given determinately.

But now consider an elementary particle. According to the uncertainty relations, it is impossible *in principle* to describe the particle's momentum with any degree of precision without a corresponding loss of precision with regards to its spatiotemporal coordinates. It follows that in order to describe it using *both* position/time (spatiotemporal) and momentum/energy (dynamical) parameters, the *spatiotemporal* parameters associated with it must be made *indeterminate*. In fact, both the spatiotemporal and dynamical parameters must be made indeterminate, but it is the fact that the spatiotemporal parameters must be made indeterminate that is the key, for now, on a Kantian picture, the dynamical principles (whether or not we ascribe determinate dynamical parameters) are strictly speaking no longer applicable, for the dynamical principles always presuppose a *determinate* appearance in space and time apprehended in accordance with the mathematical principles. The upshot of all of this is that since there is no determinate spatiotemporal magnitude to apply the dynamical principles to, we cannot complete our description of the object according to the Kantian criteria for objects of possible experience. Therefore the 'object' corresponding to our description, on Kant's view, is not *physically* real.

Bohr reaches the same conclusion regarding the physical reality of our descriptions of elementary objects:

> ... a sentence like "we cannot know both the momentum and the position of an atomic object" raises at once questions as to the physical reality of two such attributes of the object, which can be answered only by referring to the conditions for the unambiguous use of space-time concepts, on the one hand, and dynamical conservation laws, on the other hand. (1949, p. 211).

The issue is not the existence of atomic objects as such (it is undeniable that something gives rise to the phenomena we observe), but whether our fundamental spatiotemporal and dynamical concepts are literally applicable to them. Evidently, according to both Bohr and Kant, they are not. And yet these 'ordinary' concepts, for Bohr, are also *necessary* concepts. The experimental apparatus (a voltmeter, say) is always a piece of classical equipment which communicates classical information about what we assume to be (using classical criteria) an independently existing object. The concept of observation itself, therefore, presupposes the classical concepts.

> Here, it must above all be recognized that, however far quantum effects transcend the scope of classical physical analysis, the account of the experimental arrangement and the record of the observations must always be expressed in common language supplemented with the terminology of classical physics. (Bohr, 1948, p. 313).

> The main point here is the distinction between the *objects* under investigation and the *measuring instruments* which serve to define, in classical terms, the conditions under which the phenomena appear. (Bohr, 1949, pp. 221-222).

We require the classical concepts, not only to observe, but also to communicate experimental results:

> ... the requirement of communicability of the circumstances and results of experiments implies that we can speak of well defined experiences only within the framework of ordinary concepts. (Bohr, 1937, p. 293).

The situation seems hopeless. We require the classical criteria in order to observe a physical object and to communicate the experience; yet, the classical criteria cannot fulfil their intended function in the atomic domain, for they mutually exclude each other. Ironically, it is the uncertainty relations that save us. They guarantee that we *can* nevertheless achieve a unified description by 'patching together' the mutually exclusive dynamical and spatiotemporal descriptions of the object under different experimental conditions. "The apparently incompatible sorts of information about the behaviour of the object under examination which we get by different experimental arrangements can clearly not be brought into connection with each other in the usual way, but may, as equally essential for an exhaustive account of all experience, be regarded as "complementary" to each other." (Bohr, 1937, p. 291). The uncertainty relations guarantee that a causal description can never contradict a spatiotemporal description—that the two can be used in a complementary way—for any experiment intended to *determinately* establish the object's spatiotemporal coordinates *can tell us nothing* about its dynamical parameters, and vice versa.

> the proper rôle of the indeterminacy relations consists in assuring quantitatively the logical compatibility of apparently contradictory laws which appear when we use two different experimental arrangements, of which only one permits an unambiguous use of the concept of position, while only the other permits the application of the concept of momentum ... (Bohr, 1937, p. 293).

We are not licensed, however, to take the next step and ascribe physical reality to this 'patched together' object of our descriptions, for the object is not real but abstract, and its classical attributes are idealizations.

> From the above considerations it should be clear that the whole situation in atomic physics deprives of all meaning such inherent attributes as the idealizations of classical physics would ascribe to the object. (Bohr, 1937, p. 293).

It is not too difficult to make sense of this from a Kantian point of view. Again, the concept of a noumenon is the key—this time in its positive signification as an idea, or concept of reason. Kant distinguishes two kinds of concepts: "Concepts of reason serve for **comprehension**, just as concepts of the understanding serve for **understanding** (of perceptions)." (CPR, A311/B367).

Concepts of reason, or ideas, have no validity with respect to the cognition of an object of possible experience—the cognition of such objects must always refer to determinate (sensible) conditions according to which they can be given to us in experience. Nevertheless, these concepts can be used regulatively, to *connect* the understanding's concepts—in our case, the various descriptions of phenomena (the 'marks') observed in the context of individual experiments—together in a coherent way in the context of our overall experience.

> All other pure concepts the critique relegates to the ideas, which are transcendent for our theoretical cognitive power, though that certainly does not make them useless or dispensable, since they serve as regulative principles: they serve, in part, to restrain the understanding's arrogant claims, namely, that (since it can state a priori the conditions for the possibility of all things it can cognize) it has thereby circumscribed the area within which all things in general are possible; in part, they serve to guide the understanding, in its contemplation of nature, by a principle of completeness—though the understanding cannot attain this completeness—and so further the final aim of all cognition. (CJ, p. 167-168).

The classical concepts, when they transcend possible experience, become ideas—they become the classical idealizations at the heart of the mechanistic conception of nature. (*Cf.* CJ, §§69-78). In the realm of atomic physics, however, these dynamical and spatiotemporal idealizations are incompatible; we cannot use them to describe a classical object. The uncertainty relations tell us that a precise determination of one type of parameter entirely excludes any determination whatsoever of the other type; therefore, they cannot be used to determine an object of possible experience, which requires a determination of both. But precisely because they say nothing about the objects of possible experience in this sense, they are compatible with the objects of possible experience—the results of our experiments—just so long as we understand that when we use these ideas in our description of nature it is only a manner of speaking; we may only speak 'as if' these ideas apply to our observations.

> We must be clear that, when it comes to atoms, language can be used only as poetry. The poet, too, is not nearly so concerned with describing facts as with creating images and establishing mental connections (Bohr, quoted in: Heisenberg 1971, p. 41).

Those familiar with Kant should immediately recognize the strategy being employed here. When confronted with other areas (biology and ethics, for instance) of human inquiry where the mechanistic conception (on his view) is either inadequate or inappropriate, Kant appeals to his doctrine of the antinomies to show that competing conceptions (e.g., freedom and determinism, mechanism and teleology) are merely ideas, and that they are compatible with each other if treated as such (*Cf.* CPR, B566-567, B586, CJ, §§69-78).

5 Concerns and Objections

One might object that a Kantian should not feel herself committed to anything like Complementarity, for one may opt to view the uncertainty relations as an expression of the temporary state of our ignorance with regard to elementary particles, and not as a final word. This is correct. A Kantian need not follow Bohr. However, if, as a Kantian, one does accept the uncertainty relations, then something like Complementarity must be the result—this is what it was my intention to show in this paper. Indeed, as I have shown, it is because one starts from within the Kantian framework that the motivation for Complementarity arises. It is unclear what Kant himself would have thought, but the following discussion of the mechanistic versus the teleological conceptions of nature may give us a clue.

> ... I *ought* always to *reflect* on these events and forms *in terms of the principle* of the mere mechanism of nature, and hence ought to investigate this principle as far as I can, because unless we presuppose it in our investigation [of nature] we can have no cognition of nature at all in the proper sense of the term. But none of this goes against the second maxim—that on certain occasions, in dealing with certain natural forms (and, on their prompting, even with all of nature), we should probe these and reflect on them in terms of a principle that differs entirely from an explanation in terms of the mechanism of nature ... (CJ, p. 387-388).

Although both the mechanistic and the teleological conceptions are thought of as 'complementary' ideas which guide our investigation of nature, priority is clearly given, nevertheless, to the mechanistic conception. The use of the teleological conception is reserved only for 'certain occasions' in which the mechanistic conception is either inapplicable (perhaps only temporarily) or inappropriate. It is likely that Kant would have been more conservative than Bohr, i.e., that he would not have accepted the uncertainty relations as final. In that case, one way to interpret Bohr's Complementarity doctrine is as an attempted refutation of what he took to be Kantian philosophy, with its overemphasis on the mechanistic conception of nature. Indeed, this is one way to reconcile Bohr's oft-cited criticisms of 'a priorism' (*Cf.* Folse 1985, pp. 217-221) with his insistence on the bedrock-like status of the classical concepts.

Potentially problematic for my reading, however, are statements like the following: "... no experience is definable without a logical frame and ... any apparent disharmony can be removed only by an appropriate widening of the conceptual framework." (Bohr, 1958b, p. 82), which lead Kaiser to write, of Bohr's view:

> ... there is also a very un-Kantian sentiment expressed in the end of Bohr's quotation: our formal frame might need to be *altered.* ... Kant viewed this formal frame, which includes the forms of intuition and the categories, as *a priori* and unalterable. Bohr followed a two-faculty format but he rejected *a priorism.* (Kaiser, 1992, pp. 222-223)

Yet Kaiser's interpretation is misleading, at best, for it seems to conflate Bohr's view with Heisenberg's. Heisenberg maintained that the gradual evolution of scientific concepts (or even the human species) would allow us to transcend our limitation to the classical concepts (*Cf.* Heisenberg 1959, p. 83, Heisenberg 1971, p. 124). This was not Bohr's view: "... it would be a misconception to believe that the difficulties of the atomic theory may be evaded by eventually replacing the concepts of classical physics by new conceptual forms." (1934, p. 16). And again:

> We must, in fact, realise that the unambiguous interpretation of any measurement must be essentially framed in terms of the classical physical theories, and we may say that in this sense the language of Newton and Maxwell will remain the language of physicists for all time. (1931, p. 692).

What Bohr means by 'widening', then, is not a fundamental alteration of our basic conceptual framework, but an imaginative use of our framework's own resources in order to extend its reach. "Indeed, the development of atomic physics has taught us how, *without leaving common language*, it is possible to create a framework sufficiently wide for an exhaustive description of new experience." (1958a, p. 88, emphasis mine).

Specifically related to Kant, Folse objects that Kant was a 'subjectivist' philosopher, while Bohr's intention was to provide an objective description of experience. Folse writes:

> These facts have given rise to the view held by some of the most perceptive of Bohr's interpreters that his position contains Kantian elements supporting a subjectivistic reading of complementarity. Since Bohr specifically stated complementarity provides an objective description of experience, it would seem that virtually any such reading would be contrary to his intent ... (Folse, 1985, p. 217).

But this misinterprets Kant. Kant's theoretical philosophy, as we have seen, revolves around the question of how to give an objective description of experience; thus he takes great pains, for instance, to distinguish the *objective* succession of appearances from the *subjective* one. If not ascribing to naïve realism amounts to being a subjectivist then Kant is guilty on all counts, however I do not think this is the type of subjectivism that Folse is referring to, for Bohr would be guilty of this charge as well. For Kant, possible experience is constrained by the forms of our intuition, space and time, and by the concepts by which we are able to combine these intuitions into one representation of an object. But this is no different from Bohr's insistence that we require classical concepts for the unambiguous description of experience.

Bohr was known to have admired the work of the American pragmatist William James, and this has been taken by Folse (Folse 1985, p. 49-51, Folse 1985, pp. 217-221) to tell against a Kantian influence on Bohr, for James was sharply critical of Kant. As Kaiser points out, however, James' criticisms of Kant are all directed at Kant's a priorism and not at the other aspects of his philosophy. This is perfectly compatible with a picture of Bohr as accepting certain aspects of Kant's philosophy while rejecting others. It is certainly not without precedent for one philosopher to be influenced by two rivals: Kant himself was strongly influenced by both Newton and Leibniz; their rivalry did not stop him from incorporating aspects of both of their views into his own.

Indeed, many philosophers have borrowed from Kant without making themselves into carbon copies. The Neo-Kantian philosopher, Ernst Cassirer, for instance, rejects the a priori status of Kant's classical concepts (1956 [1936], pp. 194-195) while still maintaining a broadly Kantian epistemology; the intuitionist mathematician L.E.J. Brouwer was strongly influenced by Kant—Brouwer, like Kant, founds arithmetic on the pure intuition of time—yet Brouwer rejects the pure intuition of space in light of the development of non-Euclidean geometry. Both Reichenbach and Carnap began their careers as Neo-Kantians before turning towards logical empiricism in light of the developments in geometry and logic (Friedman, 2000; Glymour & Eberhardt, 2008). Frege, throughout his career, though critical of Kant's views on arithmetic, nevertheless believed Kant to be correct for the case of geometry, even in the wake of the modern developments.[9] After mercilessly skewering most of his own contemporaries and predecessors, Frege writes, of Kant: "I have no wish to incur the reproach of picking petty quarrels with a genius to whom we must all look up with grateful awe; I feel bound, therefore, to call attention also to the extent of my agreement with him, which far exceeds any disagreement." (Frege, 1980, §89). Brouwer's arch-rival, Hilbert, was also influenced by Kant. Hilbert, in the epigraph to his *Foundations of Geometry*, quotes Kant: "All human knowledge begins with intuitions, thence passes to concepts and ends with ideas." (Hilbert, 1902). All of these thinkers incorporated parts of Kantian philosophy into their own. Bohr was a contemporary of all of these men; further, he had access to Kantian ideas through his lifelong friend and mentor, Harald Høffding, who was something of a Kant scholar. Consider Høffding's analysis of Kantian philosophy, in light of our discussion of Complementarity:

> Experience not only implies that we conceive something in space and time, but likewise that we are able to combine what is given in space and time in a definite way, i.e. as indicated in the concepts of magnitude and causality. This is the only means of distinguishing between experience and mere representation or imagination. All extensive and intensive changes must proceed continuously, i.e. through every possible degree of extension and intensity, otherwise we could never be certain of having any real experience. Gaps and breaks must be impossible (non datur hiatus non datur saltus). The origin of each particular phenomenon moreover must be conditioned by certain other phenomena, ... Wherever there appear to be gaps in the series of perceptions we assume that further investigation will discover the intervening members. This demonstration of the *validity of the categories* of magnitude and causality likewise involves a limitation: The validity of the categories can only be affirmed within the range of possible experience; they cannot be applied to things which from their very

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[9] *Cf.* Merrick (2006) for more on the relation between Kant and Frege.

> nature cannot become objects of experience. (Høffding, 1922, 147-148).

A last objection that I will address, before concluding, is with regards to the common misconception of Bohr as a positivist. This conception of Bohr has been popularised by, among others, Karl Popper and Mario Bunge. I will not spend much time answering it here. In addition to directing the interested reader to Don Howard's illuminating article (2004) on the subject, I will simply point out that this is a view that Bohr (as quoted by Heisenberg) explicitly denied: "Positivist insistence on conceptual clarity is, of course, something I fully endorse, but their prohibition of any discussion of the wider issues, simply because we lack clear-cut enough concepts in this realm, does not seem very useful to me—this same ban would prevent our understanding of quantum theory." (Heisenberg, 1971, p. 208).

One may, of course, ignore Bohr's own words here and presume to understand him better than he understood himself. If one were to make such a claim, it would not be objectionable as such; however, given the current, and widely acknowledged, dearth of understanding with respect to Bohr's views on quantum mechanics, such a presumption should be regarded as highly dubious.

In this paper I have highlighted the parallels between Bohr's doctrine of Complementarity and Kant's theoretical philosophy. We have seen how Bohr's principle of complementarity and Kant's theoretical philosophy are common in their approach: that both approaches are centred around what each thinker took to be the limits of objective experience. We have seen how, in order to transcend these limits, Bohr appealed to what a Kantian would call noumena in the positive sense, or ideas of reason. We have seen how a Kantian (who does not deny the validity of the uncertainty relations), starting from the principles of Kantian philosophy, would be led to many of the same conclusions as Bohr. Finally, we have seen how the objections to the link between the two thinkers rest on either a misinterpretation of Kant, or on a misrepresentation of Bohr, or both.

Complementarity is the natural outcome of a broadly Kantian epistemological framework and a Kantian approach to natural science, conjoined with Heisenberg's uncertainty relations. There is a very strong similarity in spirit, if not in technical detail, between Bohr's and Kant's approaches to natural science, and I hope to have inspired the conviction that the further examination of these similarities (and differences) will lead us to a better understanding of both of these men.

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