Problems of the De Broglie-Bohm Theory

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Abstract

It is shown that the de Broglie-Bohm theory has a potential problem concerning the mass and charge distributions of a quantum system such as an electron. According to the de Broglie-Bohm theory, the mass and charge of an electron are localized in a position where its Bohmian particle is. However, protective measurement indicates that they are not localized in one position but distributed throughout space, and the mass and charge density of the electron in each position is proportional to the modulus square of its wave function there.

The de Broglie-Bohm theory is an alternative to standard quantum mechanics initially proposed by de Broglie (1928) and later developed by Bohm (1952). According to the theory, a complete realistic description of a quantum system is provided by the configuration defined by the positions of its particles together with its wave function. The wave function follows the linear Schrödinger equation and never collapses. The particles, called Bohmian particles, are guided by the wave function, and their motion follows the so-called guiding equation. Although the de Broglie-Bohm theory is mathematically equivalent to quantum mechanics, there is no clear consensus with regard to its physical interpretation. For example, the interpretation of the wave function in the theory has been debated by its proponents. In this paper, we will mainly analyze the physical properties of Bohmian particles, and our analysis will show that the de Broglie-Bohm theory has a potential problem concerning the mass and charge distributions of a quantum system such as an electron.

Let's first see how the mass and charge of an electron distribute according to the de Broglie-Bohm theory. In the minimum formulation of the theory, which is usually called Bohmian mechanics (Goldstein 2009), the guiding equation for the Bohmian particle of a one-particle system with mass m and charge e in the presence of an external electromagnetic field is

$$m\frac{d\boldsymbol{x}}{dt} = \hbar\Im[\frac{\nabla\psi_t}{\psi_t}] - e\boldsymbol{A}(\boldsymbol{x},t), \tag{1}$$

where $\boldsymbol{x}$ is the position of the Bohmian particle, $\boldsymbol{A}(\boldsymbol{x},t)$ is the magnetic vector potential in position $\boldsymbol{x}$, and ψ_t is the wave function of the system that obeys

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the Schrödinger equation[1]. According to this guiding equation, the motion of a Bohmian particle is not only guided by the wave function, but also influenced by the external vector potential $\boldsymbol{A}(\boldsymbol{x},t)$. The existence of the term $e\boldsymbol{A}(\boldsymbol{x},t)$ in the guiding equation indicates that the Bohmian particle has charge e, the charge of the system, and the charge is localized in its position. Besides, the appearance of the mass of the system in the equation indicates that the Bohmian particle also has the (inertial) mass of the system. Therefore, according to Bohmian mechanics, the Bohmian particle of a one-particle system such as an electron has the mass and charge of the system. For example, in the ground state of a hydrogen atom, the Bohmian particle of the electron in the atom has the mass and charge of the electron, and it is at rest in a random position relative to the nucleus.

That the Bohmian particle of a one-particle system has the mass and charge of the system can be seen more clearly from the quantum potential formulation of the de Broglie-Bohm theory. By differentiating both sides of Eq. (1) relative to time and including an external gravitational potential V_G, we obtain

$$m\frac{d\dot{\boldsymbol{x}}}{dt} = -\nabla Q - m\nabla V_G - e[\nabla A_0 + \frac{\partial \boldsymbol{A}}{\partial t} - \dot{\boldsymbol{x}} \times (\nabla \times \boldsymbol{A})], \tag{2}$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla$, A_0 is the electric scalar potential, and $Q = -\frac{\hbar^2}{2m}\frac{\nabla^2|\psi_t|}{|\psi_t|}$ is the so-called quantum potential. The electromagnetic interaction term $-e[\nabla A_0 + \frac{\partial \boldsymbol{A}}{\partial t} - \dot{\boldsymbol{x}} \times (\nabla \times \boldsymbol{A})]$ indicates that the Bohmian particle has charge e, and the gravitational interaction term $-m\nabla V_G$ indicates that the Bohmian particle also has (passive gravitational) mass m. Moreover, the mass and charge of the Bohmian particle are localized in its position.

The question is whether the mass and charge of a one-particle system such as an electron really distribute only in one position as the de Broglie-Bohm theory assumes. In the following, we will show that protective measurement (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Aharonov, Anandan and Vaidman 1996; Vaidman 2009) gives a negative answer to this question[2].

Like the conventional impulse measurement, protective measurement also uses the standard measuring procedure, but with a weak, adiabatic coupling and an appropriate protection. Its general method is to let the measured system be in a nondegenerate eigenstate of the whole Hamiltonian using a suitable protective interaction (in some situations the protection is provided by the measured system itself), and then make the measurement adiabatically so that the state of the system neither changes nor becomes entangled with the measuring device appreciably. In this way, such protective measurements can measure the expectation values of observables on a single quantum system. Since the principle of protective measurements is based on the established parts of quantum mechanics and irrelevant to the controversial process of wavefunction collapse, their results as predicted by quantum mechanics are reliable and can be used

[1]This guiding equation applies only for spin 0 particles, and for spin 1/2 particles there is also a spin-dependent term (Holland and Philippidis 2003).

[2]Note that the earlier objections to the validity and meaning of protective measurements have been answered (Aharonov, Anandan and Vaidman 1996; Dass and Qureshi 1999). A unique exception is Uffink's (1999) objection. Although Vaidman (2009) regarded this objection as a misunderstanding, he gave no concrete rebuttal. Recently we have argued in detail that Uffink's objection is invalid due to several errors in his arguments (Gao 2011a).

to investigate the mass and charge distributions of a quantum system such as an electron.

According to protective measurement, the mass and charge density of a quantum system, as well as its wave function, can be measured as expectation values of certain observables. For example, a protective measurement of the flux of the electric field of a charged quantum system out of a certain region will yield the expectation value of its charge inside this region, namely the integral of its charge density over this region. Similarly, the mass density of a quantum system can also be measured by a protective measurement of the flux of its gravitational field in principle. Here we give a simple example. Consider a quantum system in a discrete nondegenerate energy eigenstate $\psi(x)$. Let the measured observable A_n be (normalized) projection operators on small spatial regions V_n having volume v_n:

$$A_n = \begin{cases} \frac{1}{v_n}, & \text{if } x \in V_n, \\ 0, & \text{if } x \notin V_n. \end{cases} \tag{3}$$

The protective measurement of A_n yields

$$\langle A_n \rangle = \frac{1}{v_n} \int_{V_n} |\psi(x)|^2 dv = |\psi_n|^2, \tag{4}$$

where $|\psi_n|^2$ is the average of the density $\rho(x) = |\psi(x)|^2$ over the small region V_n. Then when $v_n \to 0$ and after performing measurements in sufficiently many regions V_n we can measure $\rho(x)$ everywhere in space. When the observable A_n and the corresponding interaction Hamiltonian are physically realized by the electromagnetic or gravitational interaction between the measured system and the measuring device, what the above protective measurement measures is the charge or mass density of the quantum system, and its result indicates that the mass and charge density of the system in each position x is proportional to the modulus square of its wave function there, namely the density $\rho(x)$. In the Appendix, we give a concrete example to illustrate this important result (see also Gao 2011b).

If an electron indeed has the charge of an electron as usually thought, then the de Broglie-Bohm theory will be inconsistent with the above result of protective measurement. The guiding equation in the theory requires that the mass and charge of an electron are localized in a position where its Bohmian particle is. But protective measurement shows that they are not localized in one position but distributed throughout space, and the mass and charge density in each position is proportional to the modulus square of the wave function of the electron there.

It is also worth noting that although a Bohmian particle has mass and charge, the functions of these properties are not as complete as usual. For example, in Bohmian mechanics, a charged Bohmian particle responds not to external electric scalar potential, but only to external magnetic vector potential, and it has no gravitational mass but only inertial mass. This apparent abnormality is in want of a reasonable physical explanation. In addition, in the quantum potential formulation, although the Bohmian particles of a quantum system respond to external gravitational and electromagnetic potentials, they don't have gravitational and electromagnetic influences on other charged quantum systems, including their Bohmian particles. Moreover, the Bohmian particles of a quantum system do not have gravitational and electromagnetic interactions with each other. Therefore, the (gravitational) mass and charge of a Bohmian particle are always passive, i.e., a Bohmian particle is only a receptor of gravitational and electromagnetic interactions. This characteristic may also lead to some potential problems. For one, the nonreciprocal interactions will violate the conservation of energy and momentum (except that the Bohmian particles have no momentum and energy). At the worst, it may already suggest that the hypothetical Bohmian particles are redundant entities in the theory (and their role in solving the measurement problem is ad hoc), since they have no any influence on other entities in the theory such as the wave function. Note that these problems do not exist for the wave function; the evolution of the wave function of a charged quantum system is influenced by both electric scalar potential and magnetic vector potential, as well as by gravitational potential, and the wave functions of two charged quantum systems also have gravitational and electromagnetic interactions with each other.

In fact, there is a general argument against the Bohmian-particles explanation of the guiding equation imposed by the de Broglie-Bohm theory. The guiding equation is only a mathematical transformation of the relation between the density ρ and the flux density $\mathbf{j}$ for the wave function; the relation is $\mathbf{j} = \rho\mathbf{v}$, while the guiding equation is $\mathbf{v} = \mathbf{j}/\rho$. This is necessary if the theory gives the same predictions of measurement results as quantum mechanics. Since the wave function is not merely a probability amplitude for the predictions of measurement results, but also a realistic description of the physical state of a quantum system as assumed by most interpretations of the de Broglie-Bohm theory[3], the guiding equation already has a physical explanation relating only to the realistic wave function. Inasmuch as a fundamental mathematical equation in a physical theory has a unique physical explanation, the additional explanation of the guiding equation relating to the hypothetical Bohmian particles is probably improper. Note that this conclusion may not hold true if the guiding equation is not exactly the same as the above, e.g. the equation contains an additional stochastic damping term (Valentini and Westman 2005). Although such revised theories make different predictions from quantum mechanics, they may be consistent with existing experiments.

Lastly, we briefly discuss a possible solution to the above inconsistency problem of the de Broglie-Bohm theory. As noted earlier, the argument for the inconsistency between the de Broglie-Bohm theory and the results of protective measurement relies on the common-sense assumption that an electron indeed has the charge of an electron (and the mass of an electron). A possible way to avoid the inconsistency is to assume that an electron has twice the charge of an electron: one for its wave function and the other for its Bohmian particle. In this case, since what protective measurement measures is the mass and charge distributions relating to the wave function, not the masses and charges of the Bohmian particles, the above inconsistency can be avoided. However, this theory seems too clumsy and unnatural to be true. Moreover, it will introduce more problems. For one, there is a dilemma concerning the electromagnetic interaction between the wave function and the Bohmian particle of an electron. If they do have usual electromagnetic interaction, then the theory will be inconsis-

[3]This interpretation of the wave function is also supported by protective measurement. For a detailed analysis of the implications of protective measurement for the meaning of the wave function see Gao (2011b, 2011c).

tent with quantum mechanics and experiments. If they have no electromagnetic interaction, then this will add more problems. For instance, the manifestation of the charge of a Bohmian particle will be much stranger; it is not only passive but also selective. One needs to explain why the charged Bohmian particle of an electron responds not to the magnetic vector potential generated by the wave function of this electron, but to the magnetic vector potential generated by the wave function of another electron.

In conclusion, we have shown that the de Broglie-Bohm theory is inconsistent with the results of protective measurement concerning the mass and charge distributions of a quantum system. Although the inconsistency can be avoided by dropping a common-sense assumption, the revised theory is plagued by more problems.

Appendix: Protective measurement of the charge distribution of a quantum system

Consider the spatial wave function of a one-particle system with negative charge Q (e.g. $Q = -e$):

$$\psi(x,t) = a\psi_1(x,t) + b\psi_2(x,t), \tag{5}$$

where $\psi_1(x,t)$ and $\psi_2(x,t)$ are two normalized wave functions respectively localized in their ground states in two small identical boxes 1 and 2, and $|a|^2+|b|^2 = 1$. An electron, which initial state is a small localized wave packet, is shot along a straight line near box 1 and perpendicular to the line of separation between the boxes. The electron is detected on a screen after passing by box 1. Suppose the separation between the boxes is large enough so that a charge Q in box 2 has no observable influence on the electron. Then if the system were in box 2, namely $|a|^2 = 0$, the trajectory of the electron wave packet would be a straight line as indicated by position "0" in Fig.1. By contrast, if the system were in box 1, namely $|a|^2 = 1$, the trajectory of the electron wave packet would be deviated by the electric field of the system by a maximum amount as indicated by position "Q" in Fig.1.

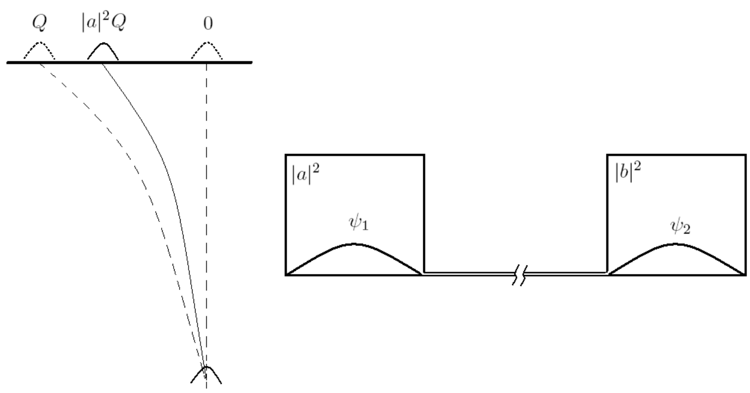

Fig.1 Scheme of a protective measurement of the charge density of a one-particle system

We first suppose that $\psi(x,t)$ is unprotected, then the wave function of the combined system after interaction will be

$$\psi(x,x',t) = a\varphi_1(x',t)\psi_1(x,t) + b\varphi_2(x',t)\psi_2(x,t), \tag{6}$$

where $\varphi_1(x',t)$ and $\varphi_2(x',t)$ are the wave functions of the electron influenced by the electric fields of the system in box 1 and box 2, respectively, the trajectory of $\varphi_1(x',t)$ is deviated by a maximum amount, and the trajectory of $\varphi_2(x',t)$ is not deviated and still a straight line. When the electron is detected on the screen, the above wave function will collapse to $\varphi_1(x',t)\psi_1(x,t)$ or $\varphi_2(x',t)\psi_2(x,t)$. As a result, the detected position of the electron will be either "Q" or "0" on the screen, indicating that the system is in box 1 or 2 *after* the detection. This is a conventional impulse measurement of the projection operator on the spatial region of box 1, denoted by A_1. A_1 has two eigenstates corresponding to the system being in box 1 and 2, respectively, and the corresponding eigenvalues are 1 and 0, respectively. Since the measurement is accomplished through the electrostatic interaction between two charges, the measured observable A_1, when multiplied by the charge Q, is actually the observable for the charge of the system in box 1, and its eigenvalues are Q and 0, corresponding to the charge Q being in box 1 and 2, respectively. Such a measurement cannot tell us the charge distribution of the system in each box *before* the measurement.

Now let's make a protective measurement of A_1. Since $\psi(x,t)$ is degenerate with its orthogonal state $\psi^{'}(x,t) = b^*\psi_1(x,t) - a^*\psi_2(x,t)$, we need an artificial protection procedure to remove the degeneracy, e.g. joining the two boxes with a long tube whose diameter is small compared to the size of the box[4]. By this protection $\psi(x,t)$ will be a nondegenerate energy eigenstate. The adiabaticity condition and the weakly interacting condition, which are required for a protective measurement, can be further satisfied when assuming that (1) the measuring time of the electron is long compared to $\hbar/\Delta E$, where ΔE is the smallest of the energy differences between $\psi(x,t)$ and the other energy eigenstates, and (2) at all times the potential energy of interaction between the electron and the system is small compared to ΔE. Then the measurement of A_1 by means of the electron trajectory is a protective measurement, and the trajectory of the electron is determined by the expectation value of the charge of the system in box 1. In particular, when the size of box 1 can be ignored compared with the separation between it and the electron wave packet, the wave function of the electron will obey the following Schrödinger equation:

$$i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t} = -\frac{\hbar^2}{2m_e}\nabla^2\psi(\vec{r},t) - k\frac{e\cdot|a|^2Q}{|\vec{r}-\vec{r_1}|}\psi(\vec{r},t), \tag{7}$$

where m_e is the mass of electron, k is the Coulomb constant, $\vec{r_1}$ is the position of the center of box 1, and $|a|^2Q$ is the expectation value of the charge Q in box 1. Correspondingly, the trajectory of the center of the electron wave packet, $\vec{r_c}(t)$, will satisfy the following equation by Ehrenfest's theorem:

$$m_e\frac{d^2\vec{r_c}}{dt^2} = -k\frac{e\cdot|a|^2Q}{|\vec{r_c}-\vec{r_1}|(\vec{r_c}-\vec{r_1})}. \tag{8}$$

[4]It is worth noting that the added protection procedure depends on the measured state, and different states need different protection procedures in general.

Then the electron wave packet will reach the position "$|a|^2Q$" between "0" and "Q" on the screen as denoted in Fig.1. This shows that the result of the protective measurement is the expectation value of the charge Q in the state $\psi_1(x,t)$ in box 1, namely the integral of the charge density $Q|\psi(x)|^2$ in the region of box 1.

The result of the above protective measurement can tell us the charge distribution of the system in each box *before* the measurement. Suppose we can continuously change the measured state from $|a|^2 = 0$ to $|a|^2 = 1$ (and adjust the protective interaction correspondingly). When $|a|^2 = 0$, the single electron will reach the position "0" of the screen one by one, and it is incontrovertible that no charge is in box 1. When $|a|^2 = 1$, the single electron will reach the position "Q" of the screen one by one, and it is also incontrovertible that there is a charge Q in box 1. Then when $|a|^2$ assumes a numerical value between 0 and 1 and the single electron reaches the position "$|a|^2Q$" between "0" and "Q" on the screen one by one, the result will similarly indicate that there is a charge $|a|^2Q$ in the box by continuity. The point is that the definite deviation of the trajectory of the electron will reflect that there exists a definite amount of charge in box 1.[5] Moreover, the above equation that determines the result of the protective measurement, namely Eq. (8), gives a more direct support for the existence of a charge $|a|^2Q$ in box 1. The r.h.s of Eq. (8) is the formula of the electric force between two charges located in different spatial regions. It is incontrovertible that e is the charge of the electron, and it exists in position $\vec{r}$. Then $|a|^2Q$ should be the other charge that exists in position $\vec{r_1}$. In other words, there exists a charge $|a|^2Q$ in box 1.

To sum up, protective measurement shows that the charge of a charged quantum system is distributed throughout space, and the charge density in each position is proportional to the modulus square of its wave function there. This conclusion is based on two established parts of quantum mechanics, namely the linear Schrödinger evolution (for microscopic systems) and the Born rule. In the above example, the linear Schrödinger evolution determines the deviation of the electron wave packet, and the Born rule is needed to obtain the information about the center of the electron wave packet detected on the screen.

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[5]Any physical measurement is necessarily based on certain interaction between the measured system and the measuring system. One basic form of interaction is the electrostatic interaction between two electric charges as in our example, and the existence of this interaction during a measurement, which is indicated by the deviation of the trajectory of the charged measuring system, means that the measured system also has the charge responsible for the interaction. Note that the arguments against the naive realism about operators and the eigenvalue realism in the quantum context are irrelevant here.

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