Bosonization and Iterative Relations Beyond Field Theories

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Solitons can be well described by the Lagrange formalism of effective field theories. But usually mass and coupling constants constitute phenomenological dimensions without any relation to the topological processes. This paper starts with a two-spinor Dirac equation in radial symmetry including vector Coulomb and scalar Lorentz potentials, and arrives after bosonization at the sine-Gordon equation. The keys of non-perturbative bosonization are in this case topological phase gradients (topological currents) that can be balanced in iterative processes providing for coupling constants driven by phase averaging and "noise reduction" in closed–loops and autoparametric resonance. A fundamental iterative spin–parity–asymmetry and dimensional shift quite near to the electron to proton mass ratio is found that can only be balanced by bosonization including Coulomb interaction.

Introduction. Solitons appear in almost all branches of physics, such as hydrodynamics, condensed matter phenomena, particle physics, plasma physics, nonlinear optics, low temperature physics, nuclear physics, biophysics and astrophysics. Solitons can be well described by the Lagrange formalism of effective field theories [1]. But usually the coupling constants are phenomenological variables without relations to the topological processes. In a previous paper it was indicated that sine–Gordon solitons (SG) [2] could be stabilized by a topological phase gradient, a field-induced shift in effective dimensionality [3]. This gradient defines topological currents that provide also for an iterative approach to topological phase coupling and coupling constants. In this paper those currents will be identified in a coupled two-spinor Dirac model including vector and scalar potentials and bosonized to a SG equation. The resulting potentials and soliton particle masses will be related to a fundamental soliton mass limit (includes mass shifts identical to the Fadeev-Bogomolny bound).

Bosonization. Similar to the work of Mandelstam and Coleman [4] we want to start with a proper Dirac fermion formalism and arrive after bosonization at the SG equation, the only non–trivial minimal quantum field theory in 1+1–dim. space–time which describe non–perturbative phenomena. With the help of the bosonization results of the massive Thirring model [5], we will start with a Dirac Hamiltonian that carries pairs of standard vector and scalar potentials. Non–perturbative bosonization will generate in the first step from two first–order Dirac ODE with fermion solutions one second order Klein–Gordon type relativistic wave equation with boson solutions. After defining topological currents in the second step we will arrive at the SG equation in accordance with exact relationships of the previous work, i.e. the fine structure iteration.

Radial symmetric Dirac equation. SG–solutions can represent a torque or spin–precession field often interpreted as continuous chain of coupled penduli initiated by spin–spin and spin–orbit coupling. This requires to compare at least two interacting states carrying spin.

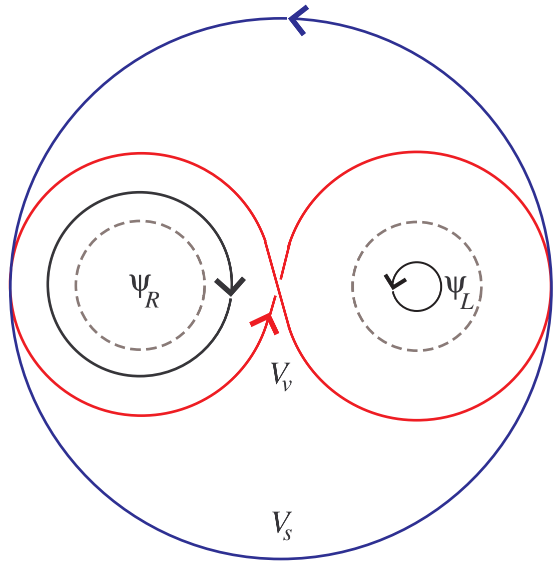

FIG. 1: Spin–asymmetry introduced by vector Coulomb (red) and scalar Lorentz (blue) currents $\psi_L/\psi_R = \alpha/\kappa$ with opposite parity states ψ_R and ψ_L and symmetry $U(1)_L \times U(1)_R$. With charge-parity symmetry both states carry opposite charges.

Consider the Dirac equation $\hat{H}_D\Psi = M_\gamma\Psi$ based on a Dirac - Hamiltonian $\hat{H}_D$ for a mixed potential consisting of a scalar potential $V_s(r)$ and a vector potential $V_v(r)$ given by

$$\hat{H}_D = c\hat{\alpha}\cdot\hat{p} + \hat{\gamma}_0\left[V_0 + V_s\right] + V_v\,, \tag{1}$$

where $\hat{\alpha}$ and $\hat{\gamma}_0$ are the usual Dirac matrices [6, 7]. The vector and scalar potentials will provide for spin–spin and spin–orbit coupling and will be necessary to bosonize the Dirac equation with two opposite parity two–spinors. In the same way the vector Coulomb potential part of V_v corresponds to the exchange of massless photons between a nucleus and leptons orbiting around it, the scalar Lorentz potential potential part of V_s with $-1/r$ characteristics corresponds to the exchange of massless scalar mesons. The resulting energy eigenvalue V_γ corresponds to opposite parity and polar two-spinors components with total mass V_0. The two-spinor wave–function in spherical symmetry is given by

$$\Psi \propto \begin{pmatrix} \psi_R(r) y^m_{jl_R} \\ i\psi_L(r) y^m_{jl_L} \end{pmatrix}, \tag{2}$$

where y^m_{jl} are the normalized spin-angular functions constructed by the addition of Pauli spinors to the spherical harmonics of order l. The spinors ψ_R and ψ_L are eigenfunctions with eigenvalues $l_R(l_R+1)$, and $l_L(l_L+1)$, respectively. A new parameter κ can be interpreted as orbital (spin) excitation between the two–spinor components, where $2\kappa = l_R(l_R+1) - l_L(l_L+1) > 0$ characterizes a left/right spin-asymmetry or difference, see fig.1. The inclusion of i with $\psi_L(r)$ is in order to make $\psi_L(r)$ and $\psi_R(r)$ real for bound-state solutions. Substituting this result back into the Dirac equation and performing some algebra [7], we arrive at two coupled first order ordinary differential equations

$$\begin{aligned} \frac{d\psi_R}{dr} + \frac{1+\kappa}{r}\psi_R &= \left[\frac{V_0 - V_\gamma + V_s - V_v}{\hbar c}\right]\psi_L, \\ \frac{d\psi_L}{dr} + \frac{1-\kappa}{r}\psi_L &= \left[\frac{V_0 + V_\gamma + V_s + V_v}{\hbar c}\right]\psi_R. \end{aligned} \tag{3}$$

The first step towards a bosonic solution can be established by the condition

$$\frac{\psi_R(r)}{\psi_L(r)} = \frac{\alpha}{\kappa} > 1\,, \tag{4}$$

this couples opposite parity radial functions, and provides for a wide variety of possible Lorentz scalar $V_s(r)$ and vector Coulombic potential $V_v(r)$ functions that have to obey the asymmetry relation

$$\frac{V_\gamma}{V_0} = \frac{V_v(r) + \frac{\hbar c\alpha}{r}}{V_s(r) + \frac{\hbar c\alpha}{r}} = \frac{\alpha^2 - \kappa^2}{\alpha^2 + \kappa^2}\,, \tag{5}$$

the asymmetry is shown in fig.1. To couple opposite parity components in eq.(3) via eq.(4) demands that both, the vector and scalar potentials in eq.(5) have to include Coulombic $-\hbar c\alpha/r$ and $-\hbar c\alpha/r$ terms, respectively. But finally those two components will merge after bosonization to one $-\hbar c\alpha/r$ potential and give the SG equation. The two first order ODE can now be combined to a Schrödinger/Klein–Gordon type relativistic wave equation that have bosonic solutions.

Towards solitons. SG–solitons correspond to a bosonic field theory, without eq.(4) the Dirac equation provides for fermionic solutions. What can we learn from a well established bosonization techniques? The Lagrangian of the massive Thirring model [5] has usually the form

$$\mathcal{L}_T = i\bar{\Psi}\gamma_\nu\partial^\nu\Psi - m_f\bar{\Psi}\Psi - \frac{1}{2}g(\bar{\Psi}\gamma^\nu\Psi)(\bar{\Psi}\gamma_\nu\Psi)\,, \tag{6}$$

where Ψ is Fermi field and γ^ν are Dirac matrices in (1 + 1) dimensions. Coleman [4] and Mandelstam have shown that the SG and the fundamental fermion of the massive Thirring model in (1+1) dimensions [5] are equivalent. The SG Lagrangian is usually given by

$$\mathcal{L}_{SG} = \frac{\mu}{2}\partial_\nu\theta\partial^\nu\theta \;- V(\theta)\,, \tag{7}$$

where V is the soliton potential and θ a field scalar. The stationary, time independent field equations simplify in one spatial dimension to

$$\partial_t V = 0, \quad \mu\partial_r^2\theta = \partial_\theta V, \quad V(\theta) = \frac{\mu}{2}(\partial_r\theta)^2\,, \tag{8}$$

with SG potential [2]

$$V(\theta) = \frac{\mu}{2\beta^2}[1 - \cos(\theta)], \tag{9}$$

where $2V_0 = \mu/\beta^2$. The standard form often corresponds to $\mu/2 = m$, $\beta\phi = \theta$. Both Lagrangians, eq.(7) and eq.(6) are equivalent if $4\pi/\beta^2 - 1 = g/\pi$. The correspondence in topological currents is established by the relation

$$-\frac{\epsilon^{\mu\nu}}{2\pi}\partial_\nu\phi = \bar{\Psi}\gamma^\nu\Psi \equiv j^\nu\,. \tag{10}$$

This means topological currents are given by phase gradients. The second bosonization relation relates mass to the trigonometric function

$$\frac{\mu}{2\beta^2}\cos(\beta\phi) = V_0\cos(\beta\phi) = -m_f\bar{\Psi}\Psi \tag{11}$$

in accordance with eq.(9). The two bosonization relations can now be used to find out the proper vector and scalar functions in eq.(3).

Topological SG currents and potentials. For SG–solitons in more than one spatial dimensions a radial topological current eq.(10) can be introduced according to

$$q(r) = \frac{\partial_r\theta}{r}, \quad \frac{2V}{\mu} = (\partial_r\theta)^2 = q(r)^2 r^2, \tag{12}$$

where $q(r)$ can be interpreted as a (fractional) radial dimension shift induced by vector and/or scalar currents [3], in this and previous papers it represents simply the electric charge (quantum). Based on these relations, an iterative and auto–parametric coupling process was found that can stabilize higher-dimensional partner solitons or pulsons by balancing the dissipative terms in [3]. For constant q eq.(12) immediately provides with eq.(8) for a harmonic oscillator coupling potential with proportionality between potential and phase

$$V(r) = \frac{\mu}{2}(qr)^2\,, \quad V(\theta) = \frac{\mu}{2}q\theta, \quad E_\mu = \mu c^2\,, \tag{13}$$

and unit condition

$$V_0 = V(r=1) = V(\theta = q) = \frac{\mu}{2} q^2 \,. \qquad (14)$$

Before we find the proper potentials in the Dirac Hamiltonian and construct the SG equivalent, it should be noted that both, the phase shift and potential shift V_0 are based on eq.(12)-eq.(14) and lead directly to an iterative condition for the spin/orbit (or fine–structure) coupling strength and will be shown later.

Charge and current density. Let the Dirac equation describe topological charge and current density, the 3-dim. case allows to introduce two orthogonal 2-dim. topological currents or phase gradients

$$\psi_L(r) = \frac{\partial_r \theta_L}{r}, \quad \psi_R(r) = \frac{\partial_r \theta_R}{r} \,. \qquad (15)$$

The path of the topological currents is given by dimensional shifts, the $SU(2) \to U(1)_L \times U(1)_R$ bosonization will describe these currents by combining eq.(15) and eq.(12) to

$$|\Psi|^2 = q(r)^2 = \psi_L(r)^2 + \psi_R(r)^2. \qquad (16)$$

The $1/r$-terms in eq.(3) carry dimensional information and can be interpreted as fractional dimension shifts, κ by spin–asymmetry (a fractional parity property) and α by vector potentials. These shifts could be algebraically assigned to topological currents, where the balancing condition is given by eq.(4) and can be approached with a proper definition of topological currents or topological phase evolutions θ_R and θ_L according to the massive Thirring bosonization. With eq.(15) both types of $1/r$–coupling, coulombic coupling and spin–asymmetry correspond to relative topological phase evolutions

$$\theta_L = \pi\kappa f(r), \quad \theta_R + \theta_0 = \pi\alpha f(r), \qquad (17)$$

where $f(r)$ will be a special function of r and θ_0 a phase offset. With eq.(17) and eq.(15) the balancing current becomes proportional to both, the rate of phase evolution and amplitude leading to eq.(4) that provides for exact analytical bosonic solutions, see also [8, 9].

Electromagnetic coupling and phase shift. For electrodynamic sources the gauge group is $U(1)$. It is convenient for our Dirac spinors to write the symmetry as $U(1)_L \times U(1)_R$. For both components single–valuedness requires an integer number M of wavelengths on an orbital loop with $U(1)$ symmetry and leads also to a topological definition of charge. This has the topology of a circular loop, on which the homotopy classes of closed curves are labelled by their winding or subloop numbers, and where the magnetic charge is quantized taking integral values [10]. This quantization can be compared in the classical sense to an orbital phase evolution of type "whispering gallery modes" (WGM) [11]. In electromagnetism the charges are multiples of a fundamental charge q with spin J, so that the wave-function transforms as

$$\psi \to e^{\pm iM\theta}\psi, \quad q = \frac{2\pi J}{M} \,, \qquad (18)$$

the unit charge corresponds to the phase sub-interval $[0, 2\pi J/M]$ and to a special topological phase evolution per loop in the interval T with loop frequency $\mu = 2\pi/T$ and M-gonal $U(1)$ symmetry. With eq.(8) and eq.(13) this periodic phase offset can be assigned to a unit potential shift, see eq.(13) and eq.(14). The topological potential shift $V(\theta) - V(q) = V(\theta_M)$ is assigned to a minimum topological phase shift (the fundamental charge q)

$$q(r) = q\psi(r) = q\frac{\partial_r f(r)}{r}, \; q = \theta - \theta_M \,, \qquad (19)$$

see eq.(14) with $V_0 = V(q)$.

Possible route to solitons. Now we can almost intuitively find the proper potentials and solutions to ψ. But the main task is to bosonize the first order ordinary differential equations eq.(3) to one SG equation with the help of eq.(12) and eq.(4). With eq.(15) the coupled equations read

$$\begin{aligned} \frac{d^2\theta_R}{dr^2} &= -\frac{\kappa}{r}\frac{d\theta_R}{dr} + \left[\frac{V_0 - V_\gamma + V_s - V_v}{\hbar c}\right]\frac{d\theta_L}{dr}, \qquad (20)\\ \frac{d^2\theta_L}{dr^2} &= +\frac{\kappa}{r}\frac{d\theta_L}{dr} + \left[\frac{V_0 + V_\gamma + V_s + V_v}{\hbar c}\right]\frac{d\theta_R}{dr}. \end{aligned}$$

The balancing bridge eq.(4) linearly couples phase gradients, where the phases can be coupled via $\alpha\theta_L = \kappa(\theta_R + \theta_0)$. With eq.(4) and eq.(5) it is sufficient to consider only one part, therefore we omit the index $\theta_R \to \beta\phi$

$$\frac{d^2\phi}{dr^2} = \frac{\kappa}{r}\frac{d\phi}{dr} + \left[\frac{V_0 + V_\gamma + V_s + V_v}{\hbar c}\right]\frac{\kappa}{\alpha}\frac{d\phi}{dr} \,. \qquad (21)$$

With eq.(5) the potentials can be written as

$$V_v = g(r)V_\gamma - \frac{\hbar c\alpha}{r}, \quad V_s = g(r)V_0 - \frac{\hbar c\alpha}{r}, \qquad (22)$$

where g is a real function. A solution to $g = 0$ can be found in [7]. The SG-formalism requires with eq.(8) and eq.(9)

$$\frac{d\phi}{dr} - 2\beta^{-1}\sin(\beta\phi/2) = 0 \,. \qquad (23)$$

Combining eq.(23) with eq.(22) and eq.(21) provides for

$$\frac{d^2\phi}{dr^2} + \frac{\kappa}{r}\frac{d\phi}{dr} - \left[(g+1)\frac{V_0}{\hbar c} + (g+1)\frac{V_\gamma}{\hbar c}\right]\frac{\kappa}{\alpha}2\beta^{-1}\sin(\beta\phi/2) = 0 \,. \qquad (24)$$

The "massless" case $g = -1$ provides for

$$\frac{d^2\phi}{dr^2} + \frac{\kappa}{r}\frac{d\phi}{dr} = 0 \,, \qquad (25)$$

with power-law radial phase solution $\phi \propto r^{1-\kappa}$. For

$$g(r) = \frac{\hbar c \alpha}{r(V_0 + V_\gamma)},\ V_v = g(r)V_0,\ V_s = g(r)V_\gamma, \quad (26)$$

the first order term in eq.(24) vanishes

$$\frac{d^2\phi}{dr^2} - \frac{(V_0 + V_\gamma)}{\hbar c}\frac{\kappa}{\alpha} 2\beta^{-1}\sin(\beta\phi/2) = 0\,. \quad (27)$$

The vector and scalar potentials necessary to bosonize the Dirac equation with two opposite parity two–spinors, merge with eq.(26) to one Coulomb-type potential

$$\begin{aligned} & V_0 + V_\gamma + V_s + V_v \\ &= (g+1)V_0 + (g+1)V_\gamma - \frac{2\hbar c\alpha}{r} \\ &= V_0 + V_\gamma - \frac{\hbar c \alpha}{r}. \end{aligned} \quad (28)$$

Eq.(23) directly leads to the SG equation

$$\frac{d^2\phi}{dr^2} - \beta^{-1}\sin(\beta\phi) = 0\,, \quad (29)$$

that can be written as

$$\frac{d^2\phi}{dr^2} - 2\beta^{-1}\cos(\beta\phi/2)\sin(\beta\phi/2) = 0\,. \quad (30)$$

Comparing eq.(30) with eq.(27) provides for the condition

$$\frac{(V_0 + V_\gamma)}{\hbar c}\frac{\kappa}{\alpha} = \cos(\beta\phi/2) = \frac{(V_0 - V_\gamma)}{\hbar c}\frac{\alpha}{\kappa}\,. \quad (31)$$

An additional trigonometric relation is given by

$$\frac{V_0^2 - V_\gamma^2}{V_0^2 + V_\gamma^2} = \tan^2(\beta\phi/2) = \frac{1 - \cos(\beta\phi)}{1 + \cos(\beta\phi)}\,, \quad (32)$$

where the SG energy can be related to the Dirac energies by

$$\frac{V(\theta)}{V_0} = 2\frac{V_0^2 - V_\gamma^2}{\hbar^2 c^2} = 1 - \cos(\beta\phi)\,. \quad (33)$$

Now we have arrived at the SG equation that provides for a compatibility condition between pseudosphere surfaces with constant negative curvature [1]. Symmetry arguments support our result. Driven by the topological phase gradient eq.(12) the local $2n$-dimensional oscillator potential eq.(13) can be mapped under $PSL(2, \mathbb{R})$ to a $n + 1$ dimensional Coulomb potential on pseudospheres [12]. This means, that local topological currents can be mapped to distant sinks/sources via vector Coulomb and scalar Lorentz potential, cylindric symmetry suggests $n = 1$.

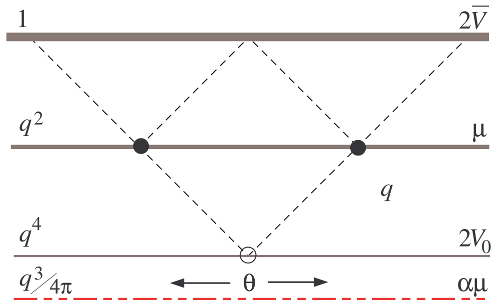

FIG. 2: Phase fluctuations and mean phase shift (linear horizontal axis) responsible for energy exchange between solitons mediated by (virtual) photons. The one-dimensional coupling strength (log. vertical axis) soliton–wave–soliton and soliton–waves shows an energy cascade $2V_0 = q^2 E_\mu = q^4 2\overline{V}$.

Background-soliton and soliton-soliton coupling. According to [13] it can be assumed, that solitons show phase fluctuations excited by background fluctuations, where the mean background radiation power energy is mediated by waves and coupled to massive solitons by a Compton–type permanent scattering process. The mean background potential for a spectral frequency c/λ_μ has the mean potential

$$2\overline{V} = q^{-2}E_\mu = q^{-2}\frac{hc}{\lambda_\mu}\,, \quad (34)$$

where soliton fluctuation amplitude of the soliton Compton wavelength λ_μ with respect to the scattering photon wavelength is considerably reduced by a factor q that can be obtained from a temporal phase averaging process providing for the potential reduction $q^{-2} = \overline{(\partial_t\theta)^2}$. In this context it was proposed that the energy reduction $q^{-2} = \beta^2$ is the Fadeev-Bogomolny bound [14, 15, 16], see fig.2. In [13] this coupling bound was obtained from an oscillator solution in 1-dim. auto-parametric resonance. The relative phase–fluctuations between two–solitons leads to soliton–soliton coupling since the fluctuations mediated by waves are reduced or averaged by another $q^2 = 2V_0/E_\mu$ factor, where q is the reduction of the phase amplitude, see eq.(13) and SG equation eq.(29). Starting with the background potential $\overline{V}$ the energy cascade leads to the soliton energy E_μ that reduces to the unit coupling energy V_0. The reduction can be simplified to

$$2V_0 = 2V(r=1) = q^2 E_\mu = 2q^4\overline{V}, \quad (35)$$

see fig.2. In Planck units ($r = c = \hbar = 1$) the cascade starts at the reference $2V_0 = 1$. This means, that soliton–soliton interaction scales with q^4.

Coulomb coupling. In this over–determined system the linear relationship between potential and phase eq.(14) provides for an iterative solution. Combining eq.(13) - eq.(19) the discrete orbital phase evolution will be constrained by a source term eq.(12) and soliton potential term according to eq.(9) and eq.(14)

$$1 - \cos(\theta_M) = \frac{V(\theta)}{V_0} = 1 + \frac{\theta_M}{q} \, . \tag{36}$$

V_0 can be assigned to a basic topological phase q. This leads quickly with eq.(36) and eq.(18) to the iterative condition for the optimum phase shift θ_M

$$\theta_M = \pi\alpha = \beta\phi \, , \tag{37}$$

$$M\theta_M = -2\pi J \cos\theta_M \, , \tag{38}$$

where α defined according to eq.(17) can be identified as a generalized fine–structure coupling constant, alternatively one could also change to the convention $\alpha \to \alpha/(2J)$. In 3d the coupling strength between half spin particles ($J = \frac{1}{2}$) provides with $1/q^2 = 12\pi^2$ and $M = [4\pi/q]$ for $M = 137$ [11]. The Sommerfeld fine structure constant can be well approached by $\alpha_{137} = 1/137.00360094...$ [17, 18]. In [19] θ_M was identified as an conic deficit angle responsible for the Aharonov–Bohm effect. This fits well to the interpretation, that the topological phase gradient or charge/current q provides for a dimensional shift.

Spin-asymmetry κ. The orbital phase shift induced by two iteratively coupling and orbiting solitons (orbital soliton–wave–soliton coupling) θ_M is different from the orbital phase shift $\theta = \pi\kappa$ characterizing the spin–asymmetry of 2 two–spinors. Consider a permanent Compton scattering process driven by a mean background radiation of strength $2\overline{V}$ and mean wavelength $\lambda_1 = 1$, where the scattered quanta with wavelenght shift $\triangle\lambda_\mu$ couple back to the scatterer, see fig.3. This provides also for a special mean scattering angle θ that should be in a constructive resonance with the excited radial modes where

$$1 - \cos(\theta) = \frac{V_r(\theta)}{\overline{V}} = \frac{V(\theta)}{2\pi\overline{V}} = f(\theta)\frac{\triangle\lambda_\mu}{2\pi\lambda_1} \, . \tag{39}$$

$\overline{V}$ replaces the relative soliton–soliton coupling energy $V_0 = V(q)$ in eq.(36). The linear relation between phase and radial potential $V_r(\theta)$ suggests an angular relation of the form $f(\theta) \propto \theta$, where $\theta = \pi$ shows the maximum wavelength shift and provides for $f(\theta) = \theta/\pi$. This provides with $\triangle\lambda_\mu(\theta = \pi) = 2\lambda_\mu$ or $\triangle\lambda_\mu(\theta = \pi)/\lambda_1 = 2q^2$ for an iterative condition for the optimum phase shift

$$1 - \cos(\theta) = \frac{\theta}{\pi}\frac{q^2}{\pi} = \frac{1}{N}, \quad \theta = \pi\kappa \, . \tag{40}$$

Alternatively, one could say that the linear relation between phase and radial potential $V_r(\theta)$ or orbital potential $V(\theta)$ with

$$V_r(\theta) = V(\theta/2\pi) = \frac{V(\theta)}{2\pi} = \frac{V(\pi)}{2\pi}\frac{\theta}{\pi} \, , \tag{41}$$

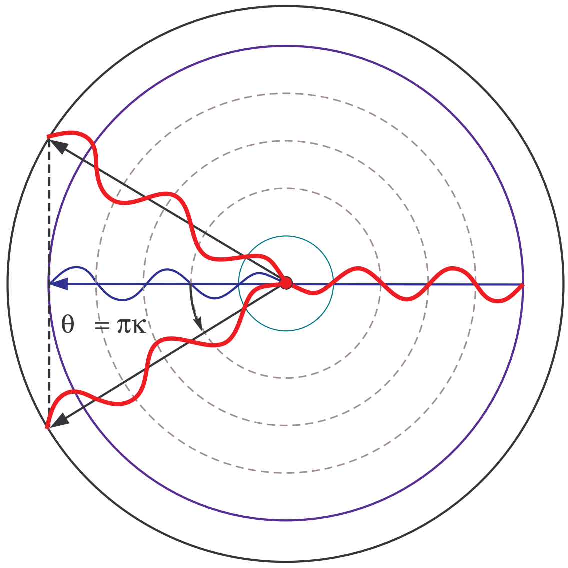

FIG. 3: The relative rotation and wavelength change due to spin–asymmetry can be assigned to a Compton type scattering angle $\pi\kappa$. Radial modes provide for a resonance condition, here $N = 6$.

leads quickly with $E_\mu = V(\pi)$, eq.(34), and eq.(39) to the iterative condition for the optimum phase shift eq.(40). $1/q^2 = 12\pi^2$ provides for the balancing phase shift in eq.(17) $\kappa = 1836.11766....$ If WGM are formed (where the epi/hypo–cycloidal round-trip path fits integer numbers of the wavelength), the number is given by $N = 683174$ with a shift $12\pi^3/\kappa/683174 - 1 = 9.7 \cdot 10^{-8}$. Integer N corresponds to both, orbital radial and standing waves. The value of κ controls the current based on spin–asymmetry.

Origin of the basic soliton energy. A re–scaling to human artificial units requires to apply the proper scaling relations. In [13] a scaling relation has been found that relates the unit values $q^2 E_\mu = \overline{V} = 1$ obtained with Planck units ($\hbar = c = \lambda_1 = 1$) with special reference distance $\lambda_1 = q^2\lambda_\mu = \hbar c/(2\overline{V})$ on one end to the familiar/measurement values based on the human artificial energy unit E_u provided by the system of units (SI) on the other end. The central role identifying the correct baryon mass scale is the shift of Planck velocity units to human artificial velocity, length, and arbitrary mass units. Planck velocity units demand that the light velocity equals the unit velocity $c = u = 1$, such that the mean background energy $\overline{V}$ scales with the square of the wave velocity and the SI unit energy scales with the square of the unit velocity u (in SI E_u =1J = 1kg m^2/s^2)

$$\frac{2\overline{V}}{E_u} = \frac{c^2}{u^2} = \Xi^2, \quad \Xi = 299792458\,. \tag{42}$$

Practical necessity motivates to choose a unit velocity $0 < u \ll c$ with $\Xi = c/u \gg 1$.

Particle and photon energies can be compared via Compton and photon wavelengths that refer to the light velocity. Planck length units demand that the 1-dimensional quantum energy of waves coupling to particles E_μ is inversely proportional to the wavelength, especially to the Compton wavelength with

$$\frac{E_\mu}{E_u} = \frac{\lambda_u}{\lambda_\mu}\,. \tag{43}$$

With Planck units we get $\lambda_1 = q^2\lambda_\mu = 1$, $q^{-2}E_\mu = 1$. As a result, the characteristic soliton wavelength of one-dimension coupling is with eq.(34), eq.(42), and eq.(43) exactly given by

$$\lambda_\mu = \frac{\lambda_u}{q^2\Xi^2} \approx 1,31777... \cdot 10^{-15}\mathrm{m}\,. \tag{44}$$

Eq.(44) provides for the basic soliton mass μ via Compton relation $\mu = h/(c\lambda_\mu) = q^2\Xi^2 h/(c\lambda_u)$. Realized in SI units the value is

$$\mu = \frac{\hbar}{c}\frac{\Xi^2}{6\pi\mathrm{m}} \approx 1.67724... \cdot 10^{-27}\mathrm{kg}\,. \tag{45}$$

The 3-dim. coupling constant. It is interesting to note, that the small synchronizing/coupling current in the standard SG formalism is given by $q_R\alpha = q_L\kappa$, compare eq.(4). The Gauss relation can connect the 1-d coupling energy E_{1d} with potential ϕ_{1d} to a 3-d coupling energy E_{3d} with a spherical symmetric potential $\phi_{3d}(r)$ such, that the radial coupling energy is defined by

$$E_{3d} = \frac{q}{\epsilon_0}\,\phi_{3d}, \quad E_{1d} = \frac{q}{\epsilon_0}\,\phi_{1d}\,, \tag{46}$$

with

$$E_{3d}(r) = -\frac{1}{\epsilon_0}\int_\infty^r \phi_{3d}^2 4\pi \frac{dr'}{\lambda_1} = \frac{q^2\lambda_1}{4\pi\epsilon_0 r},\ \phi_{3d} = \frac{q\lambda_1}{4\pi r}\,. \tag{47}$$

The fine structure constant can be defined by

$$\alpha = \frac{q^2}{4\pi\epsilon_0\hbar c} = \frac{E_{3d}(r=\lambda_\mu)}{E_{1d}(\lambda_\mu)} = \frac{\phi_{3d}(r=\lambda_\mu)}{\phi_{1d}(\lambda_\mu)}\,, \tag{48}$$

where the relations at the special reference distance $\lambda_1 = q^2\lambda_\mu = \hbar c/(2\overline{V})$ given by dimensionless Planck units $\hbar = c = \lambda_1 = 1$ must obey the unit condition

$$E_{1d}(\lambda_1) = 2\overline{V} = \phi_{1d}(\lambda_1) = 1\,, \tag{49}$$

with a length scale reference exactly given by $\lambda_1 = q^2\lambda_\mu = |c^{-2}|/(2\pi)$m. This provides for

$$\epsilon_0 = \frac{\lambda_1 q}{\hbar c} = \frac{q}{2\overline{V}}\,, \alpha = \phi_{3d}(\lambda_1) = \frac{q}{4\pi},\ M = \left[\frac{1}{\alpha}\right]\,, \tag{50}$$

where [ ] means next higher integral value [11].

Another interesting relation. Leaving Planck units by replacing q by the SI elemental charge e (and assigning the measured or defined SI values to all constants), $\epsilon_0 = \lambda_1 e/(\hbar c) \approx 8.974129 \cdot 10^{-12}(\mathrm{s/m})^2$ is slightly above the SI vacuum permittivity constant ($\approx$ 1.35%), see eq.(50). If we assume, that eq.(50) is correct, the dimension of charge becomes kilogram and we have determined the most likely charge–to–mass ratio of a fundamental baryon. This would mean that Coulomb coupling approached by topological phases seems to be somehow related to Newtonian gravity. What about the small difference? The deviation in ϵ_0 could be due to the difference in charge to mass ratio of an isolated free baryon and a nuclear clustered baryon where mass is reduced by nuclear effects and binding energies ($\approx$ 1%). In other words, nuclear mass is not simply additive, so the charge–to–mass ratio of protons depends on the context.

Conclusion. We have introduced a formalism to describe a spin–asymmetry that can be interpreted as a local dimensional shift. This leads to an orbital current based on $\kappa \neq 0$ that can be assigned after bosonization to a SG condition. This includes the compatibility between pseudospherical manifolds with local 2-dimensional oscillator potential that can be mapped under $PSL(2,\mathbb{R})$ to a 2 dimensional Coulomb potential [12, 20]. The strength of the stationary current is balanced by Coulomb interaction, where κ obtained in eq.(40) is quite near to the electron to proton mass ratio 1/1836.15... [21]. These results could approach the nature of quantum charge driven by the topological phase gradients. For bosonization and iterative relations beyond field theories it was important to assign both coupling "constants" α and κ to iterative feedback processes driven by phase averaging and "noise reduction" in closed–loops and autoparametric resonance. In subsequent papers it will be shown, that the balancing current regulated by eq.(4) can be identified with high precision as the electron.

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