On the invariance of the speed of light

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It has been argued that the existence of a minimum observable interval of space and time (MOIST) is a model-independent result of the combination of quantum field theory and general relativity. In this paper, I promote this result to a fundamental postulate, called the MOIST postulate. It is argued that the postulate leads to the existence of a maximum signal speed and its invariance. This new result may have two interesting implications. On the one hand, it suggests that the MOIST postulate can explain the invariance of the speed of light, and thus it might provide a deeper logical foundation for special relativity. Moreover, it suggests that the speed constant *c* in modern physics is not the actual speed of light in vacuum, but the ratio of the minimum observable length to the minimum observable time interval. On the other hand, the result also suggests that the existing experiments confirming the invariance of the speed of light already provide observational evidence to support the MOIST postulate.

1. Introduction

It has been widely argued that the existence of a minimum observable interval of space and time (MOIST) is a model-independent result of the proper combination of quantum field theory and general relativity (see, e.g. [1-2])[1]. This suggests that the existence of a MOIST may have a firmer basis beyond the existing theories, and it is probably a more fundamental feature of nature. On the other hand, the existing theories are still based on some *unexplained* postulates. For example, special relativity, the common basis of quantum field theory and general relativity, postulates the invariance of the speed of light in all inertial frames, but the theory does not explain why[2]. In the long run, these postulates need to be explained by some more fundamental ones. Therefore, it may be useful to examine the relationship between MOIST and the existing theories from the opposite direction.

In this paper, I will investigate the implications of the existence of a MOIST for special relativity. I will argue that it is a more fundamental postulate that can explain the invariance of the speed of light. It is well known that the key ingredients for the appearance of minimum observable space and time intervals

[1] Note that the minimum observable space and time intervals are often loosely called minimum length and minimum time in the literature.

[2] Though Einstein originally based special relativity on two postulates: the principle of relativity and the constancy of the speed of light, he later thought that the universal principle of the theory is contained only in the postulate: The laws of physics are invariant with respect to Lorentz transformations between inertial frames [3]. Note that the constancy of the speed of light denotes that the speed of light in vacuum is constant, independently of the motion of the source, in at least one inertial frame.

at the Planck scale are quantum field theory ($\hbar$, c) and general relativity (G, c), and the formulae of the Planck time and Planck length naturally contain $\hbar$, c and G, namely $t_P = \sqrt{\frac{\hbar G}{c^5}}$ and $l_P = \sqrt{\frac{\hbar G}{c^3}}$. When viewing the connection from the opposite direction, we can obtain the speed of light, c, from the minimum observable space and time intervals, t_P and l_P, namely $c = l_P / t_P$. As we will see, this is not merely a simple mathematical transform; rather, it may have some interesting implications, not only for the meaning of special relativity, but also for the experimental evidence of the existence of a MOIST.

2. The MOIST postulate

Although quantum field theory and general relativity are both based on the concept of continuous spacetime, it has been argued that their proper combination leads to the existence of the Planck scale, a lower bound to the uncertainty of distance and time measurements [1-2]. For example, when we measure a space interval near the Planck length the measurement will inevitably introduce an uncertainty comparable to the Planck length, and as a result, we cannot accurately measure a space interval shorter than the Planck length. Moreover, different approaches to quantum gravity also lead to the existence of a minimum length, a resolution limit in any experiment [1]. In this paper, I will promote this result to a fundamental postulate[3]:

The MOIST Postulate: There are minimum observable space and time intervals, which are the Planck length and Planck time respectively.

The postulate implicitly assumes the validity of the principle of relativity. It means that the minimum observable length and the minimum observable time interval are the same in all inertial frames. If the minimum observable space and time intervals are different in different inertial frames, then there will exist a preferred Lorentz frame, while this contradicts the principle of relativity.

3. Maximum signal speed and its invariance

Now I will analyze the possible consequences of the MOIST postulate. In particular, I will argue that it imposes a very stringent restriction on the continuous transmission of a physical signal, and even at the normal energy scale it also has an interesting consequence.

First of all, the MOIST postulate requires that any physical change during a time interval shorter than the Planck time, t_P, is unobservable, or in other words, a physically observable change can only happen during a time interval not shorter than the Planck time. Otherwise we can measure a time interval shorter than the Planck time by observing the physical change, which contradicts the MOIST postulate. However, the postulate does not require that a nonphysical change (e.g. movement of a shadow) or an unobservable

[3] It is worth noting that this postulate is not implied but only motivated by the existing arguments for the existence of minimum observable space and time intervals. One reason is that these arguments implicitly assume the (approximate) validity of both quantum theory and general relativity down to the Planck scale (e.g. [2]), but this assumption may be debatable (see also [4]).

physical change (e.g. see below) cannot happen during a time interval shorter than the Planck time. Next, the MOIST postulate requires that the continuous transmission of a physical change over a distance shorter than the Planck length, l_P, is unobservable[4], but it does not prohibit the happening of such transmissions either. Certainly, a transmission over a distance not shorter than the Planck length is still observable[5]. Note that the transmission of an observable physical change (e.g. change of a light pulse from being absent to being present) corresponds to the transmission of information or energy, which is usually called the transmission of a physical signal[6]. For convenience I will use this common parlance in the following discussions.

Let us consider the continuous transmission of a physical signal in an inertial frame[7]. If the signal moves with a speed larger than $c = l_P / t_P$, then it will move more than one l_P during one t_P, and thus moving one l_P, which is physically observable in principle, will correspond to a time interval shorter than one t_P during the transmission. This contradicts the MOIST postulate, which requires that a physically observable change can only happen during a time interval not shorter than the Planck time, t_P. By comparison, the continuous transmission of a physical signal with a speed smaller than c is permitted, as during the transmission the signal will move less than one l_P during a time interval shorter than one t_P, while the displacement smaller than one l_P is physically unobservable according to the MOIST postulate[8]. This argument shows that the MOIST postulate leads to the existence of a maximum signal speed for the continuous transmission of a physical signal, which is equal to the ratio of the minimum observable length to the minimum observable time interval, namely $v_{\max} = l_P / t_P = c$.

Since the minimum observable time interval and the minimum observable length are the same in all inertial frames, the maximum signal speed for the continuous transmission of a physical signal will be c in

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4 This kind of unobservability is not only at the individual level but also at the statistical level. We may understand this result by thinking that the signal (e.g. the wave function of a microscopic particle) has a spatial fuzziness not smaller than one l_P.

5 For a signal with a position uncertainty much larger than its transmission distance, which is not shorter than the Planck length, the transmission is still observable at the statistical level.

6 For the sake of latter discussions, it is also worth noting that defining the transmission speed of a physical signal is not so simple. An actual physical signal with a finite extent, e.g. a pulse of light, travels at different speeds in a media. Roughly speaking, the largest part of the pulse travels at the group velocity, and its earliest part travels at the front velocity. Under conditions of normal dispersion, the group velocity can represent the signal speed, namely the actual propagation speed of information or energy. In particular, for a microscopic particle moving in vacuum as a physical signal, the signal velocity can be defined as the group velocity of its wave packet. But in an anomalously dispersive medium where the group velocity exceeds the speed of light in vacuum [12], the group velocity no longer represents the signal velocity. For these situations, the signal velocity is usually defined as the front velocity, namely the speed of the leading edge of the signal [13]. However, this definition is not operational in actual experiments. An operational definition of signal velocity may be based on the signal-to-noise ratio, which closely relates to quantum fluctuations [14].

7 In the discussions of this section the speed of signal always denotes the two-way speed, and the speed of light is always the two-way speed of light. This two-way speed can be experimentally measured independent of any clock synchronization scheme. For an arbitrary convention of simultaneity, the usual Lorentz transformations in special relativity, which is based on the Einstein synchronization, will be replaced by the Edwards-Winnie transformations [15-16].

8 This suggests that space and time must both have a minimum observable interval; otherwise either the continuous transmission of a physical signal is impossible or the signal speed can be infinite, both of which contradict experience. For instance, consider the situation that time has a minimum observable interval but space has not. Then a physical signal can move an arbitrarily short distance that is physically observable. But for a signal moving with any finite speed v, moving an observable distance shorter than vt_P will correspond to a time interval shorter than one t_P, while this contradicts the existence of a minimum observable time interval.

every inertial frame. Now I will argue that this maximum speed *c* is invariant in all inertial frames. Suppose a physical signal moves in the x direction with speed *c* in an inertial frame S. Then its speed will be either equal to *c* or larger than *c* in another inertial frame S' with a velocity in the $-x$ direction relative to S. Since *c* is the maximum signal speed in every inertial frame, the speed of the signal in S' can only be equal to *c*. This result also means that when the signal moves in the x direction with speed *c* in the inertial frame S', its speed will be also *c* in the inertial frame S with a velocity in the x direction relative to S'. Since the inertial frames S and S' are arbitrary, we can reach the conclusion that if a signal moves with the speed *c* in an inertial frame, it will also move with the same speed *c* in all other inertial frames. This proves the invariance of speed *c*.

Here is another argument for the invariance of speed *c*. Suppose a signal moves in the x direction with speed *c* in an inertial frame S. Then its speed will be either *c* or smaller than *c* in another inertial frame S' with a velocity in the x direction relative to S. If its speed is smaller than *c* in S', say *c-v*, then there must exist a speed larger than *c-v* and a speed smaller than *c-v* in S' that correspond to the same speed in S due to the continuity of velocity transformation and the maximum of *c*. This means that when the signal moves with a certain speed in frame S its speed in frame S' will have two possible values, which is impossible. Thus the signal moving with speed *c* in S also moves with speed *c* in S', which has a velocity in the x direction relative to S. This result also means that when a signal moves in the x direction with speed *c* in S', its speed is also *c* in S with velocity in the $-x$ direction relative to S'. Since the inertial frames S and S' are arbitrary, this also proves that the maximum signal speed *c* is invariant in all inertial frames[9].

To sum up, I have argued that the MOIST postulate (i.e. assuming the existence of a minimum observable interval of space and time at the Planck scale) leads to the existence of a maximum signal speed *c*, and this speed is invariant in every inertial frame. It is well known that the invariance of the speed of light has been confirmed by experiments with very high precision [18], and no violation of Lorentz invariance has been found either [19]. Therefore, we may say that the MOIST postulate already has experimental support. On the other hand, the postulate can explain the invariance of the speed of light and thus might provide a deeper logical foundation for special relativity, the common basis of quantum field theory and general relativity. Let me give a more detailed analysis of this implication.

4. Relativity without light

Special relativity is originally based on two postulates: the principle of relativity and the constancy of the speed of light. But, as Einstein later admitted to some extent [20], it is an incoherent mixture [21]; the

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[9] Here one may object that I should first state clearly the spacetime transformations before the analysis of speed transformation. However, on the one hand, I are trying to derive the fundamental postulate that determines the spacetime transformations, and on the other hand, as I have argued above, the spacetime transformations are not needed to derive the relation between the maximum speeds in two inertial frames, and the latter can be obtained only from some basic requirements such as the continuity of speed transformation etc. Besides, it is worth noting that a similar argument for the invariance of a maximum speed was also given by Rindler in [17].

first principle is universal in scope, while the second is only a particular property of light, which has obvious electrodynamical origins in Maxwell's theory. In view of this problem, there has been a lasting attempt that tries to drop the light postulate from special relativity, which can be traced back to Ignatowski [22] (see also [23-24]). It has been found that, based only on homogeneity of space and time, isotropy of space and the principle of relativity, one can deduce Lorentz-like transformations with an undetermined invariant speed $K^{-1/2}$:

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \frac{1}{\sqrt{1-Kv^2}} \begin{pmatrix} 1 & -v \\ -Kv & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \tag{1}$$

Unlike special relativity that needs to assume the invariance of the speed of light, an invariant speed naturally appears in the theory, which is usually called relativity without light. This is a surprise indeed.

However, since the value of the invariant speed can be infinite or finite, the theory of relativity without light actually allows two possible transformations: Galilean and Lorentzian. This raises serious doubts about the connection between the theory and special relativity. Some authors doubted that the theory is indeed relativistic in nature [24], and others still insisted that the light postulate in special relativity is still needed to derive the Lorentz transformations [25-27]. Indeed, even if experience can help to determine the invariant speed and eliminate the Galilean transformations, and even if the experience may not refer to any properties of light in an essential way [28-29], there is still one mystery unexplained. It is why the invariant speed is finite. In other words, we need to further explain the finiteness of the invariant speed by some more fundamental postulates. If successful, this will establish a genuine theory of relativity without light, which may then lead us to a deeper understanding of spacetime and relativity.

As I have argued in the last section, the MOIST postulate leads to a maximum signal speed, $v_{\max} = l_P / t_P = c$, which is invariant in all inertial frames. Thus the invariant speed $K^{-1/2}$ in the existing theory of relativity without light can be determined by the minimum observable space and time intervals[10], and the determining relation is $K^{-1/2} = l_P / t_P$. Besides, this also provides a possible new explanation of the speed constant *c* in special relativity (as well as in quantum field theory and general relativity); it is not the actual speed of light in vacuum (though which may be also equal to *c*), but the ratio of the minimum observable length to the minimum observable time interval. Once we have deduced the invariance of speed *c* in terms of the MOIST postulate, special relativity will gain a new formulation. It can be based on the MOIST postulate, which states that the minimum observable space and time intervals, l_P and t_P, are invariant in all inertial frames. The theory can be taken as a more complete theory of relativity without light.

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[10] Note that the MOIST postulate can be compatible with the homogeneity of space and time and the isotropy of space, and thus the derivation of the Lorentz-like transformations in the theory of relativity without light is still valid under the postulate.

5. Further discussions

In this section, I will present some further discussions on the suggested relation between the MOIST postulate, maximum signal speed and special relativity. I will also briefly discuss other possible implications of the postulate and its relation with discrete spacetime.

First of all, although the MOIST postulate leads to the existence of a maximum signal speed *c* for continuous transmissions, it does not preclude the superluminal continuous transmissions that do not correspond to actual information or energy transmissions. Two well-known examples are superluminal light pulse propagation and the hypothetical tachyons. Experiments have shown that the group velocity of a light pulse in an anomalously dispersive media (e.g. atomic caesium gas) can be much larger than *c* [12]. But the superluminal light pulse propagation does not correspond to the superluminal transmission of a physical signal, and it can be shown that the signal speed is still equal to or smaller than *c* in this case [14]. Similarly, a consistent theory of tachyons also requires that the tachyons cannot be used to send signals with a speed larger than *c* from one place to another [30]. Besides, the MOIST postulate does not preclude the existence of superluminal nonlocal signals either. If there is some mechanism to realize nonlocal signal transmission, then its signal speed can be larger than *c*, and the nonlocal process may also violate the Lorentz invariance [31]. But the signal speed in this case also has an upper limit depending on the distance due to the limitation of the MOIST postulate, which is equal to the ratio of transmission distance to the Planck time.

Next, the MOIST postulate may have more implications. The existence of an invariant speed is only its implication for the continuous evolution of the wave function or quantum field. There may exist another discontinuous and nonlinear quantum evolution, the dynamical collapse of the wave function, and the MOIST postulate may also impose restrictions for this process [35]. Since the effect of a dynamical collapse evolution depends not only on time duration but also on the wave function itself (e.g. its energy density distribution), during an arbitrarily short time interval the effect can always be observable at the statistical level for some wave functions. However, the MOIST postulate demands that all observable processes should happen during a time interval not smaller than the Planck time, t_P, and thus each tiny collapse must happen during one t_P or more. Moreover, since there are infinitely many possible positions where the collapse can happen at any time, the duration of each tiny collapse will be exactly one t_P for most time; when the time interval becomes larger than one t_P the collapse will happen in other positions with a probability almost equal to one. This means that the dynamical collapse of the wave function cannot be continuous but be essentially discrete[11]. It has been shown that such a discrete model of dynamical collapse is consistent with the existing experiments and also has some interesting predictions [36].

Thirdly, I will briefly discuss the relation between the MOIST postulate and discrete spacetime. On the one hand, the postulate requires that a space and time interval shorter than the Planck scale is unobservable, and thus it can be regarded as a minimum requirement of spacetime discreteness in the

[11] This also implies that the main dynamical collapse models based on continuous spacetime are inconsistent with the MOIST postulate.

observational sense. If an arbitrarily short interval of space and time is always observable, then space and time will be infinitely divisible and cannot be discrete. On the other hand, the MOIST postulate does not imply that spacetime is discrete in the ontological sense[12]. It is also possible that spacetime itself is still continuous but physical laws do not permit the resolution of spacetime structures below the Planck scale. By comparison, there is a stronger requirement of spacetime discreteness, namely that spacetime itself is discrete. For instance, one may further impose a limitation stronger than the MOIST postulate, e.g., that an unobservable change does not happen or no change happens during a time interval shorter than the Planck time. However, there are at least two worries about such extension. First, it can never be tested whether any change happens or not within the Planck time due to the existence of a minimum observable time interval. Next, it seems that continuous spacetime may be still useful as a description framework, even though all observable physical changes satisfy the requirements of the MOIST postulate[13].

Lastly, it is worth noting that the MOIST postulate has no usual problem of Lorentz contraction faced by discrete space. It is well known that the length contraction in special relativity apparently contradicts the ontological discreteness of space. There are some possible approaches to solve this apparent inconsistency (see [32] for a review). For example, one may resort to the existence of a preferred Lorentz frame or still insist on the Lorentz invariance but resort to some form of deformed Lorentz transformations, as in some models of doubly special relativity [9-10][14]. By comparison, the MOIST postulate is compatible with the Lorentz contraction and does not lead to the existence of a preferred Lorentz frame. But the observability of space and time intervals will become relative. For instance, in some inertial frames, the moving distance of a signal is longer than the Planck length and is observable, while in other inertial frames the distance may be shorter than the Planck length by Lorentz contraction and thus is unobservable. Whether this is a potential problem deserves further study.

In conclusion, I have argued that the existence of a minimum observable interval of space and time (MOIST) leads to the existence of a maximum signal speed and its invariance. This result may have two interesting implications. First, it suggests that the MOIST postulate is a fundamental postulate that can explain why the speed of light is invariant in all inertial frames, and thus it might provide a deeper logical foundation for special relativity. Besides, it suggests that the speed constant c in modern physics is not the actual speed of light in vacuum, but the ratio of the minimum observable length to the minimum observable time interval. Next, the result also suggests that the existing experiments confirming the invariance of the speed of light already provide experimental support to the MOIST postulate.

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[12] There are already many models of discrete spacetime in the ontological meaning. For example, in loop quantum gravity, the discreteness of space is represented by the discrete eigenvalues of area and volume operators [5-6], while in the causal set approach to quantum gravity, one has a direct discretization of the causal structure of continuum Lorentzian manifolds [7-8]. Besides, in doubly special relativity [9-10] and triply special relativity [11] there is also an objective invariant minimum length. In these theories, the classical Minkowski spacetime is replaced by a quantum spacetime such as κ-Minkowski noncommutative spacetime.

[13] In my opinion, there are two reasons to support this view. First, it seems that there is no physical limitation on the difference of the happening times of two causally independent events, e.g., the difference is not necessarily an integral multiple of the Planck time. Next, it seems that continuous spacetime is still needed to describe nonphysical superluminal motion, e.g., superluminal light pulse propagation in an anomalously dispersive media. During such superluminal propagations, moving one l_P will correspond to a time interval smaller than one t_P.

[14] There was a recent debate on whether the model of deformed special relativity is consistent [33-34].

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Appendix: Relativity without light

There are many different deductions of the Lorentz-like transformations without resorting to the light postulate. Yet the assumptions they are based on are basically the same, namely homogeneity of space and time, isotropy of space and the principle of relativity. Here I will introduce a very clear and simple deduction (see also [37]).

Consider two inertial frames S and S', where S' moves with a speed v relative to S and when $t=0$ the origins of the two frames coincide. The space-time transformation equations in two-dimensional space-time can be written as follows:

$$x' = X(x,t,v) \qquad (1)$$

$$t' = T(x,t,v) \qquad (2)$$

where x',t' denote the space and *time coordinates* in the frame S', and x,t denote the space and *time coordinates* in the frame S. Now I will invoke the above assumptions to derive the space-time transformations.

(1) Homogeneity of space and time

The homogeneity of space requires that the length of a rod does not depend on its position in an inertial frame. Suppose there is a rod in the frame S, which ends are at positions x_1 and x_2 ($x_2 > x_1$). Due to the homogeneity of space, the length of the rod is the same when its ends are at positions $x_1 + \Delta x$ and $x_2 + \Delta x$. Correspondingly, the length of the rod in the frame S' is also the same for these two situations. Then we have:

$$X(x_2 + \Delta x, t, v) - X(x_1 + \Delta x, t, v) = X(x_2, t, v) - X(x_1, t, v) \qquad (3)$$

or

$$X(x_2 + \Delta x, t, v) - X(x_2, t, v) = X(x_1 + \Delta x, t, v) - X(x_1, t, v) \qquad (4)$$

Dividing both sides by Δx and taking the limit $\Delta x \to 0$, we get:

$$\left.\frac{\partial X(x,t,v)}{\partial x}\right|_{x_2} = \left.\frac{\partial X(x,t,v)}{\partial x}\right|_{x1} \qquad (5)$$

Since the positions x_1 and x_2 are arbitrary, the partial derivative must be constant. Therefore, the function $X(x,t,v)$ will be a linear function of x. In a similar way, $X(x,t,v)$ is also a linear function of t due to the homogeneity of time, and the same for $T(x,t,v)$. In conclusion, the homogeneity of space and time requires that the space-time transformations are linear with respect to both space and time[15].

Considering that the origins of the two frames S and S' coincide when $t=0$, we can write down the linear space-time transformations in a matrix notation:

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} A_v & B_v \\ C_v & D_v \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \qquad (6)$$

where A_v, B_v, C_v, D_v are only functions of the relative velocity v. Furthermore, since the origin of S'

[15] The idea that the homogeneity of space and time requires space-time transformations are linear can be traced back to Einstein, and was later developed by more authors (see, e.g. Terletskii 1968; Lévy-Leblond 1976; Berzi and Gorini 1969). However, it can be argued that the principle of relativity, together with the law of inertia, can also lead to the linearity of space-time transformations (Fock 1969; Torretti 1983; Brown 2005). Thus the homogeneity of space and time may be dropped from the assumptions needed for deduce a theory of relativity without light.

moves at a speed v relative to the origin of S, i.e., $x' = 0$ when $x = vt$, we also have the following relation:

$$B_v = -vA_v \qquad (7)$$

(2) Isotropy of space

The isotropy of space demands that the space-time transformations do not change when the x-axis is reversed, i.e., both x and v change sign, and so does x'. Applying this limitation to Equation (6) we have[16]:

$$\begin{cases} A_{-v} = A_v \\ B_{-v} = -B_v \\ C_{-v} = -C_v \\ D_{-v} = D_v \end{cases} \qquad (8)$$

(3) Principle of relativity

The principle of relativity requires that the inverse space-time transformations assume the same form as the original transformations. This means that the transformations from S' to S assume the same functional forms as the transformations from S to S'. Moreover, the combination of the principle of relativity with isotropy of space further implies reciprocity (Berzi and Gorini 1969; Budden 1997; Torretti 1983), namely that the speed of S' relative to S is the negative of the speed of S relative to S'. Thus we have:

$$\begin{cases} A_{-v} = \dfrac{D_v}{A_v D_v - B_v C_v} \\ B_{-v} = \dfrac{-B_v}{A_v D_v - B_v C_v} \\ C_{-v} = \dfrac{-C_v}{A_v D_v - B_v C_v} \\ D_{-v} = \dfrac{A_v}{A_v D_v - B_v C_v} \end{cases} \qquad (9)$$

Combining the conditions (8) and (9) we can get:

$$D_v = A_v \qquad (10)$$

$$C_v = \frac{A_v^2 - 1}{B_v} \qquad (11)$$

Then considering Equation (7) the space-time transformations can be formulated in terms of only one

[16] Isotropy of space plays an important role in the deduction. Since isotropy of space and its resulting condition of reciprocity hold only for the standard convention of simultaneity, we only deduce a theory of relativity without light consistent with the standard convention. If simultaneity is really a convention (for a different view see Malament 1977), then it seems that in order to have a theory of relativity without light we should deduce the general Edwards-Winnie transformations for any convention (Edwards 1963; Winnie 1970), not only the Lorentz-like transformations. But this seems to be an impossible task, as symmetries such as isotropy of space and reciprocity play an indispensable role in the deduction.

unknown function A_v, namely

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \begin{pmatrix} A_v & -vA_v \\ -\dfrac{A_v^2 - 1}{vA_v} & A_v \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \qquad (12)$$

or

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = A_v \begin{pmatrix} 1 & -v \\ -\dfrac{A_v^2 - 1}{vA_v^2} & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \qquad (13)$$

Now consider a third frame S'' which moves with a speed u relative to S', and we have:

$$\begin{aligned} \begin{pmatrix} x'' \\ t'' \end{pmatrix} &= A_u A_v \begin{pmatrix} 1 & -u \\ -\dfrac{A_u^2 - 1}{uA_u^2} & 1 \end{pmatrix} \begin{pmatrix} 1 & -v \\ -\dfrac{A_v^2 - 1}{vA_v^2} & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \\ &= A_u A_v \begin{pmatrix} 1 + u\dfrac{A_v^2 - 1}{vA_v^2} & -(u+v) \\ -\dfrac{A_u^2 - 1}{uA_u^2} - \dfrac{A_v^2 - 1}{vA_v^2} & 1 + v\dfrac{A_u^2 - 1}{uA_u^2} \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \end{aligned} \qquad (13)$$

The principle of relativity demands that this transformation assumes the same form as the transformation from S to S', and thus the two diagonal elements of the matrix also satisfy Equation (10), namely they are equal. Thus we have:

$$1 + v\frac{A_u^2 - 1}{uA_u^2} = 1 + u\frac{A_v^2 - 1}{vA_v^2} \qquad (14)$$

or

$$\frac{A_u^2 - 1}{u^2 A_u^2} = \frac{A_v^2 - 1}{v^2 A_v^2} \qquad (15)$$

Since u and v are arbitrary, this equation means that its both sides are constants. Denoting this constant by K and considering the condition $A_v = 1$ when $v = 0$, we have:

$$A_v = \frac{1}{\sqrt{1 - Kv^2}} \qquad (16)$$

Therefore, we deduce the final space-time transformations in terms of the homogeneity of space and time, isotropy of space and the principle of relativity, namely:

$$\begin{pmatrix} x' \\ t' \end{pmatrix} = \frac{1}{\sqrt{1 - Kv^2}} \begin{pmatrix} 1 & -v \\ -Kv & 1 \end{pmatrix} \begin{pmatrix} x \\ t \end{pmatrix} \qquad (17)$$

The velocity addition law can be further deduced. Suppose the speed of the frame S'' relative to S' is w. Then using Equation (16) and Equation (13), in which the first diagonal element of the matrix is A_w by definition, we can directly deduce the velocity addition law, namely:

$$w = \frac{u+v}{1+Kuv} \qquad (18)$$

It can be seen that $K^{-1/2}$ is an invariant speed, independent of any inertial frame. The possible values of K can be determined as follows. Equation (16) indicates $A_v > 0$ for any v. Moreover, the first diagonal element of the matrix in Equation (13) further demands $A_v \geq 1$, for if $A_v < 1$ then for some values of u and v (e.g. $u >> v$) we can get $A_w < 0$. Therefore, we have $K \geq 0$ according to Equation (16).

When $K = 0$ we obtain the Galileo transformations, while when $K > 0$ we obtain the Lorentz transformations. Thus the theory is the most general one consistent with the principle of relativity, which can accommodate both Galilean and Einsteinian relativity. But in this meaning it is not yet relativistic in nature, as the value of K or an invariant speed needs to be further determined in order to establish its connection with Einstein's relativity. Note that this does not mean we need to determine the concrete value of K such as $K = 1/c^2$. What we need to determine is only $K \neq 0$, as K and c are quantities with dimension and their values can assume the unit of number 1 in principle. Certainly we can resort to experience, also without light, to eliminate the possibility of $K = 0$, and we have more today indeed. This, however, is unsatisfactory in several aspects. First of all, we have not deduced a theory of relativity without light consistent with Einstein's relativity in this way. There is still one step left, which may be more important. This obviously departs from the initial aim of dropping the light postulate from special relativity. We hope that, by dropping the light postulate, we can still deduce a theory consistent with special relativity. Next, although we can determine the value of K by experience, there is still one deep mystery unexplained. It is why there exists an invariant and maximum speed, independent of any inertial frame. For Galilean relativity there is no such mystery, but for Einstein's relativity there is one. Lastly, the determination of K by theoretical considerations may lead us to a deeper understanding of space-time and relativity, and will probably bring a further development of special relativity. The existing theory of relativity without light is only a first step towards this direction.

To sum up, we have not had a theory of relativity without light consistent with Einstein's relativity yet. Only after answering why there is an invariant and maximum speed and thus determining the finiteness of K by a deeper postulate can we claim we have. I have provided a possible answer in my paper.