An Exceptionally Simple Argument Against the Many-worlds Interpretation: Further Consolidations

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Abstract

It is argued that the components of the superposed wave function of a measuring device, each of which represents a definite measurement result, do not correspond to many worlds, one of which is our world, because all components of the wave function can be measured in our world by a serious of protective measurements, and they all exist in this world.

In standard quantum mechanics, it is postulated that when the wave function of a quantum system is measured by a macroscopic device, it no longer follows the linear Schrödinger equation, but instantaneously collapses to one of the wave functions that correspond to definite measurement results. However, this collapse postulate is ad hoc, and the theory does not tell us why and how a definite measurement result appears. There are in general two ways to solve the measurement problem. The first way is to integrate the collapse evolution with the normal Schrödinger evolution into a unified dynamics, e.g. in the dynamical collapse theories (Ghirardi 2008). The second way is to reject the collapse postulate and assume that the Schrödinger equation completely describes the evolution of the wave function. There are two main alternative theories for avoiding collapse. The first one is the de Broglie-Bohm theory (de Broglie 1928; Bohm 1952), which takes the wave function as an incomplete description and adds some hidden variables to explain the emergence of definite measurement results. The second one is the many-worlds interpretation (Everett 1957; DeWitt and Graham 1973), which assumes the existence of many equally real worlds corresponding to all possible results of quantum experiments and still regards the unitarily evolving wave function as a complete description of the total worlds.

Although the many-worlds interpretation is widely acknowledged as one of the main alternatives to quantum mechanics, its fundamental issues, e.g. the preferred basis problem and the interpretation of probability, have not been

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completely solved yet (see Barrett 1999, 2011; Saunders et al 2010 and references therein). In this note, we will argue that the existence of many worlds seems inconsistent with the results of protective measurements (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Aharonov, Anandan and Vaidman 1996; Vaidman 2009; Gao 2011).

According to the many-worlds interpretation, the components of the wave function of a measuring device, each of which represents a definite measurement result, correspond to many worlds, one of which is our world (Vaidman 2008; Barrett 2011). It is unsurprising that the existence of such many worlds may be consistent with the results of conventional impulsive measurements[1], as the many-worlds interpretation is just invented to explain the emergence of these results, e.g. the definite measurement result in each world always denotes the result of a conventional impulsive measurement. However, this does not guarantee consistency for all types of measurements. It has been known that there exists another type of measurement, the protective measurement (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Aharonov, Anandan and Vaidman 1996). Like conventional impulsive measurement, protective measurement also uses the standard measuring procedure, but with a weak, adiabatic coupling and an appropriate protection. Its general method is to let the measured system be in a nondegenerate eigenstate of the whole Hamiltonian using a suitable protective interaction, and then make the measurement adiabatically. This permits protective measurement to be able to measure the expectation values of observables on a single quantum system. In particular, the wave function of the system can also be measured by protective measurement as expectation values of certain observables (see the Appendix)[2].

It can be seen that the existence of many worlds seems inconsistent with the results of protective measurements. The reason is that the whole superposed wave function of a measuring device, if it indeed exists as assumed by the many-worlds interpretation, can be directly measured by a serious of protective measurements in our world[3]. The result of the protective measurement as predicted by quantum mechanics indicates that all components of the wave function of the measuring device exist in our world. Therefore, according to protective measurements, the superposed wave function of a measuring device do not correspond to many worlds, one of which is our world. Concretely speaking, there are no many copies of the measuring device, each of which is in one world and obtains a definite result; rather, there is only one measuring device that obtains no definite result in our world. In this way, protective measurement seems to provide a strong argument against the many-worlds interpretation.

Several points needs to be clarified regarding the above argument. First of all, the above argument does not depend on how many worlds are *precisely* defined in the many-worlds interpretation. In particular, it is independent of

[1]It should be pointed out that the consistency is still debated due to the controversial interpretation of probability. For more discussions see Saunders et al (2010) and references therein.

[2]Note that the earlier objections to the validity and meaning of protective measurements have been answered (Aharonov, Anandan and Vaidman 1996; Dass and Qureshi 1999; Vaidman 2009; Gao 2012).

[3]Protective measurement generally requires that the measured wave function is known beforehand so that an appropriate protective interaction can be added. But this requirement does not influence our argument, as the superposed wave function of a measuring device can be prepared in a known form before the protective measurement.

whether worlds are fundamental or emergent, e.g. it also applies to the recent formulation of the many-worlds interpretation based on a structuralist view on macro-ontology (Wallace 2003). The key point is that all components of the superposed wave function of a measuring device can be detected by protective measurements in a single world, namely our world, and thus they all exist in this world. Therefore, it is impossible that the superposed wave function of a measuring device corresponds to many worlds, only one of which is our world. Note that this objection is more serious than the problem of approximate decoherence for the many-worlds interpretation (cf. Janssen 2008). Although the interference between the nonorthogonal components of a wave function can be detected in principle due to the unitary dynamics, it cannot be detected for individual states, but only be detected for an ensemble of identical states. Moreover, the presence of tiny interference terms in a (local) wave function in our world does not imply that all components of the wave function wholly exist in this world. For example, it is possible that each world has most of one component of the wave function that represents a definite measurement result and tiny parts of other components, and this picture is consistent with the many-worlds interpretation.

Next, the above argument is not influenced by environment-induced decoherence. Even if the superposition state of a measuring device is entangled with the states of other systems, the entangled state of the whole system can also be measured by protective measurement in principle (Anandan 1993). The method is by adding appropriate protection procedure to the whole system so that its entangled state is a nondegenerate eigenstate of the total Hamiltonian of the system together with the added potential. Then the entangled state can be protectively measured. On the other hand, we note that if environment-induced decoherence is an essential element of the many-worlds interpretation, then the theory will be inconsistent with standard quantum mechanics. When a measuring device is isolated from environment, standard quantum mechanics still predicts that the device can obtain a definite result, while the many-worlds theory will predict the opposite due to the lack of environment-induced decoherence.

Thirdly, the above argument does not require protective measurement to be able to distinguish the superposed wave function of a measuring device from one of its components, or whether the superposed wave function collapses or not during an impulsive measurement. Since the determination demands the distinguishability of two non-orthogonal states, which is prohibited by quantum mechanics, no measurements consistent with the theory including protective measurement can do this. What protective measurement tells us is that such a superposed wave function, whose existence is assumed by the many-worlds interpretation, does not correspond to many worlds as assumed by the many-worlds interpretation. In other words, protective measurement reveals inconsistency of the many-worlds interpretation. Fourthly, we stress again that the principle of protective measurement is independent of the controversial process of wavefunction collapse and only depends on the linear Schrödinger evolution and the Born rule. As a result, protective measurement can (at least) be used to examine the internal consistency of the no-collapse solutions to the measurement problem, e.g. the many-worlds interpretation, before experiments give the last verdict[4].

[4]For a more detailed analysis of the implications of protective measurement see Gao (2011).

Lastly, we discuss a possible way to refute the above argument against the many-worlds interpretation. According to the principle of protective measurements, only observers (or measuring devices) whose states are not entangled with the superposed wave function of a measuring device can make a protective measurement of the wave function, and an observer who is decoherent with respect to the outcomes obtained by the device cannot make such a measurement. Then it seems that, by insisting that there is no branching and no worlds without decoherence, one can refute the above argument. For the observers in each world must be already decoherent with respect to the outcomes obtained by the device, and thus they cannot make the protective measurement which is required by the argument[5].

However, this view contradicts the assumption that worlds, no matter they are emergent or fundamental, are objective in the many-worlds interpretation. The objectivity of worlds means that everything in the universe, whether or not it interacts with the measured system and the decoherent device or observer, has a copy in each world, though these copies may be the same[6]. In a physical theory where the minds of observers play no special role, a measurement result, once it has been recorded by a measuring device or an observer, should exist objectively, and in particular, it should exist for any observer in the world, independently of whether the observer makes a measurement or knows the result. Under this objectivity assumption, the above argument against the many-worlds interpretation is valid. For our world is also one of the assumed branching worlds represented by the components of the wave function of a measuring device, and observers in this world are not necessarily decoherent with respect to the outcomes obtained by the device, and thus those independent observers can make a protective measurement of the superposed wave function of the device, whose result will indicate that the whole superposed wave function exists in our world.

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[5]Certainly, even such a protective measurements cannot be made, it does not imply that the superposed wave function of the device does not exist wholly in our world either.

[6]In particular, the objectivity of worlds means that the emergence of distinct worlds is not merely the subjective perception of the decoherent observer in the wave function.

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Appendix: Protective measurement of the wave function of a single quantum system

As a simple example of protective measurement, consider a quantum system in a discrete nondegenerate energy eigenstate $|E_n\rangle$. In this case, the system itself supplies the protection of the state due to energy conservation and no artificial protection is needed.

The interaction Hamiltonian for a protective measurement of an observable A in this state involves the same interaction Hamiltonian as the standard measuring procedure:

$$H_I = g(t)PA, \tag{1}$$

where P is the momentum conjugate to the pointer variable X of an appropriate measuring device. The time-dependent coupling strength $g(t)$ is also a smooth function normalized to $\int dtg(t) = 1$. But different from conventional impulse measurements, for which the interaction is very strong and almost instantaneous, protective measurements make use of the opposite limit where the interaction of the measuring device with the system is weak and adiabatic. Concretely speaking, the interaction lasts for a long time T, and $g(t)$ is very small and constant for the most part, and it goes to zero gradually before and after the interaction.

Let the total Hamiltonian of the combined system be

$$H(t) = H_S + H_D + g(t)PA, \tag{2}$$

where H_S and H_D are the Hamiltonians of the measured system and the measuring device, respectively. Let the initial state of the pointer at $t = 0$ be $|\phi(x_0)\rangle$, which is a Gaussian wave packet of eigenstates of X with width w_0, centered around the eigenvalue x_0. Then the state of the combined system after T is

$$|t = T\rangle = e^{-\frac{i}{\hbar}\int_0^T H(t)dt} |E_n\rangle |\phi(x_0)\rangle \,. \tag{3}$$

By ignoring the switching on and switching off processes[7], the full Hamiltonian (with $g(t) = 1/T$) is time-independent and no time-ordering is needed. Then we obtain

$$|t = T\rangle = e^{-\frac{i}{\hbar}HT} |E_n\rangle |\phi(x_0)\rangle \,, \tag{4}$$

where $H = H_S + H_D + \frac{PA}{T}$. We further expand $|\phi(x_0)\rangle$ in the eigenstate of H_D, $|E_j^d\rangle$, and write

$$|t = T\rangle = e^{-\frac{i}{\hbar}HT} \sum_j c_j |E_n\rangle |E_j^d\rangle \,, \tag{5}$$

[7]The change in the total Hamiltonian during these processes is smaller than PA/T, and thus the adiabaticity of the interaction will not be violated and the approximate treatment given below is valid. For a more strict analysis see Dass and Qureshi (1999).

Let the exact eigenstates of H be $|\Psi_{k,m}\rangle$ and the corresponding eigenvalues be $E(k,m)$, we have

$$|t=T\rangle = \sum_j c_j \sum_{k,m} e^{-\frac{i}{\hbar}E(k,m)T} \langle \Psi_{k,m}|E_n, E_j^d\rangle |\Psi_{k,m}\rangle. \tag{6}$$

Since the interaction is very weak, the Hamiltonian H of Eq.(2) can be thought of as $H_0 = H_S + H_D$ perturbed by $\frac{PA}{T}$. Using the fact that $\frac{PA}{T}$ is a small perturbation and that the eigenstates of H_0 are of the form $|E_k\rangle \left|E_m^d\right\rangle$, the perturbation theory gives

$$\begin{aligned} |\Psi_{k,m}\rangle &= |E_k\rangle \left|E_m^d\right\rangle + O(1/T), \\ E(k,m) &= E_k + E_m^d + \frac{1}{T}\langle A\rangle_k \langle P\rangle_m + O(1/T^2). \end{aligned} \tag{7}$$

Note that it is a necessary condition for Eq.(7) to hold that $|E_k\rangle$ is a nondegenerate eigenstate of H_S. Substituting Eq.(7) in Eq.(6) and taking the large T limit yields

$$|t=T\rangle \approx \sum_j e^{-\frac{i}{\hbar}(E_n T + E_j^d T + \langle A\rangle_n \langle P\rangle_j)} c_j \, |E_n\rangle \left|E_j^d\right\rangle. \tag{8}$$

When P commutes with the free Hamiltonian of the device, i.e., $[P, H_D] = 0$, the eigenstates $\left|E_j^d\right\rangle$ of H_D are also the eigenstates of P, and thus the above equation can be rewritten as

$$|t=T\rangle \approx e^{-\frac{i}{\hbar}E_n T - \frac{i}{\hbar}H_D T - \frac{i}{\hbar}\langle A\rangle_n P} \, |E_n\rangle \, |\phi(x_0)\rangle. \tag{9}$$

It can be seen that the third term in the exponent will shift the center of the pointer $|\phi(x_0)\rangle$ by an amount $\langle A\rangle_n$:

$$|t=T\rangle \approx e^{-\frac{i}{\hbar}E_n T - \frac{i}{\hbar}H_D T} \, |E_n\rangle \, |\phi(x_0 + \langle A\rangle_n)\rangle. \tag{10}$$

This shows that the center of the pointer shifts by $\langle A\rangle_n$ at the end of the interaction. For the general case where $[P, H_D] \neq 0$, we can also obtain the similar result. Thus protective measurement can measure the expectation value of the measured observable in the measured state.

Let the explicit form of $|E_n\rangle$ be $\psi(x)$, and the measured observable A be (normalized) projection operators on small spatial regions V_n having volume v_n:

$$A = \begin{cases} \frac{1}{v_n}, & \text{if } x \in V_n, \\ 0, & \text{if } x \notin V_n. \end{cases} \tag{11}$$

The protective measurement of A then yields

$$\langle A\rangle = \frac{1}{v_n}\int_{V_n} |\psi(x)|^2 dv = |\psi_n|^2, \tag{12}$$

where $|\psi_n|^2$ is the average of the density $\rho(x) = |\psi(x)|^2$ over the small region V_n. Then when $v_n \to 0$ and after performing measurements in sufficiently many regions V_n we can measure $\rho(x)$ everywhere in space.

Similarly, let the measured observable be $B = \frac{1}{2i}(A\nabla + \nabla A)$. Then the protective measurement of B then yields

$$\langle B \rangle = \frac{1}{v_n} \int_{V_n} \frac{1}{2i}(\Psi^* \nabla \Psi - \Psi \nabla \Psi^*) dv = \frac{1}{v_n} \int_{V_n} |j(x)|^2 dv. \tag{13}$$

This is the average value of the flux density $j(x)$ in the region V_n. Then when $v_n \to 0$ and after performing measurements in sufficiently many regions V_n we can measure $j(x)$ everywhere in space.

Since the wave function $\psi(x,t)$ can be uniquely expressed by $\rho(x,t)$ and $j(x,t)$ (except for a constant phase factor):

$$\psi(x,t) = \sqrt{\rho(x,t)} e^{im \int_{-\infty}^{x} \frac{j(x',t)}{\rho(x',t)} dx' / \hbar}, \tag{14}$$

the whole wave function of the measured system can be measured by protective measurement.