Can continuous motion be an illusion?

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It is widely accepted that continuity is the most essential characteristic of motion; the motion of macroscopic objects is apparently continuous, and classical mechanics, which describes such motion, is also based on the assumption of continuous motion. But is motion really continuous in reality? In this paper I will try to answer this question through a new analysis of the cause of motion. It has been argued that the standard velocity in classical mechanics cannot fulfill the causal role required for explaining continuous motion in a deterministic way. However, there is a hot debate over the solution to this causal explanation problem. The existing solutions, i.e. the "at-at" theory, the impetus view and the "no instants" view, have serious drawbacks. After reviewing their drawbacks and presenting my own objections, I propose a new solution to this causal explanation problem. It is argued that the relativity of motion or the equivalence between motion and rest demands that motion has no deterministic cause, and thus no causal explanation is needed for motion. Based on this result, I further argue that the uncaused motion is not deterministic and continuous but essentially random and discontinuous. Moreover, objects have a potential (as probabilistic cause of motion) that determines the probabilities of their future positions. This suggests a possible connection with quantum mechanics. Lastly, I give a brief explanation for the apparent inconsistency between the suggested potential theory of motion and the appearance of continuous motion in the macroscopic world.

Key words: continuous motion; Zeno's paradoxes, cause of motion; relativity of motion; uncaused motion; probabilistic cause; random discontinuous motion

It is widely accepted that continuity is the most essential characteristic of motion; the motion of macroscopic objects is apparently continuous, and classical mechanics, which describes such motion, is also based on the assumption of continuous motion. But is motion really continuous in reality? This is still a controversial issue. On the one hand, the appearance of continuous motion cannot be simply regarded as the real picture of motion, though it certainly puts the burden of proof on those who would deny continuity of motion. On the other hand, the continuity of motion should not be *a priori* assumption; rather, it can only be a result of the complete law of motion, which states how objects move in reality. Besides, the debate about the meaning of quantum mechanics, which is considered as the most fundamental theory of nature, makes the puzzle of motion more complex.

In this paper I will try to answer this question through a new analysis of the cause of motion. It has been widely argued that the standard velocity in classical mechanics cannot fulfill the causal role required for explaining continuous motion in a deterministic way. But there is a hot debate on the solution to this causal explanation problem. The existing solutions, i.e. the "at-at" theory, the impetus view and the "no instants" view, have serious drawbacks. After reviewing their drawbacks and presenting my own objections, I propose a new solution to this causal explanation problem. It is argued that the relativity of motion demands that motion has no deterministic cause, and thus no causal explanation is needed for motion. Based on this result, I further argue that the uncaused motion is not deterministic and continuous but essentially random and discontinuous. Moreover, objects have an indeterministic propensity (as probabilistic cause of motion) that determines the probabilities of their positions at every instant. I also give a brief explanation for the appearance of continuous motion in the macroscopic world.

The plan of this paper is as follows. In Section 1, I review the latest debate on the nature of instantaneous velocity in classical mechanics (CM). The conclusion is that the standard velocity of an object cannot be an instantaneous intrinsic property of the object, and thus it is not part of the instantaneous physical state of the object. This leads to the standard "at-at" theory of motion. In Section 2, I discuss the problem of incompleteness resulted from the lack of velocity in the instantaneous state of an object. It is pointed out that although CM can account for the evolution of neighboring states, it cannot explain the change of instantaneous states (i.e. positions). As a result, CM is not yet a complete theory of motion; it does not state the evolution law of instantaneous states. This poses a serious causal explanation problem or incompleteness problem (as I shall call it in this paper). Section 3 discusses the "evading" way to solve this incompleteness problem, the so-called "no instants" view, which simply denies the actual existence of instants and instantaneous states. If instantaneous states do not exist, then CM, which does not account for the evolution of instantaneous states, can be still complete. Although this solution seems to be the simplest, it is in fact the most difficult and radical way, which requires that the existing language of both mathematics and physics should be largely revised. I discuss various existing objections and also give my own objections to this radical view. Section 4 analyzes another more direct way to complete CM, the impetus view. It endows an object with an instantaneous intrinsic velocity that does not dependent on the change of position. If there exists an intrinsic velocity, then position can be defined by the integral of this velocity, and the change of position can thus be explained by the intrinsic velocity. Some arguments for and against the impetus view are discussed, and it is pointed out that this view meets serious difficulty when considering the transformational properties of velocity.

After reviewing the existing views of motion and their drawbacks, I then propose a new way out of the dilemma of continuous motion in the second part of the paper. In Section 5, I argue that the relativity of motion or the equivalence between motion and rest demands that motion has no deterministic cause including intrinsic velocity, and thus no (deterministic) causal explanation is needed for motion. In Section 6, I further argue that the uncaused motion is essentially random and discontinuous. Section 7 fixes a loophole relating to the relativity of motion and strengthens the argument. It seems that the incompleteness problem does not exist for random discontinuous motion, as we need not to find some deterministic cause (e.g. intrinsic velocity) to explain it in a deterministic way. However, this theory is still plagued by the not-Markovian problem from an indeterministic aspect, just like the "at-at" theory of continuous motion from a deterministic aspect. That is to say, a probabilistic cause is still needed to explain the randomness of motion. This then leads us to a potential theory of motion, which is proposed in Section 8. According to the theory, objects have a potential at instants that determines the probabilities of their future position, and their motion is essentially random and discontinuous. Besides, I also present a further analysis of motion, which concentrates more on the law itself. The analysis makes the random discontinuous motion and the potential more real. Although the potential theory of motion may be promising, it seems plainly inconsistent with observations in the macroscopic world; the macroscopic objects apparently move in a continuous way. In the last section, I give a brief explanation for this apparent inconsistency, and I also point out that the theory may be consistent with observations in the microscopic world and may has some possible connections with quantum mechanics.

1. Continuous motion and instantaneous velocity

According to classical mechanics (CM), a moving object is in a definite position at every instant[1]. Moreover, objects move in continuous trajectories; an object does not move from one position to another

[1] In this paper we only consider the motion of the mass center of an object or a particle, which can be described by a material point. For simplicity, we always say the motion of an object or a particle.

without passing through the intervening space[2]. This means that the motion of an object can be described by a continuous position function $x(t)$ [3], which gives the location of the object at any time *t*.

The continuity of motion further permits that an object can also have a definite velocity at every instant when its trajectory is differential, which is defined as the first time derivative of the trajectory[4]. The mathematical formulation of this definition of instantaneous velocity can be written as follows:

$$v(t) = \frac{dx(t)}{dt} = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t}$$

Here the notion of a limit can be made rigorous as by Cauchy and Weierstrass: $\lim_{x \to a} f(x) = L$ if and only if for each positive number ε, there exists a positive number δ such that $|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$. In physics, this definition means that the instantaneous velocity of an object is the rate of change of its position.

The long-standing practice of CM is to take the instantaneous state of a system to consist of both the position and the velocity, and thus the evolution of this instantaneous state is deterministic (e.g. for constrained system)[5]. However, there has been a debate on the nature of instantaneous velocity defined as above. We also see a recent resurgence of this debate (see, e.g. Arntzenius 2000; Smith 2003). The main issue is whether the standard calculus definition of velocities allows velocities to be part of the instantaneous states of objects. Is instantaneous velocity really a property at an instant? If instantaneous velocity is a property at an instant, is it an intrinsic property or merely extrinsic/relational property? Can instantaneous velocity play the causal and explanatory roles that CM is traditionally interpreted as attributing to it[6]? In order to answer these questions, we need a more detailed analysis of instantaneous velocity.

An argument supporting the long-standing practice of CM can be formulated as follows (see also Smith 2003). According to the above definition of instantaneous velocity, an arbitrarily small neighborhood around t can be selected in defining $v(t)$. No finite neighborhood is required, and no particular point of the object's trajectory other than $x(t)$ is indispensable to the object's having $v(t)$. Thus the

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[2] The continuity of motion includes two aspects: (1). spacetime is not discrete; (2). there are no discontinuous jumps. Note that the common meaning of discontinuity relating to motion refers to the discrete but still consecutive change of position. When the discrete units of spacetime approaches zero, it will become the usual continuous motion. The discontinuity of motion I will discuss in this paper is different from this, and it denotes the essential discontinuity of the trajectory function in all positions.

[3] A function $x: A \to R$ is continuous if and only if for every $t \in A$ and every real number $\varepsilon > 0$, there exists a real number $\delta > 0$ such that whenever a point $t' \in A$ has distance less than δ to t, the point $x(t') \in R$ has distance less than ε to $x(t)$.

[4] Note that velocity may not exist for a continuous trajectory, e.g. for that of a particle in Brownian motion.

[5] It has been argued that CM may be indeterministic when the Lipschitz condition is violated, i.e. allowing non-Lipschitz forces into the theory (see, e.g. Norton 2006). However, this violation may not appear in practical physical situations.

[6] According to the traditional causal explanation of CM, instantaneous velocity can play the causal and explanatory role in explaining continuous motion; any difference between an object's position at one moment and its position at some later moment is caused by the object's having non-zero velocity at various intervening moments. Moreover, any difference between an object's velocity at one moment and its velocity at some later moment is further caused by a force. For a recent analysis and development of this explanation, see Lange (2005).

instantaneous velocity of an object cannot be a property of any finite time interval, and it must be an intrinsic property of the object at an instant. We can make this argument more specific. Given any neighborhood $(t-\Delta t, t+\Delta t)$ around t, there is always a smaller one $(t-\delta t, t+\delta t)$ where $\Delta t > \delta t > 0$ in which the instantaneous velocity $v(t)$ is still determined. As a result, the trajectory's points inside the larger interval but outside the smaller are irrelevant to $v(t)$, and in the limit $\Delta t \to 0$ every point other than $x(t)$ will be irrelevant in defining $v(t)$ (Smith 2003, p.276). This indicates that instantaneous velocity $v(t)$ is not a property of the values of $x(t)$ in any particular neighborhood around t; rather, it can only be an intrinsic property of an object at instant t, which is irrelevant to its positions at other instants. Therefore, velocity, defined as the first time derivative of the trajectory of an object, should be part of the instantaneous states of the object, and the long-standing practice of CM is perfectly sound.

An obvious objection to the above argument is as follows. Although there is indeed no minimal neighborhood of t such that the velocity is an intrinsic property of position development in that neighborhood, that is simply because there is no minimal open neighborhood of t, and this does not imply that velocity must be part of the instantaneous state at t (Arntzenius 2003, p.281). On the contrary, since velocity, the first derivative of the position function $x(t)$, is defined as the limit of $(x(t)-x(t_0))/(t-t_0)$ as t approaches t_0, and necessarily involves reference to the values of $x(t)$ at times in the neighborhood of time t_0, the velocity of an object at any time necessarily involves reference to the positions of that object at neighboring times. Accordingly, velocity is a relationally defined quantity, and it cannot consist in the possession of properties that are intrinsic to an object at a time (Tooley 1988, p.229). In the words of Butterfield (2006), the instantaneous velocity is local but temporally extrinsic. Another way to see this is to realize that a given velocity $v(t)$ at instant t will rule out many possible trajectories at times other than t, for example, those trajectories which velocities are different from $v(t)$. However, an instantaneous state at an instant should not *by logic and definition alone* constrain histories of instantaneous states at other times (Albert 2000, p.17; Arntzenius 2003, p.281). Laws of nature and contingent relations may imply such constraints, but logic and definition should not. Thus, velocity, by its standard calculus definition, cannot be part of the instantaneous state of an object.

One can further argue that, since velocity is a relationally defined quantity, which involves reference to the positions of other neighboring instants, it is not really instantaneous (Arntzenius 2000, p.195). Therefore, the instantaneous velocity of an object is neither an intrinsic property nor a property at an instant. This view has more support from the latest infinitesimal conception made rigorous by Robinson (1966). According to the theory, infinitesimal is a quantity neither finite nor zero, and the instantaneous velocity of an object is then the ratio of the infinitesimal distance covered to the infinitesimal period of time elapsed. One might think that a moving object at an instant travels an infinitesimal distance, and endures for an infinitesimal period of time, and thus the instantaneous velocity of an object will be its instantaneous property. In fact, if there are infinitesimal distances, an object does not travel such a distance during an instant, as infinitesimal distances are not covered in an instant. Rather, it travels an infinitesimal distance during an infinitesimal interval (Tooley 1988, p.233). Therefore, instantaneous velocity, defined as the ratio of these intervals, is a property dependent upon the locations of an object throughout an infinitesimal time interval, which may be called interval property or neighboring property according to Arntzenius (2000).

As Lange (2005) pointed out, the intrinsic-relational division may be fuzzy when it comes to properties defined in terms of limits such as instantaneous velocity. Indeed, if we are restricted in the understanding of the limit aspect of instantaneous velocity, it might be difficult to settle the intrinsic-relational debate. In fact, there exist other features of velocity which can determine whether it is intrinsic or relational, and which do not depend on the understanding of limits. One such feature is the relativity of velocity. The velocity of an object is always defined relative to a reference frame. Therefore, velocity is a relation between the moving object and the external reference frame, and it is impossible to view velocity as an intrinsic property of an object. This relational characteristic still sustains in the limiting process from average velocity to instantaneous velocity. As a result, instantaneous velocity cannot be an intrinsic property of an object anyway. Moreover, this relational characteristic may further entail that the velocity of an object at an instant is also a relational property involving reference to the positions of that object at neighboring times, since an instantaneous intrinsic property of an object is irrelevant to other objects such as external reference frames.

To sum up, the instantaneous velocity of an object cannot be an instantaneous intrinsic property of the object, and thus it is not part of the instantaneous state of the object. As we will see, the lack of velocity in the instantaneous state of an object will result in a serious problem for CM, which may further endanger continuous motion and determinism.

2. The problem of incompleteness

If velocity is not part of the instantaneous physical state of an object, the only (relevant) intrinsic property that the object has at an instant is its position: its kinematic state at an instant is its position. This will lead to the so-called "at-at" theory of motion. In his book, *The Principles of Mathematics*, Bertrand Russell set out such a theory of motion, which is succinctly described in the final paragraph of the chapter on motion (see also Tooley 1988, p.226):

> Motion consists merely in the occupation of different places at different times, subject to continuity as explained in Part V. There is no transition from place to place, no consecutive moment or consecutive position, no such thing as velocity except in the sense of a real number which is the limit of a certain set of quotients. The rejection of velocity and acceleration as physical facts (i.e. as properties belonging at each instant to a moving point, and not merely real numbers expressing limits of certain ratios) involves, as we shall see, some difficulties in the statement of the laws of motion; but the reform introduced by Weierstrass in the infinitesimal calculus has rendered this rejection imperative.[7]

The "at-at" theory of motion was originally proposed to solve Zeno's arrow paradox. The paradox can be basically formulated as follows. Assume that time is composed of durationless instants. At every instant the arrow does not have time to move and is at rest during that instant. Thus the flying arrow cannot be moving at any time. This conclusion also holds true when the instant has finite duration, which is the smallest part of time. Suppose that the arrow actually moved during such an instant. It would be at different locations at the start and end of the instant, which implies that the instant has a 'start' and an 'end', which in turn implies that it contains at least two parts, and thus is not the smallest part of time at all. This leads to a contradiction. Thus, the flying arrow cannot be moving even when the instant has finite duration. According to the "at-at" theory, it is fallacious to conclude from the fact that the arrow does not travel any distance in an instant that it is at rest. Motion has nothing at all to do with what happens during instants; it has instead to do with what happens between instants. In short, motion is merely being in different locations at different times, and that is that. If an object has the same location at the instants immediately neighboring, then we say it is at rest; otherwise it is in motion. Therefore, since the arrow in flight has different positions at different instants, it is surely moving.

The "at-at" theory is a static theory of motion. In Henri Bergson's cynical words, "movement is composed of immobilities." (Bergson 1911, p.308) Continuous motion is simply the occupation, by an object, of a continuous series of places at a continuous series of times. There are no states of motion at an instant, and no instantaneous properties indicate that an object is moving or not. However, velocity had

[7] Russell (1903), p.473.

been taken as an instantaneous state of motion in CM, which can indicate whether an object is moving or not at an instant, and determinism also refers to the evolution of the instantaneous state including both position and velocity. This is quite understandable, as the equation of motion (i.e. Newton's second law) is second order in CM. Now if the instantaneous state no longer includes velocity, then how to understand CM and determinism? Is the long-standing practice of CM wholly wrong? Surely not; the practical application of CM is undoubtedly successful. The key lies in the re-understanding of the explanatory ability of CM. As we know, velocity at an instant is defined in terms of limit of position developments in a (finite or infinitesimal) neighborhood of this instant. Thus, although velocity is not part of the instantaneous state of a system, it is part of the neighborhood state of the system (i.e. the state of a system throughout any finite, or infinitesimal, time interval around an instant, see also Arntzenius 2000, p.194), or we can say, it is one kind of neighborhood property[8]. The neighborhood states are still acceptable states that can be non-trivially used in explanations and predictions of future behavior. In this way, CM can get back determinism in the meaning that the neighborhood state at an instant determines the states at all times and hence the neighborhood states at all times. This is actually what the equation of motion in CM says. In practical application, we can only measure average velocity during a finite time interval, and thus this re-interpretation of determinism will not influence the successful applications (including explanations and predictions etc) of CM.

However, although the "at-at" theory can be consistent with the long-standing practice of CM when considering the deterministic evolution of neighborhood states, it does have a problem in accounting for the evolution of instantaneous states. Imagine an object (taken as a mass point) moves from A to B and then proceeds from B back to A (see also the ball example in Arntzenius 2000, p.191). Suppose it moves in a straight line in both directions and so occupies each point between A and B twice. At each one of these points, what is the difference between the object's moving from A to B and its moving from B to A? Since the object lacks an intrinsic instantaneous velocity at each instant, its instantaneous states are the same for these two situations. Then how to explain why the rightward moving object continues to move right and the leftward moving object continues to move left? To speak in an anthropopathic way, the object does not "know" along which direction it should move at all. CM usually resorts to the law of inertia to explain the continued motion. A freely moving object can sustain its velocity, and thus the rightward moving object is in ever more rightward positions because at earlier times it was moving rightward. But this demands that velocity should be an intrinsic property of the object at instants. Since this is not right, the law of inertia cannot help as a matter of fact, and thus CM will lose its explanatory ability here. In other words, the above question is beyond the scope of CM.

This problem has been called causal explanation problem (Lange 2005, p.438)[9]. I shall call it

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[8] Neighborhood properties are not intrinsic fundamental properties at a time. Rather, they are properties of finite, or infinitesimal, developments of states. Since there are definitional relations between the neighborhood states at different times, they are not physical states but features of finite developments of physical states; a particular physical state at some time does not *by definition and logic alone* impose any constraints on the physical states at other times. A more detailed discussion about neighborhood properties has been given in Arntzenius (2000).

[9] Most people may think that the causal explanation problem, which concerns the interpretation of CM, should be solved only within the framework of CM. This view, however, might not be wholly right. It is well known that CM have many flaws, some of which have been widely studied, notably by Mach (1893) in the end of 19th century, and their solutions have leaded to new theories such as (special and general) relativity. Similarly, it is very likely that there also exist some unnoticed flaws of CM, which may point to quantum mechanics (QM), of which CM is only an approximation. If such flaws really exist, it can be expected that they probably relate to the problems of continuity and determinism, as randomness (or indeterminism) and discontinuity are generally regarded as the essential features of QM at least according to its standard explanation. As we will see, the causal explanation problem or the incompleteness problem (as I shall call it) is just one of such flaws, and its solution may not only require us to go beyond the framework of CM, but also suggest a realistic way to QM, the more complete and precise description of the actual world. Historically, it is the inability of CM to explain the experimental facts such as black-body radiation that leads to the founding of QM. In fact, the logical analysis of the concepts of CM might also lead us to QM, as to relativity.

incompleteness problem here, as it may be irrelevant to causation. In fact, the problem can be formulated more generally. In CM the evolution equation of position is second order[10]. In order to solve this equation, two initial conditions are needed; one is initial position, and the other is initial velocity, namely the initial first time derivative of position. However, the initial velocity is not an intrinsic property at the initial instant, and it also depends on the evolution of position before the instant, while the evolution of position can only be known after solving the equation. This is a vicious circle, which indicates that CM is incomplete in describing the evolution of instantaneous states. Note that, because we can measure the initial value of velocity as a time-averaged value, the CM equation for neighboring states can still be applied in practical situations.

To sum up, although CM can account for the evolution of neighboring states, it cannot explain the change of instantaneous states (i.e. positions). Thus understood, CM is not yet a complete theory of motion; it does not state the evolution law of instantaneous states, that is, it does not tell us how position changes for a moving object. How to complete CM then? Or the instantaneous states simply do not exist?

3. No instants?

An “evading” way to solve the incompleteness problem of CM is to simply deny the actual existence of instants and instantaneous states. If instantaneous states do not exist, then CM, which does not account for the evolution of instantaneous states, can be still complete. This seems to be the simplest solution, but as we will see, it is in fact the most difficult and radical way, which requires that the existing language of both mathematics and physics should be largely revised.

The “no instants” view can be traced back to Aristotle. He used it to refute Zeno’s paradox of the arrow. If time is infinitely divisible and instants don’t exist at all, then Zeno’s argument will naturally collapse, as one cannot say that the arrow is at rest at every instant of its flight. In fact, motion is simply irreducible on the “no instants” view. In order to assess this view, let’s first look at the instants more closely. On one account, the notion of an instant is derived from a process of dividing an interval into smaller and smaller parts. Obviously, this process will have no end when time is infinitely divisible. So if an instant is defined as the smallest part of an interval, where a part is itself defined in terms of dividing that interval, such an instant does not exist indeed. The continuity of time entails that there is no smallest part of an interval. However, this unsuccessful extrapolation does not imply that instants cannot exist. We can have another different conception of an instant; it is regarded not as a part of an interval, but as an extensionless boundary between two parts of an interval (Le Poidevin 1997, p.180). The present moment, for example, may be thought of as a boundary between past and future. Note that this seems to have been Aristotle’s view: “The now is a link of time....for it links together past and future, since it is a beginning of one and an end of another.”(Aristotle 1996, Book IV, 222a10) Interestingly, on this view, although instants do exist, time is not composed of instants[11].

In fact, it is perfectly consistent in mathematics that a continuous line is composed of points and can even be formed by points in view of the modern set theory. In the words of Grunbaum, “It is a commonplace in the analytic geometry of physical space and time that an extended straight-line segment, having positive length, is treated as consisting of unextended points, each of which has zero length. Analogously, time intervals of positive duration are postulated to be aggregates of instants, each of which has zero duration.’ (Grunbaum 1952, p.288; Grunbaum 1967, p.121) On this view, instants are indeed parts of a time interval. Moreover, they are more basic than time intervals; the aggregates of uncountably infinite instants can form a continuous time interval. Time can then be described by a real line, each point of which, represented by a real number, represents each instant. Indeed, this is just the standard assumption in both CM and quantum mechanics (QM).

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[10] Note that this problem does not exist in Aristotle’s theory of motion where the equation of motion is first order.

[11] This is presumably why Aristotle says that “Time is not composed of indivisible nows” (Aristotle 1996, Book VI, 239b9) in refuting Zeno’s arrow paradox, though he believed the existence of nows. See also Le Poidevin (1997), p.180.

However, as already pointed out by Arntzenius (2000), there are some measure theoretic paradoxes in a continuous, pointy space. For example, there must exist regions that have no well-defined measure in a continuous pointy space when assuming the axiom of choice and that the measure is countably additive (see e.g. Skyrms 1983 and Wagon 1985). This may further yield many paradoxical results, one of which is the well-known Banach-Tarski paradox, according to which the volume of the fusion of a finite number of disjoint regions can be altered by rearranging them without stretching or distorting the shape of each region. A possible response to this objection to points is that, though these paradoxes imply that the basis of modern mathematics might have some flaws, they may not essentially relate to the existence of points, let alone provide a devastating argument against points. One might deny the axiom of choice and re-write mathematics based on points (though this is still a controversial issue and surely a very large project in mathematics), and one may also simply disregard these paradoxes in physics, as they do not appear in practical physical situations (even in mathematics one can only prove the existence of measureless regions with the help of the axiom of choice, and one can not have explicit construction of measureless regions).

A more relevant argument against points is related to the physical measurability of points. It says that points and measure 0 differences between regions in space and time have no physical effects, and thus a physical theory should be set in a no-points or gunky space, not in a pointy space (Arntzenius 2004, p.7). For example, it is a standard practice in QM that the wave functions are regarded as equivalence classes of (square integrable) functions that differ up to Lebesque measure 0. However, one the one hand, the unobservable may still exist in nature; one the other hand, one can simply insist that a physical theory needs not to be constructed solely on observable quantities. For example, even if instants do not exist, one can also first assume them in the theory, and then "filter" their unobservability by hands in the final results. By comparison, if instants (0-sized or finite-sized) do exist, the "no instants" view will be completely wrong.

As I think, these arguments against the existence of durationless points in continuous spacetime do not really favor the "no instants" view; rather, they actually favor the discreteness of spacetime much more. In discrete spacetime, there is a minimum finite part of spacetime (but it does not require that time is simply composed of these minimum time units). For one reason, the infinite divisibility of spacetime assumed by the "no instants" view, like the durationless instants it opposes, cannot be observed and confirmed in principle either; only finite time intervals can be measured. If it is better to construct a physical theory based on observable quantities, as Heisenberg did in successfully establishing the matrix formulation of QM, then both durationless points and the gunky spacetime should be rejected, and only discrete spacetime can be one basis for a fundamental physical theory, though its precise meaning still needs to be studied. In fact, it has been generally argued that a proper combination of quantum theory and general relativity, two results of which are the formula of black hole entropy and the generalized uncertainty principle (see, e.g. Garay 1995; Smolin 2001), may imply that space and time are not continuous but discrete. In particular, the hologaphical principle, which is a more general principle and widely regarded as one of the bases of a complete theory of quantum gravity ('t Hooft 1993; Susskind 1995), indicates that the information inside a finite spacetime region is also finite. This again implies that the features of large spacetime regions are determined by the features of minimal-sized spacetime regions, and thus spacetime will have ultimate finite parts.

Therefore, it seems that every problem associated with the existence of points can be overcome; there is no single devastating argument that space and time have to be gunky. Moreover, some objections to durationless instants can be equally taken not only as objections to the gunky spacetime, but also as support for the discrete spacetime.

Lastly, let's briefly see some difficulties that the "no instants" view has to meet. Although these difficulties might be not insurmountable, they do show that what a big price we need to pay for this view and how radical it is to do physics in gunky spacetime. On the "no instants" view, there are neither velocities nor positions at times for a moving object, as there are simply no instants[12]. Then how can one conceive of motion in features other than as being in different positions at different instants? Imagine an object changing its state from rest to motion. Exactly when does the object begin to move, i.e. change its

[12] No nows exist either; there are only past and future. This seems to be more counterintuitive.

position? And when does its velocity change from zero to non-zero? It seems that the object is always both at rest and moving during any time interval around the changing time, no matter how small it is. How to understand this changing process then? It seems irreducible and unanalyzable, as no minimum temporal part is available to bear neither position and velocity nor the change of velocity in the gunky spacetime.

As another more illustrating example, consider a wheel rotating around a center (see also Arntzenius and Hawthorne 2005, p.446). Rest can only be attributed to the center of the wheel, and its cannot be a property attributed to any spatial region larger than the center point itself, as any such regions will have moving parts. As Barrow (1678) claimed, "Rest is often peculiar to them [i.e., points] ... as ... to the center of a wheel." Then consider a gunky or non-pointy wheel that is rotating. How can we represent the fact that the center is at rest? We cannot attribute rest to any extended portion of the wheel, since any such part will have moving parts. We might simply deny the existence of such a center. However, the point that Barrow raised for the center of the wheel can be raised for any given point. For at any point there is a velocity (composed of a magnitude and a direction) that obtains there and not elsewhere within the neighborhood.

Thus it is extremely difficult, though not impossible, to describe simple continuous variation in gunky spacetime due to the lack of point-sized bearers such as instants and locations (see Arntzenius 2005 and Arntzenius and Hawthorne 2005 for some attempts). Admittedly, there have been a number of approaches to the mathematics of gunky spaces. Moreover, as pointed out by Arntzenius (2004), these approaches can generally be divided into three categories: the measure theoretic approach (Sikorski 1964; Skyrms 1993), the topological approach (Roeper 1997), and the metric approach (Gerla 1990). However, on the one hand, some approaches are not suitable to describe physical situations; on the other hand, although some approaches may be suitable, it is very complex and cumbersome to do physics by using them. Here it is also worth noting that in physics the gunky spacetime and the pointy spacetime cannot be distinguished in principle. So why not directly use the simple pointy geometry to do physics?

To sum up, instants still do well in both mathematics and physics, and there is no compelling reason to reject them in our physical theories. By comparison, the "no instants" view not only has no firm basis, but also requires a radical revision of modern mathematics and physics, while the feasibility of this revision is still unknown, though we already know it will lead to an unwieldy and unintuitive formalism. In addition, motion is simply irreducible on the "no instants" view. This seemingly makes motion unintelligible. So, before accepting these strong claims, we had better first examine whether motion can be further analyzed, and whether there is some other ways to complete CM by stating how the position of a moving object changes.

4. Intrinsic velocity: the cause of motion?

In fact, there is a very direct way to complete CM. It is to endow an object with an instantaneous intrinsic velocity that does not dependent on the change of position (Bigelow and Pargetter 1989; Lange 2005; Tooley 1988). The standard velocity is defined by the change rate of position, so it is not surprising that it alone cannot be used to explain the change of position. If there exists an intrinsic velocity, then position can be defined by the integral of this velocity, e.g. as Tooley (1988) argued, and the change of position can thus be explained by the intrinsic velocity. Due to the existence of intrinsic velocity, the instantaneous states of an object will be different for different velocities, e.g. for the moving state and rest state of an object. In the example given in Sec. 3, the object moving from A to B and the object moving from B to A will have different intrinsic velocities at each instant, and the intrinsic velocity can cause the object to subsequently move in the direction in which it is pointing. Thus, we can explain why the rightward moving object continues to move right and the leftward moving object continues to move left by resorting to the introduced intrinsic velocity.

A similar view, called the impetus theory, already appeared before the founding of CM[13]. The idea of

[13] The idea can even be traced back to the Christian philosopher Philoponus who lived in the sixth century. For an introduction of Philoponus' theory of impetus, see, for example, Sorabji (1988).

impetus was originally presented, notably by Buridan, to avoid the difficulty for explaining continued motion in Aristotle's theory of motion, which assumes external force is the cause of motion. Aristotle could not explain in a satisfying way why a projectile keeps flying after the external force is no longer pushing on it[14], which situation is similar to the example given in Sec. 3. According to the impetus view, a projectile continues in motion because of the force transmitted to it by the agent that launched it. This force internal to objects is called impetus. Therefore, the inertial motion, namely the motion that occurs without any external force, is sustained by an internal motive force, the impetus, which can be transferred from an external propelling agent that initializes the motion. In this sense, impetus, an internal force, not external force, is the cause of motion. Since the amount of impetus, according to Buridan, is proportional to both the mass and the velocity of an object, one can similarly define an intrinsic velocity by dividing impetus by mass. This instantaneous intrinsic velocity, though essentially different from the standard velocity defined in terms of change of position, has the same value as the standard velocity at each instant, and can be regarded as the cause of motion. Therefore, it seems that the impetus or intrinsic velocity can fulfill the causal role required for explaining continuous motion in CM, and hence the impetus view can solve the causal explanation problem or incompleteness problem for continuous motion in a satisfying way.

The impetus theory was very popular in the pre-Newton times. In fact, Newton was also its adherent at one time before he founded CM. He called it "the inherent, innate and essential force of a body"[15]. Even after the founding of CM, the standard velocity was also explained as something like impetus, e.g. power or tendency, in order to make it play the explanatory role required by the traditional causal interpretation of CM (see, e.g. Maclaurin 1742, p.53-55; Walton 1735, p.47). More surprisingly, many contemporary students still have such an impetus belief[16]. The remarkable popularity of the impetus belief strongly implies that it has some reasonable elements. To begin with, this belief might be derived from a lifetime of kinesthetic experience, and seems consistent with everyday experience. Secondly, the belief is natural and intelligible. Motion involves change of position, and a change requires a cause according to the principle of causality — thus, motion should have a cause. Since continued motion (e.g. the flight of an arrow) occurs without any external force in the direction of motion, there must exist an internal motive force that moves the object in this direction, which is just the impetus. This line of reasoning seems quite justifiable as the change of position is primary for motion.

Unfortunately, this complete version of CM based on intrinsic velocity or impetus has serious drawbacks. In this theory, we need to assume a further quantity, intrinsic velocity, to our ontology, and also need to add a natural law which states that the intrinsic velocity always has the same value as the standard velocity at each instant. However, it seems that this larger ontology might be implausible and unnecessary when considering Ockham's razor. More importantly, these two velocities are related only by natural law, not by metaphysical necessity, and thus it is conceptually possible that they would not be equal[17] (Arntzenius 2000, p.206). This poses a serious problem for the impetus theory. The theory requires that intrinsic velocity is not merely nomologically connected to change of position, but rather is essentially something to do with change of position (e.g. defined in purely kinematic terms). But it is very difficult to capture the essentially kinematic character of intrinsic velocity along with its explanatory role; whatever the intrinsic velocity is, it does not seem to be a velocity nor is it properly connected to velocity in general.

Lange (2005) presented a more sophisticated impetus theory to solve the problem. He proposed that intrinsic velocity is something like a dispositional property, and it can be interpreted as a tendency, a power, or a propensity of an object to follow a particular future trajectory, as determined by the first time

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[14] For a detailed discussion of Aristotle's explanations of continued motion, see Sorabji (1988).

[15] In a short paper sent to Edmond Halley in the spring of 1685, Newton said: "The inherent and innate force of a body is the power by which it preserves in its state of rest or of moving uniformly in a straight line." (Herivel 1965, p.331)

[16] Clement (1982) showed that nearly 80% of a group of engineering freshmen had the impetus belief. They thought the force from hand pushes up on the tossed coin when it is on the way up.

[17] In my opinion, there exist serious doubts about the existence of such a natural law since, as we will see, intrinsic velocities and standard velocities satisfy different velocity transformations; intrinsic velocities should be unchanged in different reference frames, while the standard velocities will be different in different reference frames.

derivative from above of its trajectory at each point. This suggestion can avoid the above criticisms since the intrinsic velocity as the relevant disposition is defined in a purely kinematical way. Moreover, as long as the trajectory is smooth the intrinsic velocity will match the standard velocity according to the prior trajectory. Therefore, although a natural law is required to link intrinsic velocity to standard velocity, it is the one that already exists. This follows from the fact that CM implies that all trajectories are continuous and smooth (see Lange 2005, Sec.2).

However, as Harrington (2008) already pointed out, the purely subjunctive definition of intrinsic velocity also has physical and metaphysical implausibility. First, such subjunctive properties are not only never actualized in the actual world, it is nomologically impossible that they could be actualized in any world governed by CM and containing at least two objects. The intrinsic velocity is defined according to the path that the object would follow if no forces were present, but in the actual world and any possible world containing at least two objects and obeying CM, all inertial trajectories are empty. Next, the definition fails to account for the transformational properties of velocity. Although intrinsic velocity is causally relevant, its value is not causally relevant since the value can be zero in an appropriate inertial frame. Moreover, it seems very odd that the intrinsic velocity of an object is only defined relative to other objects and is constrained to vary with them. As we will see in the next section, these issues relating to relativity of motion may block any attempt to complete CM by adding deterministic cause (e.g. intrinsic velocity) to the motion of objects.

5. Motion has no deterministic cause

Motion involves change in position. If we can find the actual cause of the change, we may discover how objects move, and as a result, a satisfying way to complete CM will naturally appear. The analysis in the last section has already suggested that intrinsic velocity probably does not exist, and there may not exist any instantaneous condition or cause to determine the motion of an object in a definite way[18]. In this section, we will further examine whether motion has deterministic cause (e.g. intrinsic velocity) or not, in particular, with the help of the relativity of motion.

Force is the cause of the change of motion (i.e. change of velocity). What is the cause of motion (i.e. change of position) then? I will first give a heuristic answer, based on a very simple but informal argument. Motion and rest are equivalent (i.e. motion is relative). No cause exists for rest. Thus motion also has no cause[19]. A more detailed argument can be given as follows. Imagine an object being initially at rest in an inertial frame. After imposing an influence (e.g. external force) on the frame but not on the object, the object will move with a constant velocity in the inertial frame. Since no cause results in the position change of the object when it is at rest, and no influence is added to act upon the object during the process from rest to motion, no cause results in the position change of the object when it is moving with a constant velocity either. Therefore, no cause determines the position change of a freely moving object. In short, free motion has no cause. Does the motion under the influence of an external force have a cause? Is force the cause of such motion? CM already gives a negative answer; force is not the cause of motion but the cause of the change of motion. To sum up, motion has no (deterministic) cause[20].

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[18] Note that indeterminism as a possible way out has been briefly referred to but immediately rejected by Arntzenius (2000). He simply said that determinism should not be bound to fail just because of Zeno’s arrow argument, though we should have no a priori commitment to determinism. In Arntzenius (2004), he further claimed that the atomicity of the structure of time alone should not imply that the world cannot be deterministic. These reasons are both right, but as we will see, indeterminism may have a firm basis in other aspects, e.g. in the relativity of motion.

[19] Historically, the consideration of the equivalence between (uniform rectilinear) motion and rest also made Newton finally transform impetus into inertia, and further founded CM (see, e.g. Westfall 1983). However, as I have argued above, this is an incomplete revolution. There is still an incompleteness problem left, which needs to be solved by further analyzing the cause of motion.

[20] As commonly used, the word “cause” always refers to deterministic cause in this paper. I will discuss probabilistic cause

Although the above argument seems plain, there may exist some possible objections to it. I will discuss them one by one in the following.

Objection 1: There might still exist a certain cause for an object being at rest.

It is possible that both the object and the inertial frame have the same cause to make them move relative to a common frame. So they move with the same velocity relative to it and are at rest relative to each other. However, we don't care about the motion situation of the object in other frames, and we only care about the cause of its motion in the inertial frame we study. In this frame, no cause results in the motion of the object when it is at rest. In fact, we don't care about whether the motion of the object is apparent motion or true motion either; our argument is irrelevant to the possible existence of absolute frame and absolute motion.

Objection 2: Some sort of cause may respect the equivalence between motion and rest.

This is impossible, as there is no cause for rest. Thus, no cause can respect the equivalence.

Objection 3: Is there really no influence added to act upon the object during its state changing from rest to motion?

According to CM, when an external force is imposed on the inertial frame but not on the object, the velocity of the object relative to the frame will change, and this change can be explained by the relativity of motion. During this process, there is no physical influence (instantaneously) transferred from the frame to the object. This is well understood. So, although some influence is indeed added to the frame, no physical influence, either from the frame or from other objects, is added to act upon the object during its state changing from rest to motion.

After answering these objections, I will further purify the argument and try to make the conclusion based on the least assumptions and the firmest experience. It can be seen that the conclusion that motion has no cause is mainly based on the relativity of motion. Here the relativity of motion does not denote the principle of relativity, Galilean relativity or Einstein's relativity. Rather, it only means that velocity, or position change in general, is relative. The motion of an object is relative to other objects or reference frames; an object can be at rest in one reference frame, but it can be moving in another reference frame at the same time. The motion is not restricted to uniform rectilinear motion or uniform accelerating motion etc, and it can be any sort of motion. In fact, the above argument can be formulated in a more general way, independent of the existence of inertial frames and uniform rectilinear motions. Suppose an object is at rest in a reference frame. After adding an influence (e.g. external force) to the frame but not to the object, the object will move in the frame. Since no cause results in the position change of the object when it is at rest in the frame, and no influence is added to act upon the object during the above process, no cause results in the position change of the object when it is moving in the frame either. Thus, the motion of the object, which can be any sort of motion, has no cause. Here we need not resort to Newton's second law to demonstrate that the forced motion has no cause, as I argued above. In addition, I stress again that the relativity of motion here is irrelevant to the existence of absolute motion. For example, if there exists an absolute frame, the relativity of motion will denote the relativity of apparent motion, and the conclusion will be that apparent motion has no cause.

Therefore, the conclusion that motion has no cause does not depend on the laws of CM, and it is only based on the relativity of motion, which is one of our firm experience about motion and has also been confirmed in both macroscopic and microscopic domains. At first sight, this conclusion seems trivial and is already implied by CM. According to Aristotle, motion has a cause, and the cause is force. But CM utterly rejects this view, and it leaves no cause for motion. Thus motion will have no cause according to CM. However, a further analysis will show that it is actually inconsistent with CM[21]. In order to see this, let's

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later.

[21] The existing solutions are still in the framework of CM, which assumes both continuous motion and determinism. However, the conclusion that motion has no cause obviously contradicts the principle of causality with which CM is expected to comply. Moreover, as we will see, the uncaused motion will be essentially random and discontinuous, as no deterministic cause or instantaneous condition determines the position change of a moving object. This is further inconsistent with the core of CM, the assumptions of continuous motion and determinism.

look at one of the inferences of this conclusion.

6. Random discontinuous motion

Consider a free particle moves in one-dimensional space. The particle is in one position at an instant, and spontaneously appears in another position at another instant. The position of the particle is constantly changing, but no deterministic cause or instantaneous condition determines the position change of the particle. How does the particle move then? An intuitive answer is that, since the particle doesn't "know" how to move at each instant, it can only move in a random way. In the following, I will present a more detailed argument for this conclusion.

First, the free particle can be in any position of the one-dimensional space. Nothing restricts the range of its possible position. By comparison, for a particle limited in an infinite potential well with width *L*, it can only move in a finite-sized space. Next, no deterministic cause determines the position change of the particle. This means that its positions at different instants are irrelevant, and no instantaneous condition determines the position of the particle at each instant (after the initial time) either. Therefore, the particle will be randomly in one position of the one-dimensional space at each instant[22]. We can understand this conclusion more visually by a selecting process. At each instant the particle selects one position from all possible positions in the whole one-dimensional space, but no rule restricts the selecting process and the particle can select any position each time. So the position of the particle must be random in the whole one-dimensional space at each instant, and the position change or motion of the particle must be also random.

It can be further seen that this random motion must be discontinuous everywhere. The particle is in one position at an instant, and at the instant immediately neighboring it randomly appears in another position, which is irrelevant to the previous position. The new position is probably not in the neighborhood of the previous position, as the range of positions outside the neighborhood is much large than that near the neighborhood. In a word, the randomness at instants will "tear" apart the continuity of motion and make motion essentially discontinuous[23]. This seems plainly inconsistent with the apparent continuous motion of macroscopic objects. We will briefly explain this apparent inconsistency in the last section, and a more detailed explanation can be found in Gao (2006a, 2006b).

It should be stressed that the random motion here is different from the process of playing dice. In each run of playing dice, there exist some physical causes that determine the number of the thrown dice in principle. But we don't know the causes and think the process of playing dice is random. This randomness comes from our ignorance of the actual causes determining each result of playing dice. By comparison, the above randomness in motion is objective and independent of our knowledge. There is no cause at all for motion in nature. Moreover, this objective randomness further implies that we can never predict the position change of a particle, even in principle.

7. More about relativity of motion

The conclusion that motion has no (deterministic) cause is based on relativity of motion. This relativity of motion is only the relativity of continuous motion. But if motion has no cause, then continuous

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[22] If there is no any cause including probabilistic cause, then the probability of the particle appearing in each position will be the same. On the other hand, if there is probabilistic cause, then the probabilities of the particle appearing in different positions may be different. We will discuss the probabilistic cause in more detail later.

[23] It can be argued that the uncaused motion should be also random and discontinuous even in gunky spacetime, as no matter how to define the position of an object according to the "no instants" view, its change should also have no cause. However, it seems that the gunky spacetime cannot accommodate such motion, which requires that there is still discontinuity during an infinitesimal time interval, and as a result, the conclusion that motion has no cause may also refute the "no instants" view.

motion, as well as its relativity, will not exist, as motion will be discontinuous and random, as argued above. So it seems that there exists a problem in the above argument. Let's make the argument clearer. It first assumes that motion is continuous, and then, based on the relativity of continuous motion, it reaches the conclusion that motion has no cause, and thus motion is not continuous but discontinuous and random. This is a contradiction; the assumption of continuous motion leads to discontinuous motion. What we can get from this contradiction is only the conclusion that the assumption of continuous motion is wrong, not the conclusion that motion has no cause. If motion is not continuous, then it must be discontinuous everywhere in space. However, the discontinuous motion may be not random. Thus it seems that there is indeed a loophole in the above argument.

The crux lies in whether the motion-has-no-cause conclusion depends on the assumption of continuous motion, that is to say, whether discontinuous motion also has no deterministic cause. If discontinuous motion is also relative, then it should also have no cause, as we have argued above. Is discontinuous motion relative then? At first sight, discontinuous motion is not relative, as we cannot make it at rest, as for continuous motion, by selecting a proper reference frame. However, this only indicates that discontinuous motion is absolute relative to continuous motion, which the reference frame undergoes[24]. In fact, discontinuous motion is also relative. For an object undergoing discontinuous motion, if another object also moves in the same discontinuous way, then the object will be at rest relative to the other. In other words, we can also make discontinuous motion at rest by selecting a special reference frame which undergoes discontinuous motion. Therefore, all possible motion, including both continuous motion and discontinuous motion, is relative and thus has no cause. In particular, discontinuous motion also has no (deterministic) cause, and thus it must be random.

Yet there is a possible problem in the above analysis, that is, whether two independent objects can move in the precisely identical discontinuous way (Note that reference frame, especially its state of motion, should be independent of the object). For continuous motion this poses no problem, as it only requires that these two objects move with the same velocity. For discontinuous motion we need to consider two different situations. If the discontinuous motion of an object is determined by a deterministic cause, then it is indeed possible to make another independent object move in the same discontinuous way, simply by imposing the same deterministic cause on it. Since the cause is deterministic, we can do this in principle. Thus the above analysis holds true for this situation. However, if the discontinuous motion of an object has no deterministic cause, then the discontinuous motion will be essentially random. For this situation, it seems impossible to make another independent object also move in the same random way[25], as two independent random processes cannot be the same at every instant. Therefore, the discontinuous motion that has no deterministic cause is not relative in the usual meaning. Although the above analysis does not apply to this situation, the conclusion that discontinuous motion is random still holds true. In a word, the discontinuous motion has no (deterministic) cause, and it must be random. As a result, motion is both discontinuous and random.

8. Making potential real

If motion is really random and discontinuous, then the incompleteness problem or causal explanation problem plaguing the "at-at" theory of continuous motion will not exist. Motion is not continuous but discontinuous and random, and hence we need not to find some cause (e.g. intrinsic velocity) to explain it in a deterministic way. The "at-at" theory of random discontinuous motion is already complete in this sense. However, this theory still has a serious drawback, which can be seen from an example similar to that discussed in Sec 1.3. Imagine an object moving in a random and discontinuous way, and it has different

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[24] Note that, even though motion is essentially discontinuous, the reference frame as a macroscopic system may move in a continuous way in the meaning of time average, which will be explained in brief in the last section.

[25] It is worth noting that if two particles are permitted to interact, then they can move in the precisely same discontinuous way. As a result, the objects will be at rest relative to each other. In fact, the motion of the two particles is no longer independent, and they can be taken as an inseparable whole.

probabilities to appear in different positions. Then how can the object "know" where it should appear in a larger probability (or more frequently) and where it should appear in a smaller probability (or less frequently)? In other words, no instantaneous condition or cause determines the probabilities of the future position of the object. To speak in a more technical language, the theory is not Markovian[26]. In fact, the causal explanation problem is only the deterministic aspect of the not-Markovian problem, and the problem still has an indeterministic aspect, as shown in the above example.

This not-Markovian problem can be solved from the start, irrespective of the random discontinuous motion, and we will see that the solution can also lead us to the random discontinuous motion, moreover in a more satisfactory form. The line of reasoning is as follows. In order to make the "at-at" theory of motion be Markovian, some sort of intrinsic property existing at instants is still needed. Since the (deterministic) cause such as impetus or intrinsic velocity is not available due to the relativity of motion, only probabilistic cause or indeterministic potential[27], which determines the probabilities of the future position of a moving object, can help to complete the "at-at" theory and make it Markovian. This again means that motion is not continuous but random and discontinuous. When taking seriously the drawbacks of the existing views of motion (i.e. the "at-at" theory, the "no instants" view and the impetus theory), this potential theory of motion seems to be a promising way to obtain a complete Markovian theory of motion.

It seems that CM, by rejecting Aristotle's view of motion, already implies that motion has no deterministic cause such as force. However, the uncaused motion obviously contradicts the principle of causality with which CM is expected to comply. Moreover, the uncaused motion is both random and discontinuous, as we have argued above. This is further inconsistent with the core of CM, the assumption of continuous motion. Thus, it is very natural to raise some doubts about the above argument. In particular, the argument seems to depend on the analysis of causation, which concept should be avoided in physics according to some philosophers[28]. In the following, I will present a further analysis of motion, which avoids the talk of causation and concentrates more on the law itself. As we will see, the analysis will make the random discontinuous motion and the potential more real.

The continuous motion requires that there exists a cause or intrinsic property, e.g. intrinsic velocity $v(t)$, that can determine the change of position after an infinitesimal time interval *dt* (see, e.g. Bigelow and Pargetter 1989; Carroll 2002; Lange 2005; Tooley 1988). This can be represented by the formula $x(t+dt)=x(t)+v(t)dt$ in mathematics (see also Tooley 1988). However, as I have argued, the relativity of motion permits no existence of deterministic cause like the intrinsic velocity. If there is no cause at all, $x(t+dt)$ will not be determined by $x(t)$ and can assume any possible value with the same probability, irrelevant to $x(t)$, and thus the change of position will be not deterministic and continuous but random and discontinuous everywhere. On the other hand, if there is probabilistic cause or potential, represented by $\rho(x,t)$, which determines the probabilities of the future position of the object, the motion of the object, represented by its trajectory $x(t)$, will be still random and discontinuous[29]. In this situation,

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26 It is generally believed that the actual world and the theory describing it should be Markovian, i.e. that the future state of the world, at any given moment, depends only on its present state in a deterministic or probabilistic way, and not on any past states, in short, all influences that the past has upon the future run through the present (see, e.g. Arntzenius 2000, p.191).

27 A probabilistic cause determines the probability of its effect. For a detailed introduction of probabilistic causation, see Suppes (1970) and Eells (1991). In addition, it should be pointed out that, although the word "potential" used here may remind us of Aristotle's potentiality in his theory of motion, there is no direct connection between them.

28 It has been a controversial issue whether causal relations are objective features of reality, and whether physical theories should describe causal relations. I shall not discuss them here.

29 Note that the no-cause situation can be seen as a special situation of probabilistic cause in which $\rho(x,t)=1$.

the above formula can still exist, but in the meaning of time average. It is: $\bar{x}(t+dt)=\bar{x}(t)+\bar{v}(t)dt$, where $\bar{x}(t)=\int x\rho(x,t)dx$, $\bar{v}(t)=\int v(x,t)\rho(x,t)dx$, $v(x,t)$ is the local velocity of potential. Here the change of the time-averaged position is also continuous. However, this time-averaged formula should not be understood as above; the time-averaged velocity is not the deterministic cause that determines the change of position, while the actual probabilistic cause determining the random change of position hides inside the time average. If the evolution of potential satisfies a certain law, which will be discussed in detail later, then the continuous motion described by CM can be regarded as the ti me-averaged display of random discontinuous motion, and the above formula will be used to define the standard temporally extrinsic velocity of a moving object in CM, namely $v(t)=dx(t)/dt$.

The potential, represented by $\rho(x,t)$, can be defined as an instantaneous intrinsic property of an object, which determines the probabilities of the future position of the object during an infinitesimal time interval dt near instant t. During the infinitesimal time interval the potential is realized and the object appears in every possible position x with probability density $\rho(x,t)$. The potential as an intrinsic property does not change with the selection of reference frame, and it is absolute in this sense.

Let's look at some properties of potential. First of all, the definition of potential means that during an infinitesimal time interval the change of $\rho(x,t)$ can only be infinitesimal, which further means that the evolution of the potential is continuous. Thus, potential itself already implies the continuity of its evolution. Next, the infinitesimal change of potential during an infinitesimal time interval can be written as $d\rho=\frac{d(\rho v)}{dx}dt$, satisfying the continuity condition or potential conservation relation. The potential, as an instantaneous intrinsic property of an object, will conserve during its evolution. Then during an infinitesimal time interval there is also another quantity, $j(x,t)\equiv\rho(x,t)v(x,t)$, which describes the change rate of potential and may be called potential flux. Potential and potential flux, i.e. $\rho(x,t)$ and $j(x,t)$, will constitute the complete state of motion for a moving object during an infinitesimal time interval near instant *t*. They satisfy a certain equation of evolution, which initial condition includes both potential and potential flux. As a result, the evolution of potential (as an instantaneous state) alone is not Markovian[30].

9. Consistency with experience: a primary analysis

It is well known that CM, which assumes motion is continuous, is only an approximate description of nature, and it has been replaced by QM, a more fundamental theory, which recognizes the randomness and discontinuity of natural processes (at least according to its standard explanation). Therefore, it seems not beyond expectations that CM is plagued by the causal explanation problem, and the lack of a causal explanation may imply that the actual motion, of which classical mechanics is an incomplete description, indeed has no deterministic cause. Therefore, motion is not deterministic and continuous but random and

30 This may be not unexpected. Although potential itself is an instantaneous state, it determines the probabilities of the future position of an object during an infinitesimal time interval (not at an instant). In some sense, because the change of position, determined by potential in a probabilistic way, is Markovian, the evolution of the potential is not Markovian.

discontinuous. However, the random discontinuous motion (RDM in brief) seems plainly inconsistent with observations in the macroscopic world; the macroscopic objects apparently move in a continuous way. In order to make RDM acceptable and proceed with our analysis, we need to first assure that it is possible to explain away this apparent inconsistency at least.

The picture of RDM is as follows. A particle is in one position at one instant. Then it will still stay there or stochastically appear in another position, which is probably far from the original one, at another instant. During a time interval, the particle will move throughout the whole space with a certain position probability distribution. How can RDM generate the appearance of continuous motion then? A possible answer is that RDM may happen in extremely short space and time intervals for macroscopic objects, in particular, the position probability distribution may always concentrate in a very tiny local region for a macroscopic object. This may be due to the evolution law of RDM as well as environmental influences. As a result, a macroscopic object will be in a local region at each moment, and can only be at rest or move continuously in appearance. Moreover, velocity will appear in the meaning of time average, and we can get $v = dx/dt$, the definition of velocity, from $< j/\rho >= d < x > /dt$ (a result of potential conservation relation), where $< x >= \int x\rho(x,t)dx$. The law of continuous motion may also be derived from the law of RDM by time average, and we may get $dp/dt = -\partial U/\partial x$, the motion equation, from $d\langle p\rangle/dt = \langle -\partial U/\partial x\rangle$, where $p = mv$ is momentum, and m is mass. In this way, the time-averaged display of a large number of minute discontinuous motions can generate the apparent continuous motion in the macroscopic world. Furthermore, CM is also an almost exact description of the apparent continuous motion of macroscopic objects.

The above explanation, though very primary, provides a possible way to reconcile random discontinuous motion and the apparent continuous motion for macroscopic objects. Certainly, in order to know how random discontinuous motion generates the appearance of continuous motion, we must know the precise law of random discontinuous motion, which is still unknown[31]. On the other hand, random discontinuous motion seems quite consistent with observations in the microscopic world. It seems that a microscopic particle can be easily in different positions at the same time, as the "Schrödinger cat" superposition states of a single atom already implies, though people have different opinions on the explanation of these phenomena. Besides, quantum mechanics, of which CM is only an approximate, also recognizes the randomness and discontinuity of natural processes (at least according to its standard explanation), and thus there may exist some possible connections between random discontinuous motion and quantum mechanics (Gao 2011). I shall leave this bigger problem for future study.

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