Reconsidering Born's Postulate and Collapse Theory

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Abstract
We investigate the possibility that the amplitude of quantum wave can have a deeper physical meaning other than the probabilistic interpretation base on Born's postulate. We find that the probabilistic nature of a quantum system can be explained if matter has fluctuations in space and time. The basic properties of a zero spin bosonic field (e.g., Schrödinger's equation, Klein-Gordon equation, probability density, second quantization) are derived. In addition, the properties of this system with fluctuations in space and time can be applied to explain the wave packet collapse in quantum measurements.

Keywords: Foundations of quantum mechanics; Born's postulate; fluctuations in space and time; wave collapse; hidden variables.

1. Introduction

The ontological and epistemological implications of quantum mechanics have been the central debates of the theory since its inception. For example, in the Copenhagen interpretation, a quantum system can exhibit contradictory properties. It can appear to be continuously distributed in some cases and localized in others: wave-particle duality [1]. A phenomenon can be observed one way or another, but never simultaneously. This wave-particle complementarity principle is deeply embedded in the fundamental concepts of quantum mechanics.

In the formulation of quantum mechanics, the Schrödinger equation is a partial differential wave equation that describes the (deterministic) evolution of the wave function. All the information of a quantum system is believed to be encoded in this function, and its propagation gives rise to the wave-like behavior of a particle. According to Born's postulate [2], the amplitude of the wave function has no physical meaning other than its probabilistic interpretation. The statistical nature of quantum mechanics originates from this assumption but there is no explanation on how and why these probabilities are generated.

The particle-like nature of matter wave is evident when the location of a particle is measured; the effect is always localized. After the measurement, the wave function undergoes a random collapse to a more localized state. Different results can be yielded upon measuring superposition states that are created from identical copies of a system. Although time evolution of the superposed quantum states is determined by a unitary operator, only a definite state can be measured.

The collapse of wave is a key problem in understanding quantum measurement. There is nothing in the Schrödinger equation that allows such transition. Von Neumann added this dynamically discontinuous collapse in his formulation [3] as one of the two processes by which quantum systems evolve over time. However, the theory does not specify a dynamical mechanism how this collapse takes place. The lack of precise division when the different evolutions shall occur has created a constellation of puzzles. This ambiguity often leads to the discomforting assertion that reality does not exist unless observations are made with privileged status given to an observer.

There are in general two groups of approaches proposed as solutions for the problem. One type of approaches search answers within the framework of existing quantum mechanics, e.g. decoherence [4,5,6], many world interpretation [7,8], consistent history [9,10]. Rather than being an epiphenomenon of some other process, the second type of approaches try to look for solutions by adding new features to the theory, e.g. Bohmian mechanics [11,12,13], spontaneous collapse models [14,15,16], and other deterministic models [17]. A brief summary and difficulties for the different approaches can be found in Ref. [16]. As of today, the ad hoc assumption for wave collapse still has no fully satisfactory explanation.

To explore the probabilistic origin of quantum theory, we investigate the possibility that the amplitude of wave can have a deeper physical meaning other than the probabilistic interpretation base on Born's postulate. We find that the properties of a non-interacting spin-zero matter wave (e.g., Schrödinger's equation, Klein-Gordon equation, probability density, second quantization etc.) can be reconciled from a system with fluctuations in space and time; an explanation of how and why probabilities are generated can be provided. By studying the properties of its Hamiltonian equation, we show that matter inside the wave can only be observed as quantized oscillators which can be treated as point particles. In addition, only a probability can be assigned at a location for the particle to materialize which is generated from the "quantization potentials" in the field. These potentials and the quantized particles are real and physically present as parts of the matter wave. The probabilistic nature of a quantum field can be explained if matter has fluctuations in time and space.

The quantization potential has its unique properties. It carries information about the probabilities for a particle to materialize but does not have real energy. Sudden transfer of quantization potentials between distant locations does not require superluminal transportation of energy; instantaneous collapse of wave will not violate the principles of relativity. These unique properties allow us to explain the mechanism that is required to trigger a non-local wave collapse by measurement. Unlike in the standard interpretations of quantum mechanics, collapse is induced by interaction in a measuring process. Information of the system to be measured can be obtained from results of the interactions if one desires to make an observation. There is no prestigious status for the observer. However, not all interactions can induce collapse. The reasons why collapse cannot be induced by interactions that do not provide effective information for measurements, e.g. deflection by biprism, magnet, capacitor etc., are discussed.

Apart from the collapse by measurements, we have also investigated a different type of mechanism that may trigger collapse. Recent proposal by Xiong has suggested the possibility that wave collapse can be induced when part of the quantum wave is trapped in a closed box – topological disconnectivity [18]. Several experiments have been proposed. This concept can be understood within the framework of quantum mechanics which can also be explained in terms of the quantization potentials. Although this idea is still subject to experimental validation and we cannot eliminate the possibility that topological disconnectvity alone may not be sufficient to trigger a collapse, the verification of this concept can provide support for the collapse process we have proposed with measurements.

Our concept in this paper is formulated in the standard model energy scale. In fact, it has been suggested in emergent quantum mechanics [19,20,21,22,23,24,25,26] that determinism and reality can be returned at the high energy level. This is based on the idea that a physical system is deterministic at the Planck scale but the quantum mechanical nature of our world is due to information loss/dissipation [27,28,29,30,31] at larger scales. The question whether the field with fluctuations in space and time can have a deeper reality as proposed in Ref. [32] is not part of this discussion.

This paper is organized in the following manner: Section 2 outlines the dynamic properties of the wave with displacements in time and space. The displacement in proper time is defined as the difference between the time within the wave and the time of an inertial frame. The fluctuations can be described by a scalar field. Section 3 investigates the general properties of the quantization potentials that generate the probabilistic nature of the quantum wave. Section 4 derives the basic equations for a non-interacting spin-zero particle field (e.g., Schrödinger's equation, the Klein-Gordon equation, probability density, second quantization). Section 5 discusses two possible properties of the quantization potential field that are not necessarily restricted by relativity. Section 6 studies the collapse by topological disconnectivity. A modified experiment using single source electron is suggested for validating this proposal. Section 7 and 8 extend the collapse theory to more general measurement processes. The last section is reserved for discussing further implications of the theory.

2. Plane waves with fluctuations in space and time

In quantum mechanics, the squared amplitude of a matter wave is the probability density of locating a particle base on Born's postulate. Unlike classical waves, this amplitude has no other physical interpretation. To find an explanation for quantum statistics, we will first identify a possible alternate meaning for this amplitude.

Consider the following example: unlike general relativity, quantum theory does not establish a direct relationship between energy and space-time. Since we know that matter can affect the geometry of space-time, however, is it possible that the amplitude of matter wave can have a physical interpretation relates to the principle of relativity?

In the theory of relativity, there are four basic physical quantities: energy (E), momentum ($\vec{p}$), time (t), and distance ($\vec{x}$). All matter particles have energy, momentum, and a location in space-time whether or not they have mass or any of the various charges. These four physical quantities are sufficient to describe the propagation of a free particle. It is worth stressing that while any free particle has energy and momentum, it needs not have charge. It is therefore reasonable to assume that the physical amplitude of a free particle wave cannot be defined by a force field.

Among these four basic quantities, space-time and energy-momentum are 4-vectors. It is therefore possible to write down the amplitude of a physically vibrating plane wave as a 4-vector displacement $(\mathrm{T},\vec{\mathrm{X}})$. This 4-displacement is in effect space-time interval that can be seen as a Lorentz transformation of some proper time displacement (T_0).

$$(\mathrm{T}_0,0,0,0)\rightarrow(\mathrm{T},\vec{\mathrm{X}}), \tag{1}$$

where

$$\mathrm{T}^2=\mathrm{T}_0^2+\left|\vec{\mathrm{X}}\right|^2. \tag{2}$$

Therefore, matter inside this plane wave will have fluctuations in time and space with amplitudes T and $\vec{\mathrm{X}}$ respectively. We will investigate how this wave with fluctuations in space and time can be related to quantum theory.

Consider the background coordinates $(t_0,\vec{x}_0)$ for the flat space-time as observed in an inertial frame $'O_0'$. Assume that in this frame, a plane wave exists with fluctuations in time only, i.e. no fluctuation in space. We will use the background coordinates $(t_0,\vec{x}_0)$ as references for measuring the fluctuations that take place inside the wave.

In classical mechanics, the amplitude of a flexible string under tension is the maximum displacement of a segment of the string from its equilibrium coordinate in space. Using a similar concept, we can define the wave's displacement amplitude (T_0) as the maximum difference between the time observed inside the wave (t_0') and the time (t_0) observed outside the wave within an inertial frame. We may then write

$$t_0'=t_0+\mathrm{Re}(\zeta_{0t}), \tag{3a}$$

where

$$\zeta_{0t}=-i\mathrm{T}_0e^{-i\omega_0t_0}. \tag{3b}$$

Thus, the plane wave will have this temporal fluctuation when observed with respect to an inertial frame outside. In addition, time inside the wave passes at the rate $1-\omega_0\mathrm{T}_0\cos(\omega_0t_0)$ relative to time in the outside inertial frame. This ratio has an average value of 1. Matter will therefore still appear to travel along a time-like geodesic when observed over many cycles.

We will consider ζ_{0t} to be a field that generates fluctuations in proper time. Since matter inside this plane wave has no fluctuation in space, the spatial coordinates in the wave frame $(\vec{x}_0')$ are the same as those in the inertial frame $(\vec{x}_0)$.

$$\vec{x}_0'=\vec{x}_0. \tag{4}$$

We now study how this plane wave will appear in another frame of reference. By an appropriate Lorentz transformation, the background coordinates $(t_0,\vec{x}_0)$ of inertial frame $'O_0'$ can be related to the background coordinates $(t,\vec{x})$ for the flat space-time observed in another frame $'O'$. We assume that frame $'O_0'$ travels with velocity $\vec{v}$ relative to frame $'O'$. Similarly, the coordinates of the fluctuation $(t_0',\vec{x}_0')$ can be Lorentz transformed to the coordinates of fluctuation $(t',\vec{x}')$ as observed in frame $'O'$. We can thus relate the fluctuation coordinates $(t',\vec{x}')$ to the background coordinates $(t,\vec{x})$:

$$t' = t + \mathrm{Re}(\zeta_t), \tag{5a}$$

$$\vec{x}' = \vec{x} + \mathrm{Re}(\vec{\zeta}_x), \tag{5b}$$

where

$$\zeta_t = -i\mathrm{T}e^{i(\vec{k}\cdot\vec{x}-\omega t)}, \qquad \vec{\zeta}_x = -i\vec{\mathrm{X}}e^{i(\vec{k}\cdot\vec{x}-\omega t)}. \tag{5c}$$

The fields ζ_t and $\vec{\zeta}_x$ thus generate fluctuations in time and space respectively. The amplitude $\vec{\mathrm{X}} = (\vec{k}/\omega_0)\mathrm{T}_0$ is the maximum displacement of the wave from its equilibrium coordinate $\vec{x}$, and $\mathrm{T} = (\omega/\omega_0)\mathrm{T}_0$ is its maximum displacement from the time t. In frame $'O'$, matter in the plane wave experiences fluctuations in both space and time.

We can unify these ideas by defining a scalar field ζ for the fluctuations in the plane wave:

$$\zeta = \frac{\mathrm{T}_0}{\omega_0} e^{i(\vec{k}\cdot\vec{x}-\omega t)}. \tag{6}$$

The fields ζ_t and $\vec{\zeta}_x$ given in Eq. (5c) for the fluctuations in time and space can be obtained from ζ as follows:

$$\zeta_t = \frac{\partial \zeta}{\partial t} = -i\mathrm{T}e^{i(\vec{k}\cdot\vec{x}-\omega t)}, \tag{7a}$$

$$\vec{\zeta}_x = -\vec{\nabla}\zeta = -i\vec{\mathrm{X}}e^{i(\vec{k}\cdot\vec{x}-\omega t)}. \tag{7b}$$

The wave fluctuations can thus be described by a single scalar function.

3. Quantization of wave fluctuations

Consider the scalar field ζ and its complex conjugate ζ^*. Both functions satisfy the wave equation:

$$\partial_u \partial^u \zeta + \omega_0^2 \zeta = 0, \tag{8a}$$

$$\partial_u \partial^u \zeta^* + \omega_0^2 \zeta^* = 0. \tag{8b}$$

Eqs. (8a) and (8b) are similar to the Klein-Gordon equation, except that we have yet to understand how the scalar field ζ can be related to the zero spin particle field in quantum theory. The corresponding Lagrangian density for the equations of motion is $\mathcal{L} = K[(\partial^u \zeta^*)(\partial_u \zeta) - \omega_0^2 \zeta^* \zeta]$, and the Hamiltonian density is

$$\mathcal{H} = K[(\partial_0 \zeta^*)(\partial_0 \zeta) + (\vec{\nabla}\zeta^*)\cdot(\vec{\nabla}\zeta) + \omega_0^2 \zeta^* \zeta], \tag{9}$$

where K is a constant for the matter field and invariant under Lorentz transformation.

Let us examine the properties of this Hamiltonian density equation. Substitute $\zeta = (\mathrm{T}_0/\omega_0)e^{-i\omega_0 t}$ into Eq. (9), the Hamiltonian density of a plane wave with fluctuations in proper time only is:

$$\mathcal{H}_0 = 2K\mathrm{T}_0^2. \tag{10}$$

This result is similar to the Hamiltonian density of a harmonic oscillating system in classical mechanics, except the fluctuations are in time and not in space. In analogous to its classical counterpart, we can write constant K in terms of the angular frequency ω_0

and a mass constant m_0 per unit volume V_0, i.e. $K = m_0\omega_0^2/(2V_0)$. As we will demonstrate in the next few paragraphs, our choice for the constant K is not arbitrary.

Under Lorentz transformation, the plane wave will have fluctuations in space and time. From Eqs. (5b) and (7b), the fluctuations in space for matter in the plane wave are $\mathrm{Re}(-\vec{\nabla}\zeta)$. The Hamiltonian density per Eq. (9) corresponding to these fluctuations in space is given by the term $K\left(\vec{\nabla}\zeta *\right)\cdot\left(\vec{\nabla}\zeta\right) = m_0\omega_0^2\left|\vec{\mathrm{X}}\right|^2/(2V_0)$. This is the expected result for a classical system with mass m_0 and angular frequency ω_0 at the non-relativistic level.

Substitute K into Eq. (10), the Hamiltonian density of a plane wave with fluctuations in proper time becomes $\mathcal{H}_0 = m_0\omega_0^2\mathrm{T}_0^2/V_0$. The energy generated inside a unit volume is $E = m_0\omega_0^2\mathrm{T}_0^2$ of a harmonic oscillator in proper time. As shown in Eq. (4), matters in this plane wave are stationary in space. If Eq. (10) is the total Hamiltonian density of a matter wave, energy E shall correspond to the total internal energy of matter with mass m_0 at rest. This internal energy varies with amplitude T_0.

As we have learnt from relativity, matter with mass m_0 must have internal energy $E = m_0$ when observed. This is equivalent to the energy generated by a proper time harmonic oscillator with amplitude $\mathrm{T}_0 = 1/\omega_0$ which satisfies the condition $E = m_0 = m_0\omega_0^2\mathrm{T}_0^2$. The energy-mass equivalence in relativity imposes a constraint on the energy of a proper time oscillator that can be observed. Thus, only an oscillator with quantized amplitude $\mathrm{T}_0 = 1/\omega_0$ can materialize which can be treated as a point particle[1]. The system with fluctuations in space and time shall be a quantized particle field.

From Eq. (10), the proper time plane wave with amplitude $\mathrm{T}_0 = 1/\omega_0$ has sufficient energy for one quantized oscillator per unit volume. However, the appearance of an oscillator is random. We can only assign a probability for quantization at a particular location. In fact, these probabilities shall depend on the Hamiltonian density from Eq. (9), i.e. $\rho_{\mathrm{Pr}} \approx \mathcal{H}/m_0$ in the non-relativistic limit. A region with higher Hamiltonian density shall have more chance for an oscillator to materialize. The Hamiltonian from Eq. (9) is a potential for quantization.

In the system with fluctuations in space and time, only the quantized oscillator has energy. Other regions without the particle are "vacuum" but have the potentials for quantization. These "quantization potentials" do not possess real energy. The only information they carry are their ability to materialize into particles. We know of the their presence only by repeated measurements of the quantized energy appearing in a unit volume. Thus, the quantization potential can be the generator of the probability wave in quantum mechanics; its relation with the wave function in quantum mechanics will be discussed in the next section. The quantized oscillator and quantization potentials are intrinsic part of the matter field that has fluctuations in space and time.

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[1] This quantized oscillator will appear to travel along a time-like geodesic when averaged over many cycles as discussed in Section 2.

A particle can disappear and reappear at a distant location. Multiple particles can also be created/annihilated simultaneously. Unlike standard quantum theory, the world line of a particle in this model describes what might actually happen. The particle is real with a well defined location. The quantization potential field defines the probability that a particle will jump, where to jump and how it will move between jumps, such as in a Bell type quantum field theory [33,34]. Taking mass m_0 as the de Broglie mass-energy, we have the final form for the constant K of the system,

$$K = \frac{\omega_0^3}{2V_0}. \tag{11}$$

These comments may remind us of the virtual states that exist only for a limited time, e.g. particles can be created out of vacuum. However, virtual particles do not have a permanent existence; they arise from fluctuations of vacuum energy, and can be understood as a manifestation of time-energy uncertainty principle. On the other hand, a normalized matter wave always has sufficient energy for one particle to appear. The appearance of a particle at a particular location in the potential field is temporary but the existence of the particle in the system is real. This is due to insufficient internal energy in a region that cannot materialize fully as restricted by the energy-mass equivalence. Even in a region with insufficient energy to quantize into an oscillator, there is a potential for the creation of a particle.

Under Lorentz transformation, a particle in the plane wave with angular frequency ω and wave vector $\vec{k}$ will travel at a velocity $\vec{v} = \vec{k} / \omega$. It also has fluctuations in time and space with amplitudes $\mathrm{T} = \omega / \omega_0^2$ and $\vec{\mathrm{X}} = \vec{k} / \omega_0^2$ from the Lorentz transform of amplitude $\mathrm{T}_0 = 1/\omega_0$. We can calculate the quantized fluctuation amplitudes of a particle. For example, we can estimate the spatial fluctuation amplitude of an electron ($\omega_0 = 7.6x10^{20} rad./s$):

$$|\vec{v}| = 0.99999 \Rightarrow |\vec{\mathrm{X}}| = 8.6x10^{-9}\,\mathrm{cm},$$

$$|\vec{v}| = 0.001 \Rightarrow |\vec{\mathrm{X}}| = 3.9x10^{-14}\,\mathrm{cm}.$$

4. Quantum properties of the quantization potential field

In the non-relativistic limit, the wave functions of quantum mechanics can be derived directly from the quantization potential field. As discussed, the location for a particle to materialize is random. A region with higher potentials shall have more chance to locate a particle. We can define a probability density ρ_{Pr} base on the Hamiltonian density, i.e., $\rho_{\mathrm{Pr}} \approx \mathcal{H} / \omega_0$. By taking the approximations $\omega_0^2 \zeta^* \zeta >> (\vec{\nabla}\zeta^*) \cdot (\vec{\nabla}\zeta)$ and $(\partial_0 \zeta^*)(\partial_0 \zeta) \approx \omega_0^2 \zeta^* \zeta$ in the non-relativistic limit, the Hamiltonian density $\mathcal{H}$ from Eq. (9) becomes:

$$\mathcal{H} \approx 2K\omega_0^2 \zeta^* \zeta = \frac{\omega_0^5}{V_0} \zeta^* \zeta. \tag{12}$$

Thus, the probability density ρ_{Pr} of finding a particle within a region with Hamiltonian density $\mathcal{H}$ is:

$$\rho_{\mathrm{Pr}} \approx \frac{\mathcal{H}}{\omega_0} \approx \frac{\omega_0^4}{V_0} \zeta^* \zeta . \tag{13}$$

Base on this probability density, we can establish a relationship between the potential field ζ from Eq. (6) and the quantum mechanical wave function ψ for a plane wave within a cube of volume V :

$$\psi = \left(\frac{\omega_0^2}{\sqrt{V_0}} e^{i(\omega_0 t + \chi)} \right) \zeta = \frac{a}{\sqrt{V}} e^{i(\vec{k}\cdot\vec{x} - \tilde{\omega} t + \chi)} , \tag{14}$$

where

$$a = \omega_0 \mathrm{T}_0 \frac{\sqrt{V}}{\sqrt{V_0}} , \tag{15}$$

$$\tilde{\omega} = \vec{k} \cdot \vec{k} /(2\omega_0) \approx \omega - \omega_0 , \tag{16}$$

and $e^{i\chi}$ is a phase factor. Eq. (13) can then be written as:

$$\rho_{\mathrm{Pr}} \approx \psi^* \psi . \tag{17}$$

Using the superposition principle and taking the volume V approaches infinity, we can write

$$\psi = \frac{1}{(2\pi)^{3/2}} \int a(\vec{k}) e^{i(\vec{k}\cdot\vec{x} - \tilde{\omega}\, t + \chi)} d\vec{k} . \tag{18}$$

By substituting ζ with ψ in Eq. (8a) and taking the non-relativistic limit, we obtain the Schrödinger equation for a free particle in quantum mechanics.

As we can see, the phase factor $e^{i\chi}$ in Eqs. (14) and (18) does not change the probability density. In fact, as demonstrated in quantum mechanics, the theory developed with wave functions ψ is invariant under global phase transformation but the relative phase factors are physical. Here the wave function ψ serves as a mathematical tool for describing the quantization of the potential density. A system with wave function ψ from the superposed plane waves can have a global phase shift χ without changing the results in quantum theory. As a result, ψ is not required to have the same phase as the potential field ζ . Despite what is commonly believed, matter waves can have a physical interpretation even though their overall phase for the wave function ψ is unobservable.

The above analysis is based on a single particle system in the non-relativistic limit where approximations are taken to obtain the Schrödinger equation. As it is well known in quantum theory, when the Klein-Gordon equation is treated as a single particle equation in a relativistic theory, one will encounter the difficulties of negative energy solutions. Since the quantization potential field ζ satisfies an equation similar to the Klein-Gordon equation, we expect the system with fluctuations in space and time shall have the same properties of a bosonic field in quantum theory.

The fluctuations in space and time are real physical quantities. As shown in Eqs. (5a) and (5b), only the real component of ζ is relevant for obtaining these physical quantities. We retained the complex component of ζ in previous analysis to simplify the derivation of the complex wave function. Here, the quantization potential plane wave from Eq. (6) can be combined with its complex conjugate[2]. However, in the following analysis, we will switch to the use of a field φ for describing the potential field[3] i.e.,

$$\zeta = \varphi\sqrt{\frac{V_0}{\omega_0^3}}\,. \tag{19}$$

A real scalar field for a system within a cube of volume V can be expressed as:

$$\varphi = \frac{1}{\sqrt{2\omega V}}\left(ae^{-ikx} + a^* e^{ikx}\right), \tag{20}$$

where a is defined in Eq. (15) but with an added phase factor. By using the principle of superposition and treating the volume V approaches infinity, we find that

$$\varphi(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2\omega}}\left[a(\vec{k})e^{-ikx} + a^*(\vec{k})e^{ikx}\right]. \tag{21}$$

Substitute Eq. (19) into Eq. (9) and taking φ as a real scalar field, the Hamiltonian density equation becomes

$$\mathcal{H} = \frac{1}{2}[(\partial_0\varphi)^2 + (\vec{\nabla}\varphi)^2 + \omega_0^2\varphi^2]. \tag{22}$$

As in quantum field theory, the transition to a quantum field can be done via canonical quantization. Therefore, $a(\vec{k})$ shall be taken as the annihilation operator and its hermitian conjugate $a^+(\vec{k})$ as the creation operator in the emergent field. Comparing Eqs. (20), (21) and (22) with the results from quantum field theory, the quantization potential field has the same properties of a bosonic field.

5. Effects not limited by relativity

In non-relativistic limit, the quantization potential wave propagates base on the Schrodinger's equation which is deterministic. It can spread over a far apart spatial distance but its evolution is continuous and restricted by relativity. On the other hand, what happens inside the wave is random. As discussed briefly in Section 3, a particle can disappear at one location and reappear at a distant location. The probability for a particle to materialize is depending on the quantization potentials. A particle can appear to travel a long distance instantaneously without violating relativity. The presence of quantization potentials everywhere in the wave gives an impression that a particle can be in different

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[2] As shown in this paper, the use of a classical real or complex scalar field can both describe the system. However, we must treat the emergent quantum scalar field as real since we are dealing with non-charged particles. As we have learnt from quantum field theory, a complex quantum scalar field describes a field with charge.

[3] The conversion is straightforward and will facilitate our demonstration using the convention in quantum field theory.

places at the same time. The quantization potentials and particles are integral parts of the matter field. However, only the particles have energies that are detectable.

Take for example two normalized wave packets of the quantization potential fields ζ_a and ζ_b which are highly localized in space and their centers are far apart. We will consider the one particle coherent superposition state $|\zeta\rangle = \alpha|\zeta_a\rangle + \beta|\zeta_b\rangle$ with packets a and b. The two highly localized packets are superposed and remain connected even when they are spatially far apart. As discussed before, a particle can appear randomly in either packet as long as there is no restriction imposed on the particle or quantization potentials. This jump does not require superluminal transportation of energy. A particle can appear at one packet, disappear, and reappear in another packet instantaneously. This special property of the quantization potential field does not violate the principles of relativity.

The next effect not restricted by relativity involves collapse of wave. In non-relativistic limit, the probability of locating a particle in a normalized wave is:

$$\int \frac{\mathcal{H}}{m_0} d^3x \approx \int \rho_{pr} d^3x = 1, \tag{23}$$

The total quantization potentials $\int \mathcal{H} d^3x$ are just sufficient to allow one particle with mass/energy m_0 to materialize. This condition is conserved in a one particle system. Any excessive or deficient quantization potentials in a one particle system will require redistribution as constrained by Eq. (23). This redistribution can be induced under the following conditions:

Condition I - When part of the quantization potential field in a region Σ_1 is disconnected from the particle such that the disconnected potentials no longer contribute to the total potentials of the one particle system, there will be deficient potentials in the isolated one particle system. The disconnected potentials in Σ_1 shall dissipate allowing the one particle system to restore the balance in Eq. (23).

Condition II – When a particle is disconnected from part of the quantization potential field such that the disconnected particle loses its ability to materialize in a region Σ_2, there will be excessive potentials in this isolated region which cannot have any particle. The potentials in Σ_2 shall dissipate while the one particle system restore the balance in Eq. (23).

The implications of Conditions I and II will be clarified in Sections 6 and 7.

As we shall recall, quantization potentials do not carry energy. Although they are physically present, the only information they carry is the probability for a particle to materialize. Subtraction and addition of quantization potentials in a region do not require transfer of energy. The energy in a one particle system is at the quantized oscillator. However, how fast these quantization potentials can transfer is not known but not necessarily limited by relativity. The theory has no restriction even if the redistribution/collapse described is instantaneous. Sudden collapse of the quantization potentials between distant locations will not violate relativity. Its effect can be non-local.

We will apply the above concepts to the trapping of potentials in a closed box in the next section. It has been suggested that topological disconnectivity can induce possible wave packet collapse [18]. This idea is based on the wave-particle complementarity principle which can be tested with several suggested experiments. Here, we will take a different approach by investigating this proposal with the quantization potential field.

6. Topological disconnectivity

Let us assume one of the two highly localized packets in the above described one particle wave has entered a box through an opening as shown in Fig. 1. The box and its opening are large such that direct contact between the wave and the box can be omitted. After the wave packet has entered the box, the shutter is closed. Direct contact between the particle and shutter can also be omitted if the two highly localized packets are far from the shutter when it is closed.

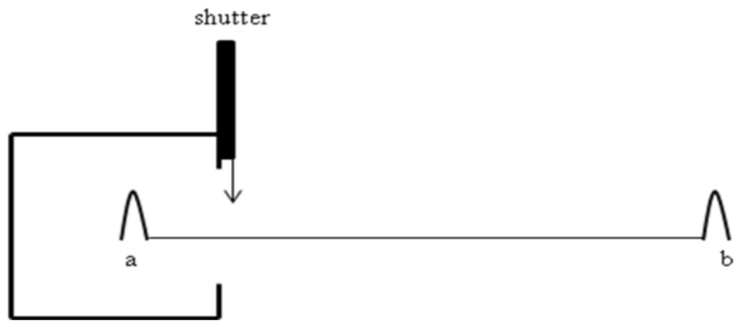

Fig. 1 The shutter is closed after one of the wave packets enters the box creating a completely closed environment.

Before the shutter is closed, the wave is in a coherent superposition state $|\zeta\rangle = \alpha|\zeta_a\rangle + \beta|\zeta_b\rangle$. After closure, wave packet a is trapped inside the box. However, there are two possible outcomes:

Scenario 1- A classical wave does not undergo sudden collapse if part of it becomes trapped in a closed environment. In analogous to the classical wave, the closure of the shutter can have no effect to the coherent superposition state $|\zeta\rangle = \alpha|\zeta_a\rangle + \beta|\zeta_b\rangle$. As we shall recall, the propagation of quantization potential wave is governed by Schrödinger equation which is a wave equation with no provision for collapse. Direct contact with the box and shutter are omitted that shall have no influence on the particle. Therefore, the two wave packets shall remain undisturbed after the shutter is closed; their total quantization potentials are conserved in Eq. (23). The trapping of a wave packet in a closed box will not induce wave collapse.

Scenario 2- The closure of shutter can block communication between the two highly localized packets. The wave packet inside the box is, therefore, disconnected from the outside. If the particle is in wave packet a when the shutter is closed, the quantization potentials outside the box cannot contribute to the isolated one particle system within. In order to restore the balance in Eq.

(23), the potentials outside the box must dissipate according to Condition I of the previous section:

$$\left|\zeta_i\right\rangle = \left|\zeta\right\rangle \Rightarrow \left|\zeta_f\right\rangle = \left|\zeta_a\right\rangle.$$

The coherent superposition state $\left|\zeta_i\right\rangle$ shall collapse to the final state $\left|\zeta_f\right\rangle$ as a result of topological disconnectivity. Likewise, if the particle is outside the box in wave packet b when the shutter is closed, the quantization potentials inside the box cannot contribute to the disconnected one particle system outside. The balance in Eq. (23) must again be restored under Condition I leading to wave collapse:

$$\left|\zeta_i\right\rangle = \left|\zeta\right\rangle \Rightarrow \left|\zeta_f\right\rangle = \left|\zeta_b\right\rangle.$$

Supporting arguments for Scenario 2 base on the wave-particle duality principle can be found in [18] which will be explained in terms of the quantization potentials in here. For example, the two wave packets will remain undisturbed after closure of shutter under Scenario 1. If we switch on a detector inside the box and find a particle, wave packet b shall collapse according to quantum mechanics. However, there is no means to induce such collapse if communication between the wave packets is blocked. This will result in excessive quantization potentials outside the box and thus a possibility to detect more than one particle in a single particle system. To resolve such dilemma, the reasonable choice is that the assumption in Scenario 1 is invalid. The wave packets shall be collapsed by topological disconnectivity as in Scenario 2. The detection inside the box is just a subsequent measurement to confirm the collapse that has already happened. Therefore, communication with the region outside the box is not necessary. However, we cannot eliminate the possibility that connection can still be maintained between the wave packets despite one of them is in a closed box.

As illustrated in quantum mechanics, a quantum wave can tunnel through physical barriers. In addition, we have shown that the quantization potentials have no energy and their properties are not necessarily restricted by relativity. This unique feature may allow continuous connection between the wave packets by tunneling through barriers. The balance in Eq. (23) will therefore be maintained even after the shutter is closed which can avoid a wave collapse as predicted in Scenario 1. In order to determine which scenario is valid, we will have to rely on experiments.

Several experiments have been proposed in [18] for testing the induced collapse by topological disconnectivity. However, there are a few technical difficulties, e.g. lack of efficient shutters. This can be challenging for fast moving photons in a double slit experiment. The use of atom interferometer has been suggested. Here, we demonstrate a similar thought experiment with the use of a single electron source[4] as shown in Fig. 2.

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[4] Although the theory is developed for a zero spin particle field, the quantization potentials obey the Schrodinger equation and have similar formulations for an electron field in the non-relativistic limit. The properties of spin can be neglected in this experiment.

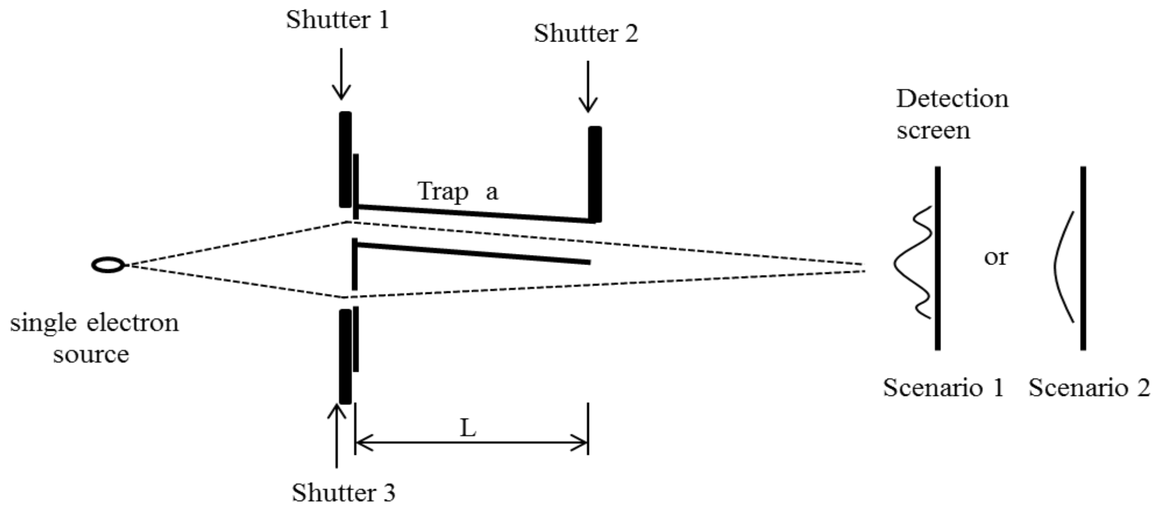

Fig. 2 Schematic diagram for a double slit thought experiment using single electron source. When the wave packet is inside Trap a, Shutters 1 and 2 are closed to create the condition of a trapped box.

Shutters 1, 2 and 3 are initially closed in this double slit experiment which may be realized using an electron biprism. Electrons are emitted one by one from the source. The sequence of operations is as follow:

1. Shutters 1 and 3 are opened to allow one electron to pass through.
2. Shutter 1 is then closed to create a trapped box condition in Trap a.
3. Shutter 3 is closed simultaneously with Shutter 1 to allow only one electron to pass through.
4. Shutter 2 is opened before the possible single electron reaches the end of Trap a.
5. After the electron is detected on the screen, Shutter 2 is closed ready for the next cycle.
6. Steps 1 through 5 are repeated until sufficient data are collected.

The synchronization and efficiency of the shutters are the major challenges in the experiment. Assuming the wave packets shall be at mid-point of Trap a when Shutter 1 is closed allowing disconnectivity to take full effect. An electron speed of 10^4 m/s and trap length of $L = 2\text{cm}$ will require the shutters to be able to open (or close) in less than one microsecond.

Under Scenario 1, the one particle system will remain as coherent superposition state $|\zeta\rangle = (|\zeta_a\rangle + |\zeta_b\rangle)/\sqrt{2}$ after trapped. The density matrix ρ_1 and density distribution n_1 on the screen are:

$$\rho_1 = |\zeta\rangle\langle\zeta|,$$

$$n_1 = N[|\langle\vec{r}\,|\zeta_a\rangle|^2/2 + |\langle\vec{r}\,|\zeta_b\rangle|^2/2 + \mathrm{Re}(\langle\vec{r}\,|\zeta_a\rangle\langle\zeta_b|\vec{r}\rangle],$$

where N is the overall electron number.

In Scenario 2, Trap a is topologically disconnected resulting in wave collapse. The density matrix ρ_2 and density distribution n_2 on the screen are:

$$\rho_2 = |\zeta_a\rangle\langle\zeta_a|/2 + |\zeta_b\rangle\langle\zeta_b|/2,$$

$$n_2 = N[|\langle\vec{r}\,|\zeta_a\rangle|^2/2 + |\langle\vec{r}\,|\zeta_b\rangle|^2/2].$$

The difference between the two scenarios is the additional interference term in n_1. Interference effect will be observed if there is no wave collapse induced by topological disconnectivity.

The results from these new experiments may provide new interesting physics. If Scenario 1 is confirmed, this will mean that the connections between parts of a quantum system can be maintained even if they are separated by physical barriers. On the other hand, verification of Scenario 2 will introduce a new kind of wave collapse which is not triggered by an ordinary measurement. This will provide support for another kind of wave collapse that we will discuss in the next section.

7. Collapse of Wave in Measurements

In the standard interpretation of quantum mechanics, measurement leads to wave collapse. To detect the presence of a particle, the measurement will require interaction with the quantization potential wave. A particle can be trapped by the interaction during this process. The trap condition shall trigger a wave collapse as discussed in Section 5, provided that the region where an interaction can take place is smaller than the spread of the particle wave. We can verify the presence of a particle from the interaction results if one makes an observation. However, observation is not a necessary step. It is the disconnection by interaction that triggers the collapse. In addition, an interaction that has large interaction region will not trigger a wave collapse and cannot effectively provide information for measurement which will be further discussed in Section 8.

Let us consider the same one particle wave with two highly localized packets as shown in Fig. 1 but replace the box with a measuring device. We will first assume the particle is within wave packet a. A measurement will begin with the interaction between part of the measuring device and the particle. During the interaction process, both the particle and quantization potentials in wave packet a are trapped. This is necessary to allow the effect of interaction to be transferred. No particle can materialize outside the interaction; the particle is disconnected from the quantization potentials outside. Thus, wave packet b will have excessive potentials. The coherent superposition state $|\zeta_i\rangle = \alpha|\zeta_a\rangle + \beta|\zeta_b\rangle$ shall collapse to the final state $|\zeta_f\rangle = |\zeta_a\rangle$ by disconnectivity under Condition II of Section 5.

Similar arguments can be made for the collapse process when the particle is in wave packet b. During the interaction, the quantization potentials in wave packet a are trapped and cannot contribute to the total quantization potentials for the one particle system outside. As a result, this deficit in potentials outside the interaction shall collapse the coherent superposition state $|\zeta_i\rangle = \alpha|\zeta_a\rangle + \beta|\zeta_b\rangle$ to a final state $|\zeta_f\rangle = |\zeta_b\rangle$ by disconnectivity under Condition I of Section 5.

The process described can reproduce the results in quantum mechanics where a measurement collapses the initial probability density ρ_i to the final probability density ρ_f, i.e.

$$\rho_i = |\zeta\rangle\langle\zeta| \Rightarrow \rho_f = \alpha^2 |\zeta_a\rangle\langle\zeta_a| + \beta^2 |\zeta_b\rangle\langle\zeta_b|$$

However, the collapse process proposed is induced by interactions in a measurement. Observation of the results from an interaction is just a subsequent step that confirms the presence of a particle.

We shall recall that the propagation of a quantization potential wave packet is determined by the Schrödinger equation. Although our formulation so far is for free particles, we can incorporate the effects of a force field by similar procedures adopted in quantum mechanics. A wave packet, with or without particle, shall obey the same wave equation when they interact with the force field; their reactions are identical until collapse. However, the interaction process is different once collapse has taken place. For instance, in the presence of a particle, interaction needs time for energy transfer. This will alter the physical state of the measuring device which can trigger a reading for measurement. On the other hand, any interaction with the potentials alone does not involve transfer of energy. The collapse can be accomplished immediately after the start of interaction without changing the state of the measuring device. The completion of the whole interaction process can be instantaneous.

The collapse process induced by interaction and topological disconnectivity are alike but the mechanism that triggers the collapse is fundamentally different. As discussed in Section 6, collapse by topological disconnectivity is subject to experimental verifications and we cannot rule out the possibility that connection can be maintained even with physical obstructions. Trapping in a box may not be sufficient to cause disconnection. On the contrary, collapse by measurement is observed daily in the laboratory and is a fundamental postulate of quantum mechanics. Unlike trapping in a box where there can be loopholes to avoid disconnection, a particle must remain disconnected with the potentials outside during an interaction. Trapping by interaction can be the mechanics that triggers the collapse in measurement. To better illustrate the concept, we will examine a laboratory example.

In a double slit experiment, an electron wave is split into two localized packets by the biprsim. We can detect which path an electron takes by probing it with another particle, e.g. photon. Deflection of probe particle in one of the path will indicate the presence of an electron which is observable. During the interaction, the probe particle and electron waves will suffer an uncontrollable change in momentum which we will assume to have a magnitude Δp. According to the Heisenberg uncertainty principle, this condition will introduce an uncertainty to the position of interaction $\Delta x \cong \hbar / \Delta p$. If an electron is present at the path detected, the electron and its associated wave packet must stay with the interaction to allow full transfer of momentum and are disconnected from the quantization potentials outside; the electron is trapped. Consequently, any potential outside the interaction shall dissipate leading to overall collapse under Condition II of Section 5 if the uncertainty Δx is smaller than the spatial distance between the two localized electron wave packets.

In the same experiment, wave collapse will also take place if the electron is not in the path detected. During an interaction with the probe particle, a wave packet without electron will be trapped and cannot contribute to the total quantization potentials of the one particle system outside. As a result, the wave packet shall collapse under Condition I of Section 5, provided that the distance between the two localized electron wave packets is larger than the uncertainty Δx.

We shall note that the interaction between the probe particle and electron's quantization potentials alone does not involve energy transfer. The whole collapse and interaction process can be accomplished instantaneously when an electron is not present. The probe particle can pass through the empty electron wave packet as if it is in a vacuum space but simultaneously induce a collapse. These results can explain the reasons of why and how a wave collapse is induced if we know which path an electron has taken in a double slit experiment. The concepts can be applied to other measurement processes.

8. Interactions without Collapse

Not all interactions can cause wave collapse. For example, if an electron wave passes through a biprism in a double slit experiment, the wave splits into two paths after interacting with the electromagnetic field. Interference effect can be observed after the two paths are merged. There is no collapse as long as we do not know which path the electron has taken. On the other hand, a measuring device is nothing more than the interaction of the electron wave with the electromagnetic field of the atoms. How can two processes with similar qualities produce totally different results?

For simplicity, we will consider a rigid plate made up of n negatively charged atoms bonded together. As one of the two highly localized wave packets from the previous examples is approaching the plate, it will be deflected by the electromagnetic field. Suppose an interaction can impart a momentum change of Δp to the plate which we will assume equally shared by all atoms. The electron wave packet will interact with n atoms each with a momentum change of $\Delta p_n = \Delta p / n$. There are, in fact, n individual interactions instead of one. The degree of uncertainty for the location of each interaction is $\Delta x_n \cong n\hbar / \Delta p$. These uncertainties are overlapping and describe the same approximate region that an interaction shall take place. During this interaction, the electron wave packet must stay within the uncertainty limit Δx_n when interact with all n atoms. For a plate made up of only a few atoms, the uncertainty of the interaction location is small. Wave collapse can occur following the same process discussed in the above examples. However, as $n \to \infty$ for a macroscopic object, the uncertainty $\Delta x \to \infty$ becomes so large that it cannot effectively trap the electron wave. The interacting electron wave packet can remain connected with the rest of its system as long as they are within the uncertainty limit. As a result, no collapse of wave will occur. We can apply the same concepts to the deflection in other electromagnetic fields generated by biprism, magnet, capacitor etc.

This idealized example outlines an approach on how to determine the outcome of an interaction. To distinguish the kind of interactions that can induce wave collapse, we need to ask whether such interaction can cause disconnectivity in any part of the quantum system. In general, a large uncertainty in the location of interaction cannot induce disconnectivity nor produce meaningful results in measurement. For example, the presence of a particle in one of the path in the double slit experiment cannot be determined if the highest accuracy of measurement is the size of the whole experiment set up. On the other hand, we can measure the location of the particle with higher accuracy if the interaction has small uncertainty and induce a collapse. This concept can reproduce the same interpretation from quantum mechanics that measurements lead to collapses.

9. Conclusions and Discussions

In this paper, we show that a quantized oscillator with fluctuations in space and time can have the same properties of a zero spin boson in quantum theory. The quantized particle is real and its world line is defined. However, the appearance of a particle is random and depends on the quantization potentials of the matter field. These potentials have a unique feature that they do not carry any energy; their properties are not necessarily restricted by relativity. How information is communicated in this quantization potential field can be very different from the currently known fundamental theories.

So far, we have demonstrated two possible effects in the quantization potential field that are not necessarily restricted by relativity: the apparent ability of the quantized oscillator to jump to a distant location instantaneously (Section 5) and the spontaneous collapse of wave (Sections 6 and 7). The system we have considered is a single particle wave. Two highly localized potential wave packets remain connected even if they are spatially separated. Interaction with one potential wave packet can cause collapse to the other under Condition I or II of Section 5. The effect can be non-local. As in the case for topological disconnectivity, the collapse process by interaction can also be explained in terms of the probabilistic wave function. However, the quantization potentials and the quantum mechanical wave have fundamental differences. The former is generated from the fluctuations of matter in space and time; it is physically present and co-exists with the particle.

In an EPR experiment, two particles emitted from a source remain connected even if they are distant apart. The quantization potentials of the two particles are entangled. Interaction with one particle can collapse not only its own potentials but also the entangled superposition states leading to the collapse of potentials for the second particle. This is possible with the new model due to the unique property that the quantization potentials carry no energy. The theory has no restriction on how fast the two entangled particles can communicate. Instantaneous collapses of the potentials for both particles do not require superluminal transfer of energy and will not violate the principles of relativity. The quantization potentials can have the non-local properties expected in an entangled quantum system. The unusual properties of the new model may open a new avenue on how to approach some of the questions in EPR paradox.

Apart from these results, two future applications may provide support for this theory:

- As discussed in Section 3, a particle has quantized fluctuation in proper time; it is actually an integral part of the space-time continuum. The curved space-time geometry arising from this vibration can be calculated and compared with the gravitational field of a point mass.

- When the quantized proper time fluctuation is transformed to another frame of reference, the particle will have fluctuations in time and space with amplitudes $\mathrm{T} = \omega / \omega_0^2$ and $\vec{\mathrm{X}} = \vec{k} / \omega_0^2$ respectively. The examples given in Section 3 provide estimates for the spatial fluctuation amplitudes of an electron. In the non-relativistic example, the amplitude of the spatial fluctuation is approximately equal to the diameter of a nucleus. However, this fluctuation also has a very short time scale ($\approx 10^{-21} s$ for electron). Whether such effects can be observed in the laboratory will require our further exploration.

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