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# The Rise of Relationals

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We begin by criticising an elaboration of an argument in this journal due to K. Hawley (2009), who argued that, when Leibniz's Principle of the Identity of Indiscernibles (PII) faces counter-examples, invoking relations to save PII fails. We argue that insufficient attention has been paid to a particular distinction. We proceed by demonstrating that in most putative counter-examples to PII (due to Immanuel Kant, Max Black, Alfred Julius Ayer, Peter Frederick Strawson, Hermann Weyl, Christian Wüthrich), the so-called Discerning Defence trumps the Summing Defence of PII. The general kind of objects that do the discerning in all cases form a category that has received little if any attention in metaphysics. This category of objects lies between indiscernibles and individuals and is called relationals — objects that can be discerned by means of relations only and not by properties. Remarkably, relationals turn out to populate the universe.

## Contents

## 1 Leibniz's Principle of the Identity of Indiscernibles

A famous principle of Leibniz "has fallen on hard times", M. Della Rocca (2005: 48) reports, because "most philosophers nowadays seem not to accept this principle"; and he continues: "The primary reason is, of course, the great *intuitive plausibility* of certain well-known counter-examples." This famous principle of Leibniz is the metaphysical

> **Principle of the Identity of Indiscernibles (PII).** *Necessarily, for every two objects, if they are indiscernible then they are identical, i.e. there are not two objects but there is only one object; with self-evident abbreviations*[1]:
>
> $\Box\big(\forall\, a, \forall\, b:\ \neg\mathrm{Disc}(a,b) \ \longrightarrow\ a = b\big)\,;$
>
> *or contra-posing: necessarily, every two distinct (i.e. not identical) objects are discernible*:
>
> $\Box\big(\forall\, a, \forall\, b:\ a \neq b \ \longrightarrow\ \mathrm{Disc}(a,b)\big)\,;$
>
> *or in yet another form: distinction without a difference is impossible*:
>
> $\neg\Diamond\big(\exists\, a, \exists\, b:\ a \neq b\ \wedge\ \neg\mathrm{Disc}(a,b)\big)\,.$

Here is a triumvirate of reasons for caring about PII.

*First*, PII teaches us an ontological lesson: indiscernibles cannot and do not exist in the universe we inhabit. Modern physics does *not* teach us otherwise; modern physics does not refute PII, in contradiction to what many philosophers of physics have claimed.[2] Rather, the ultimate constituents of physical reality are *relationals*; that is, entities that are discernible by relations but not by properties. We teach this lesson in this paper.

*Secondly*, when one holds, with Lowe (2006, pp. 3–7), that one of the central aims of contemporary metaphysics is to erect a framework of concepts to unify and embed all scientific knowledge we have gathered, or when one holds, with Cocchiarella (2007, p. 4), that one of the centrail aims is the study of ontological categories, then the logical-metaphysical category of a *relational* ought to figure prominently on the stage of metaphysics. The current paper gives reasons why: physical reality is full of them and it is relationals that make the metaphysical PII stand its ground. The current situation in metaphysics is that *relationals* are wholly absent from all discussions about objects, entities, metaphysical frameworks, ontological categories and what have you. This situation should change.

*Thirdly*, the issue of PII is intimately connected to the venerable Fregean issue of *Identity Criteria* (or what comes down to the same thing: *difference criteria*) for $F$s:[3]

$$\big(F(a) \wedge F(b)\big) \ \longrightarrow\ \big(\mathrm{IdC}_F(a,b) \ \longleftrightarrow\ a = b\big)\,. \tag{1}$$

Frege's insight was that our use of conceptions of generality and existence, and our meaningful use of words like 'all', 'most', 'several', 'some', 'one', 'two', etc. presuppose the presence of identity

[1] Russell (1937, pp. 54–58) collected places in Leibniz's *opera* where Leibniz discusses PII. Our *default* reading of modalities, e.g. $\Box$ and $\Diamond$ in PII, is nomic.

[2] Weyl (1928), Cortes (1976), French and Redhead (1988), French (1989), Butterfield (1993), Schrödinger (1996); see Muller and Saunders (2008) for an analysis of their arguments. Title of French (1989): 'Why the Principle of the Identity of Indiscernibles is not contingently True Either'.

[3] See Lowe (1989, p. 6), Horsten (2010, p. 414); our default presuppositioin is that monadic predicate $F$ is a *sortal*; see Westerhoff (2005, pp. 62–63), Wiggins (2012).

conditions. Then in any field of inquiry about $F$s that is rigorous by Fregean standards, $\mathrm{IdC}_F(a, b)$ must be found. *Leibniz's Law* (a theorem of logic) states the semantic indiscernibility of identicals: if $a = b$, then everything that is true of $a$ is also true of $b$, and *vice versa*. This implies the converse of PII. When we take the conjunction of PII and this converse, and relativize the conjunction thus obtained to $F$s, we obtain an identity-criterion for $F$s, so that Leibnizian indiscernibility criteria and Fregean identity-citeria are one and the same thing:

$$\big(F(a) \wedge F(b)\big) \;\longrightarrow\; \big(\mathrm{IdC}_F(a, b) \;\longleftrightarrow\; \neg\mathrm{Disc}(a, b)\big) \,. \tag{2}$$

So much for this interlude about why we should care about PII. Let us return to Della Rocca, whom we quoted above as asserting that the primary reason for rejecting PII is "the great *intuitive plausibility* of certain well-known counter-examples." Now, counter-examples to PII have also been based on modern-physical theory rather than on philosopher's imagination, and modern-physical theory is an onslaught on *intuitive plausibility*.[4] Thus attacks on PII come from metaphysics as well as from modern physics. Recently some philosophers of physics have however argued that, on closer inspection, modern-physical theories vindicate rather that violate PII.[5]

Putative counter-examples to PII, intuitive and unintuitive, include the following well-known and perhaps less well-known cases, all of which we shall address.[6]

- *Kant's Droplets.* Two droplets of water exactly similar in every respect.
- *Black's Spheres.* Two black iron solid spheres of a 1 mile diameter being 2 miles apart in otherwise empty space.
- *Ayer's Sound-Tokens.* An infinite sequence of the same group of four different sound-tokens of equal duration, each one separated from its neighbours by an equal interval of time:
  ...... $A\,B\,C\,D \quad A\,B\,C\,D \quad A\,B\,C\,D$ ......
- *Strawson's Chessboard.* Most of the black squares and most of the white squares of a chessboard universe, whose boundaries are the edges of the board, are indiscernible yet distinct.
- *Weyl's Quantum Particles.* Composite systems of 'identical' yet distinct particles when described by quantum mechanics, with its postulate of permutation-symmetry, leading to Pauli's exclusion principle in the case of fermions.
- *Wüthrich's Space-Time Points.* All space-time points in symmetric solutions of the gravitational field equations of the General Theory of Relativity are indiscernible yet distinct.

In a searching paper, K. Hawley (2009) lays down ground rules for considerations about putative counter-examples to PII. Hawley submits that the reasoning leading to the judgement that PII stands refuted in some given "qualitative arrangement", as in the cases on our list above, is best broken into two Steps:[7]

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[4]See for instance: Butterfield (1993), Cortes (1976), French and Redhead (1988), French (1989), Schrödinger (1996), Wüthrich (2010).

[5]For elementary particles, see Saunders (2003a), (2003b), (2006), Muller and Saunders (2008), Muller and Seevinck (2009); for space-time points, see Muller (2011).

[6]Kant (1787, p. B319), Black (1952, p. 153), Ayer (1954, p. 32), Strawson (1959, p. 122), Weyl (1928, IV.C, Section 9), Wüthrich (2010).

[7]Hawley (2009, p. 102): a "qualitative arrangement consists of those facts about the world which do not immediately settle questions about identity and parthood", where 'settle' here "is an imprecise epistemic notion, not a matter of metaphysical determination". Most other philosophers will speak of possible worlds. Qualitative arrangements include relationships.

Step 1. A description of a qualitative arrangement.
Step 3. An argument that in this qualitative arrangement we have distinct but indiscernible objects, in the plural.

But, as will become clear when we proceed, another step must be inserted, which consists in answering the following three questions:

Step 2a. What does PII meaningfully apply to in this qualitative arrangement (Step 1)?
Step 2b. What sort of features are permitted to discern?
Step 2c. What sort of features are forbidden to discern?

Different answers to questions in Step 2 may lead to different judgements about whether PII holds or fails in some qualitative arrangement. When one leaves questions in Step 2 implicit, as Hawley to a certain extent seems to have done when moving from Step 1 to Step 3 without a pause, one does not explicitly address the issue of the permissibility of the features that may or may not discern (as Step 2 commands) and thereby runs the danger of drawing unwarranted conclusions (in Step 3).

Two kinds of defence of PII were typically appealed to when faced with putative counter-examples; Hawley considers a third, novel one, the 'Summing Defence':

1. *Identity Defence*: there are not two (or more) objects, but there is one object of the *same* kind as the alleged two (or more) objects belong to.
2. *Discerning Defence*: there is, on closer inspection, some qualitative difference between the two (or more) distinct objects.
3. *Summing Defence*: there are not two (or more) objects, there is one object of a kind that is *different* from the kind the alleged two (or more) objects belong to, and that one object has no parts (so that the alleged objects also are not *parts* of it), or in current mereological terminology: it is a *simple*.

The paper is organised as follows. In Section 3, we present the 'circularity argument' against the Discerning Defence of PII in the context of Black's spheres, and argue that *it* proceeds by tacitly glossing over he questions in Step 2; we then argue that Hawley's elaboration of the circularity charge proceeds on forbidden terrain, which provides a good reason for rejecting it. The Discerning Defence of PII then will stand vindicated. In Section 4, we show that the same Discerning Defence succeeds in the remaining putative counter-examples to PII listed above. In Section 5, concerning Weyl's quantum case, we argue that recent arguments in the philosophy of physics undermine a 'uniformity argument' of Hawley's in favour of the Summing Defence over the Discerning Defence of PII in quantum mechanics, and we propound another 'uniformity argument' in favour of the Discerning Defence so as to argue for the superiority of the Discerning Defence here; we shall also briefly address quantum field theory, where things become more complicated. But first, in the next Section 2, we take care of our terminology.

## 2 Varieties of Discernibility

Throughout this paper we consider mostly cases of two objects. We take the notion of 'object' to be extremely encompassing (a purely logical notion of object, metaphysically thin): anything we can meaningfully quantify over qualifies as an *object* (iron spheres, elementary particles, planets, humans, dreams, novels, tree leaves, numbers, sets, structures, space-time points, &c.) — perhaps *entity* would have been a better word, but that usually also includes universals, properties, tropes, and more, which we shall not address and therefore want to exclude here. As we pointed out above, the formulation of PII for $F$s is strongly related to identity criteria for $F$s (1): *objects are identical iff they are indiscernible*, or in other words, *objects are distinct iff they are discernible*. Unpacking 'discernibility' will depend heavily on what kind of objects we are dealing with — which is flagged in (1) by writing '$\mathrm{IdC}_F(a,b)$'. We also could write '$\mathrm{Disc}(F;a,b)$' rather than '$\mathrm{Disc}(a,b)$', but shall generally not do so. So PII applies to objects as construed above, in every qualitative arrangement we shall meet (on our list of the previous **Section**). This parenthetically answers the question of Step 2a in full generality.

We now rehearse and extend the terminology of Muller and Saunders (2008, pp. 503–505). Often only *properties*, expressed by monadic predicates, are permitted to occur in PII, presumably due to the fact that Leibniz held that relations are reducible to properties and that, therefore, relations need not be mentioned in PII.[8] We call an object *absolutely qualitatively discernible* from other objects, or an *individual*, iff there is at least one permitted property that the object has and the other objects lack. An object *has an individuality* iff it is absolutely discernible; its *individuality*, then is the property, or properties, it has and does not share with any other object. Thus objects that are not absolutely discernible do not have an individuality; all and only individuals have an individuality. Objects are *quantitatively discernible* (or synonymously *numerically discernible*) iff they are *distinct*, which we define as not being identical:

$$\mathrm{Dist}(a,b) \ \text{ iff } \ a \neq b \,. \tag{3}$$

Relations, expressed by polyadic predicates, can and should be considered too in order not to be tacitly committed to the (untenable) Leibnizian thesis that all relations are reducible to properties. We restrict ourselves to binary relations, which are expressed by dyadic predicates. Call an object *relationally qualitatively discernible* from other objects iff there is some permitted relation that discerns it from the other objects. Further, call an object *not discernible*, or *indiscernible* from another, iff it is neither absolutely nor relationally discernible:[9]

$$\neg\mathrm{Disc}(a,b) \ \text{ iff } \ \big(\neg\mathrm{AbsDisc}(a,b) \wedge \neg\mathrm{RelDisc}(a,b)\big) \,. \tag{4}$$

We call an object that is relationally but not absolutely discernible a *relational*. Then indiscernibles are objects that are neither individuals nor relationals. Just like absolute *indiscernibles*, relationals do not have an individuality. W.v.O. Quine (1976) was the first to inquire into different kinds of discernibility; against the background of classical logic, he discovered there are *only two* different categories of relational discernibility (by means of a binary relation): either (i) the relation is irreflexive and asymmetric, in which case we speak of *relative discernibility*; or (ii) the relation is irreflexive and symmetric, in which case we speak of *weak discernibility*. S.W. Saunders (2003a; 2003b; 2006) brought Quine's distinctions and results to bear on discussions in the philosophy of physics, from which they have entered and are entering contemporary metaphysics as well as the philosophy of mathematics.[10] In the current terminology, PII says that *necessarily, objects are*

[8] See Russell (1937, pp. 13–15) and Ishiguro (1990, pp. 118–122, 130–142) for Leibniz's struggle with relations.

[9] Caulton and Butterfield (2012) then speak of *utter* indiscernibility, which we shall do only when emphasis is needed.

[10] See Saunders (2006), Hawley (2008) and references therein; and Ladyman, Pettigrew and Linnebo (2012) and references therein.

*identical if they are absolutely and relationally indiscernible*:

$$\Box\big(\forall\, a, \forall\, b : \neg\text{AbsDisc}(a,b) \wedge \neg\text{RelDisc}(a,b) \;\longrightarrow\; a = b\big)\,. \tag{5}$$

For the sake of clarity, we spell out relational indiscernibility as a schema: $\neg\text{RelDisc}(a,b)$ iff for any dyadic predicate $R$ (expressing a binary relation):

$$\forall\, c\big(R(a,c) \longleftrightarrow R(b,c)\big) \;\wedge\; \forall\, d\big(R(d,a) \longleftrightarrow R(d,b)\big)\;. \tag{6}$$

We point out that logically speaking, (6) encompasses absolute indiscernibility, which is the antecedent of PII as it traditionally has been conceived, because for every monadic predicate $F$, there is a dyadic predicate $R_F$ that is logically equivalent to it: $F(a) \wedge \big(F(b) \vee \neg F(b)\big)$. Then:

$$\vdash\; \forall\, a\,\big(F(a) \;\longleftrightarrow\; \forall\, b : R_F(a,b)\big)\;. \tag{7}$$

Ladyman, Pettigrew and Linnebo (2012, Sect. 5) establish the following implications, again with self-evident abbreviations (RvDisc: relatively discernible):

$$\vdash\; \text{AbsDisc}(a,b) \longrightarrow \text{RvDisc}(a,b) \longrightarrow \text{WkDisc}(a,b) \longrightarrow \text{Dist}(a,b)\;. \tag{8}$$

All converse implications of (8) fail. Since '$\neq$' is a weakly discerning relation, so is the relevant discernibility relation $\text{Disc}(a,b)$ from PII.

Both being an *extrinsic* absolute discernible as well as being a relational rely on the presence of other objects. What is the difference? The difference is that the afore-mentioned belongs to a subspecies of *absolute* discernibility, expressed by some monadic predicate that singles out one particular extrinsically *absolutely* discernible object, whereas the last-mentioned is an object that is *not absolutely* discernible — relationals are discerned by some dyadic predicate that does not lead to a monadic predicate that singles out one relational. Thus extrinsic absolute discernibles are relational discernibles too, but they are not relationals: the two categories of extrinsic absolute discernibility and being a relational are mutually exclusive — but not jointly exhaustive.

All identity talk so far has been and will be talk of *synchronic* identity; *diachronic* identity, for which usually *persistence conditions* are sought, lies beyond the scope of this paper (but see the last **Section**).

R. Barcan Marcus (1993, p. 200) asserted that "individuals must be there before they enter into any relations, even relations of self-identity", and did not feel the need to argue for this assertion because of its self-evident truth. "No identity without entity", she declared (*ibidem*), thereby reversing Quine's celebrated slogan 'No entity without identity'. S. French and D. Krause (2006, pp. 167–172) argue more specifically that two objects can only be discerned by some relation on pain of circularity: one cannot demonstrate in this fashion that there are *relationals*, in the plural. This is essentially the same criticism as Russell propounded more than a century ago in *The Principles of Mathematics* (1903: 458):

> Again, two terms cannot be distinguished in the first instance by difference of relation to other terms; for difference of relation presupposes two distinct terms, and cannot therefore be the ground of their distinctness. Thus if there is to be any diversity at all, there must be immediate diversity (...).

Let us get to the bottom of this.

## 3 The Circularity Charge

### 3.1 Epistemic and Metaphysical Questions

The circularity argument against relational discernment is general and applies to all cases listed in **Section 1**; for the sake of concreteness, we shall treat it in the context of Black's spheres, and then show how a similar treatment applies to the other cases on our list. We remark that Black's spheres case is the same case as Kant's droplets (i.e. two absolutely indiscernible material objects), which is why we gloss over Kant's droplets altogether. Before we embark on Black's case, we call attention to three questions that need to be distinguished sharply:

*Q1*. Is there in a given qualitative arrangement one object or there are more objects? Is there quantitative identity or diversity?

*Q2*. How can we find out whether in a given qualitative arrangement there is quantitative identity or diversity?

*Q3*. Is there, in a given qualitative arrangment where there is *quantitative* diversity, also *qualitative* diversity?

In order for a putative challenge for PII to arise, question *Q1* has to be answered in favour of quantitative diversity. Only then does it make sense to raise question *Q3*. If *Q3* is answered in favour of qualitative diversity, then PII is safe (Discerning Defence); if *Q3* is answered against qualitative diversity, then PII is in immanent danger. Question *Q2* is an *epistemic* question, not a metaphysical one, as *Q1* and *Q3* are. We ought to find out the answer to *Q1* from the description of the qualitative arrangment; if not, the case is *under-described* and our inquiry into the case under consideration stops before it has really begun. The critic of PII then has not done their job properly. We shall see that questions *Q1*, *Q2* and *Q3* are frequently confused.

We now have three Steps and three Questions. How are they related? Step 1 leads to an answer to *Q1*, and Steps 2 and 3 lead to answer to *Q3*. Since *Q2* asks how to find the answer to *Q1*, the generic answer to *Q2* is: by reading the description of the qualitative arrangement under consideration.

### 3.2 Black's Spheres

[Step 1]. We have two solid black spheres 2 miles apart, for the sake of convenience baptised 'Castor' and 'Pollux'[11]; they share all their properties (colour black, mass $m$, spherical shape, diameter of 1 mile, constitution of iron, ...) and are the only material objects in this universe, which we, like Black, take to be a 3-dimensional Euclidean space ($\mathbb{E}^3$).

[Step 2a] We recall here our general answer to the question in Step 2a: PII can be meaningfully considered for all sorts of objects in our broad logical and thin metaphysical sense, which notably includes water droplets, solid iron spheres, sound-tokens, chessboard squares, elementary particles and space-time points.

[Step 3]. Let '$a$', '$b$' and '$c$' be sphere-variables ranging over set {Castor, Pollux}; this fixes the interpretation of the quantifiers. Define this binary Distance-relation:

$$D(a, b) \text{ iff sphere } a \text{ is 2 miles apart from sphere } b, \tag{9}$$

[11] Black (1952) lets a space traveller appear in Black's universe to baptize these spheres and then the traveller disappears.

which obviously is *grounded in the structure of space* and therefore is not ungrounded. Clearly $D(a,b)$ is symmetric and irreflexive. We have:

$$D(\mathrm{C},\mathrm{P}), \quad D(\mathrm{P},\mathrm{C}), \quad \neg D(\mathrm{C},\mathrm{C}), \quad \neg D(\mathrm{P},\mathrm{P})\ . \tag{10}$$

Hence Castor (C) and Pollux (P) are absolute indiscernibles but relational discernibles, hence relationals, of the weak kind.[12]

This argument is circular, French and Krause (2006, pp. 169–171) submit; Hawley (2009, pp. 109–111) follows suit. The question whether Castor and Pollux are identical or distinct is the same as the question whether there is one sphere or there are two spheres, and also the same as whether we have quantitative diversity or not. So far so good. The argument above tacitly assumes that Castor and Pollux are distinct, i.e.

$$\text{Castor} \neq \text{Pollux}\ ; \tag{11}$$

otherwise, i.e. when leaving it open whether Castor $=$ Pollux (let alone when *assuming* that Castor $=$ Pollux, in which case there is *one* object bearing two different names), one could not possibly deduce there are *two* objects, *two* weak discernibles. Consequently a case against PII cannot take off. To argue, when (11) is assumed, that one cannot deduce, by means of PII, that Castor and Pollux are distinct because they are weakly discernible (so that the antecedent of PII is false) is circular, precisely because of (11). Quantitative diversity (11) is assumed from the outset, and therefore demonstrations with weakly discerning relations to reach the conclusion that we have quantitative diversity (11) have become superfluous: the conclusion was a premiss. *Petitio principii.*

Clearly questions *Q1* and *Q2* (p. 7) have been confused. Question *Q1* has been answered *ab initio* in favour of quantitative diversity. It is definitely not circular to assume that answering a question (*Q1*) is the only way for *another* question to make sense, (*Q3*, concerning the truth of PII). Let us continue and focus our attention on the means of discernment (Step 2).

In the circularity criticism, little if any attention has been paid to questions in Steps 2b and 2c (**Section 1**). As a consequence, no clear view has been obtained about what is permitted and what is forbidden to discern. Elaborating on (Muller and Saunders 2008, p. 527), we shall argue that not every predicate is permitted to discern and some predicates are forbidden to discern. Before doing so, we report that it has been generally acknowledged that so-called *trivialising* predicates have to be forbidden to occur in PII — they make PII hold *trivially*. The problem of characterizing what trivialising predicates are has turned out to be far from trivial.[13] We can say this much without entering the realm of controversy: identity ($=$) trivialises predicates in which it occurs, for if identity were permitted to occur in the sufficient condition of PII, then the dyadic predicate '$a \neq b$' would be enough to conclude that $a \neq b$, and the truth of PII would become as trivial as any tautology. Thus part of our general answer to the question in Step 2c will be: distinctness (3) is forbidden to discern.

[Step 2b]. Predicates in the language appropriate to describe the geometrical structure of $\mathbb{E}^3$ (the language of Euclidean geometry) that express *spatial properties* and *spatial relations* are *permit-*

[12] Black (1952, p. 158) knew of spatial relations; he came to reject their discerning power because they cannot be used to give each sphere individuality. With the wisdom of hindsight, we say: Black was unaware of the distinction between absolute and relational discernibility, and that lacking individuality does not entail indiscernibility. Read on.

[13] see Katz (1983), and Rodriguez-Pereyra (2006, p. 29), who ends with the following definition: $F$ expresses a *trivializing* property iff differing with respect to $F$ is or may be differing quantitatively.

*ted* to discern, because they only rely on $\mathbb{E}^3$, which is part and parcel of the qualitative arrangement in Black's case. Relations that employ the intrinsic properties of the spheres (colour, mass, constitution, volume, ...) are *permitted* because these also are part and parcel of the qualitative arrangement. Let us summarize this by saying that the properties and relations that one can infer from the description of the qualitative arrangement (Step 1) are all *permitted* to discern. To find out *whether* they do or do not discern is the content of Step 3 (see above: distance discerns).

[Step 2c]. Take this monadic predicate:

$$N(a) \quad \text{iff} \quad a = \text{Castor}\,. \tag{12}$$

Then $N$(C) and $\neg N$(P), because of (11). Should we now conclude that Castor and Pollux are *absolutely* discernible after all because discerned absolutely by monadic predicate $N$ (12)? No, we should not, primarily because '=' occurs in it, and secondarily because predicate $N$ (12) employs *only the fact that* the spheres bear names.

Now we consider a familiar Cartesian co-ordinate chart

$$\mathbb{E}^3 \to \mathbb{R}^3, \quad p \mapsto \big(x(p),\, y(p),\, z(p)\big)\ , \tag{13}$$

such that Castor lies in its origin $(0,0,0) \in \mathbb{R}^3$ and Pollux on the $Y$-axis, in $(0,2,0) \in \mathbb{R}^3$.

Consider these monadic predicates:

$$\begin{aligned} O(a) \ &\text{iff} \quad \text{sphere } a \text{ lies in the origin } (0,0,0). \\ Z(a) \ &\text{iff} \quad \text{sphere } a \text{ lies on the } Z\text{-axis}\,. \end{aligned} \tag{14}$$

Then again $O$(C) and $\neg O$(P), and $Z$(C) and $\neg Z$(P). Absolute discernibles after all? This time we have neither used the names of the spheres — they do not occur in (14), in contradistinction to (12) — nor identity, but their *position*, which is something spatial and part of the qualitative arrangement. Still unacceptable. Why? One reason is as before: we, human beings, assign arbitrary triples of real numbers to spatial points, to members of $\mathbb{E}^3$, and exploit these numbers to discern, in particular the *name* '$(0,0,0)$' of the location of Castor, and the *name* '$Z$-axis' for a set of points.

Another reason for why discernment by predicates $O$ and $Z$ (14) is unacceptable, we advance, is that spatial relations, or more generally, predicates relying on the presence of $\mathbb{E}^3$, that *break the symmetry* of the qualitative arrangement, or the symmetry of the theory involved in describing it (here: Euclidean geometry), should be *forbidden*. The continuous symmetry group of $\mathbb{E}^3$ is generated by rotations and displacements, the so-called *Euclidean group*; the structure of $\mathbb{E}^3$ is also invariant under reflections in an arbitrary point, which form its discrete symmetries. When we take *time* to be included in the qualitative arrangement, we obtain Galilean space-time, which has the *Galilei-group* as its symmetry group; it consists of the rotations, displacements, time translations and boosts (rectilinear motions with constant veclocity); spatial and temporal reflections can be added. Then predicates $O$ and $Z$ (14) are forbidden, because they are not displacement-invariant and therefore violate the Eulidean and the Galilean symmetry.[14]

Notice that the names 'Castor' and 'Pollux', used in the description of Black's case, cannot be eliminated in the familiar Quinean-Russellian manner. If each sphere had a *definite description*, the spheres would be absolutely discernible. But they aren't. Predicates $C$ and $N$ (12) do not

[14]Displace Castor and Pollux 2 miles in the negative direction of the $Y$-axis, and we have $\neg O$(Castor), and $O$(Pollux), by (14). Or rotate Castor and Pollux 90° clockwise in the $ZOY$-plane around the $X$-axis, and Pollux lies on the $Z$-axis, in $(0,0,-2)$, rather than on the $Y$-axis, so that $Z$(Pollux), by (14).

count as definite descriptions. The question *which* sphere is Castor and *which* one is Pollux is not meaningful because it asks for something that is not to be had — namely, definite descriptions.

Let us now return again to the circularity charge. In a nutshell, the circularity charge fails because questions *Q1* and *Q3* (p. 7) have been confused. The question *Q1* reads whether there is *quantitative* diversity, in our case whether Castor $\neq$ Pollux (11) — which is answered in the affirmative in order to address *Q3* meaningfully; and *Q3* reads whether there is also qualitative diversity besides quantitative diversity. We have answered *Q3* also in the affirmative, on the basis of a demonstration relying on the features of the qualitative arrangement, notably the structure of space, and using a permitted relation: distance relation $D$ (9), which discerns the spheres weakly and is invariant under the relevant spatial symmetry transformations. (Notice that this is the Discerning Defence in action.)

The charge of the demonstration being a *petitio principii* is wrong, for the *conclusion* of our argument is not merely *that* Castor $\neq$ Pollux, not merely *that* there is quantitative diversity, but that *the spheres are weakly discernible by means of a permitted relation*. This conclusion certainly goes beyond stating mere quantitative diversity (11), because it states that there is qualitative diversity too, a specific kind of qualitative diversity in addition. When the conclusion is not the same as the premiss, the connecting argument cannot be a *petitio principii*. We begin with two distinct objects (11) and we end with their distinctness being grounded by a permitted and weakly discerning relation: *that* was not *assumed*, but rather *demonstrated*.

### 3.3 Elaboration of the Circularity Charge

If our analysis of the previous **Section** is correct, the failure to consider what is forbidden and what is permitted to discern [Step 2] should show up in Hawley's elaboration of her vindiction of the circularity charge. We shall see this is indeed the case.

Hawley (2009, p. 109) starts by raising the following question: "granted the assumption that some facts ground others, can facts about the weak discernibility of objects ground their distinctness?" Hawley considers the following two properties (adapted to the current context):

$$\begin{aligned} &H_1(a) \ \text{ iff } \ \text{sphere } a \text{ is 2 miles from Castor ;} \\ &H_2(a) \ \text{ iff } \ \text{sphere } a \text{ is 2 miles from Pollux .} \end{aligned} \tag{15}$$

Then

$$H_1(\mathrm{P}),\ \neg H_1(\mathrm{C}),\ \neg H_2(\mathrm{P}),\ \ H_2(\mathrm{C})\ . \tag{16}$$

Recalling (11), if Castor $\neq$ Pollux, then $H_1$ and $H_2$ (15) express different properties.[15] She (*ibid.*) then goes on to ask: "But what grounds the fact they are distinct properties?" She considers two options (2009, pp. 109–110; our interjections between square brackets):

> The first [A] is that the distinction between the properties [$H_1$ and $H_2$ (15)] is grounded in the distinction between Castor and Pollux; that is, the monadic property *being two miles from Pollux* [$H_2$] depends for its identity upon the two-place relation *being two miles from* [essentially our $D$ (9)] and the object Pollux (and similarly for the property *being two miles from Castor* [$H_1$]). The second option [B] is that the distinction between the two monadic properties is somehow more fundamental than the distinction between Castor and Pollux themselves.

[15] With Hawley, we gloss over the thorny issue of identity-criteria for predicates.

By considering a third object, Hawley then argues that option [A] is superior to option [B]. We agree and follow Hawley in further ignoring the somewhat silly option [B]. Then Hawley argues that option [A] shows that if we use the different properties expressed by $H_1$ and $H_2$ (15) as the grounds for the distinctness of Castor and Pollux (11), we are trapped in a circle, because [A] says that the difference between these properties is grounded in (11) and the spatial relation $D$ (9). She concludes: "French's concerns on behalf of those who seek to ground identity facts in facts about the indiscernibility are vindicated."

We beg to disagree. One reason — which should not come as a surprise at this stage — is that considering monadic predicates to discern and then fail to do so is a false start, because advocates of relational discernment have taken precisely this failure as a reason to look beyond absolute discernment. This has led to relational discernment, and if Hawley wants to attack the Discerning Defence for PII, then the defending arguments involving relational discernment ought to have been the target.

Another reason for disagreement is that if *monadic* predicates $H_1$ and $H_2$ (15) were permitted to discern Castor and Pollux, the spheres would be extrinsically *absolutely discernible*, in contradiction to the description of the qualitative arrangement of Black's case, which has set the entire debate into motion and was intended and taken by all commentators as a challenge to PII. Never mind this. Yet another and better reason to disagree is that predicates $H_1$ and $H_2$ (15) are forbidden because they break the Euclidean symmetry and only exploit the fact that the spheres bear names. This pre-empts Hawley's discussion: it proceeds on forbidden terrain. We emphasize again that the distinctness of the spheres, the numerical diversity, is grounded in a permitted physico-geometric relation (9) that demonstrably holds between the spheres; this relation makes the spheres relationals, and the discerning relation, in turn, is grounded in the structure of space: there is no need to invoke the permitted predicates $H_1$ and $H_2$ (15) in the first place.

### 3.4 Scattered Systematic Remarks

In this **Section**, we address several issues and worries concerning PII in Black's case insofar as our Discerning Defence by an appeal to relationals bears on these.

**A. Redescription.** Consider the following *re-description* of Black's case in order to avoid a clash with PII.[16] We begin with a solid iron sphere 'Castor' and a solid iron sphere 'Pollux' of equal shape and size, in otherwise empty Euclidean space. We neither assume that they are distinct nor that they are identical. Thus a circularity charge does not even get off the ground.

From this re-description, we can only trivially deduce that the *names* are distinct:

$$\text{'Castor'} \neq \text{'Pollux'}\ . \tag{17}$$

We now ask whether we have a case of quantitative diversity without qualitative diversity on our hands. If so, then PII stands refuted.

(i) If Castor is located at some distance from Pollux, then relation $D$ (9) makes a qualitative difference, so that by contraposing Leibniz's Law, we deduce there is quantitative diversity based on qualitative diversity. PII is safe.

(ii) If Castor is at no distance from Pollux, then there is a single object bearing two different names (17). PII is safe.

[16] Brought to my attention by S.W. Saunders, private communication by e-mail, 8 June 2010.

*Objection 1.* This re-description of Black's case is an *impoverished* re-description, an under-description, one may object, because neither the antecedent of (i) nor the antecedent of (ii) follows from *it*. This does not matter, one could respond, because in each of the two cases, (i) and (ii), the conclusion is that PII is safe; and since the disjunction 'Castor is located at some distance from Pollux or is not' is a theorem of logic, the conclusion that PII is safe holds unconditionally. So *Objection 1* does not endanger the conclusion that PII is safe.

*Objection 2.* Agreed, PII is safe so far. Yet one may go on to object that in (ii), PII is taken for granted to reach the conclusion there is a single object: without PII, nothing forbids there being two spheres located in exactly the same place (see further remark **C** for collocated objects). Now recall that Black's aim was to pose a challenge for PII. Then taking for granted what is challenged (PII) is refusing to confront the challenge. Holding on to PII in *this* fashion, come what may, turns every challenge of PII *ab ovo* into a failure. So *Objection 2* is more serious: *without assuming* PII, the conclusion that PII is safe in case (ii) is unavailable. Therefore we do not endorse this re-description of Black's case: although no circularity charge is possible against this impoverished re-description, it also ducks the challenge of PII rather than faces it and meets it.

**B. Generalised distance relations.** Notice that in all spatially symmetric arrangements of an appropriate number of absolutely indiscernible spheres, the spheres are weak relationals due to distance-relation $D$ (9), e.g. 3 spheres at the corners of an equilateral triangle or of a tetrahedron. The triangle case has come up in recent discussions about the bundle theory of objects. We shall not delve into this topic, but we do want to address the problem how to discern Black's *Two-Case*, i.e. the case of two spheres, from a case of three spheres on the corners of an equilateral triangle, which we call the *Three-Case*.

Demirli (2010, p. 9) points out that binary spatial relations like $D$ (9) do not enable one to distinguish the Two-Case from the Three-Case, and suggests a primitive $n$-ary distance relation to do the job, which is not reducible to a binary one. We think there is no need for that. Iron spheres have mass; let $m_2 > 0$ be the mass of the whole in the Two-Case and $m_3 > 0$ of the whole in the Three-Case.[17] We consider two possibilities, which are exhaustive. (a) If $m_2 \neq m_3$, the two wholes are absolutely discernible. No problem. A single sphere of one whole is discerned extrinsically and absolutely from any other sphere of the other whole. No problem. No need in possibility (a) for Demirli's primitive $n$-ary distance relation. (b) If $m_2 = m_3$, and the spheres per case have an equal mass (otherwise we are done because then spheres belonging to different wholes are absolutely discernible from each other), every sphere in the Two-Case becomes absolutely discernible from every sphere in the Three-Case because their masses differ: $m_2/2$ is the mass per sphere in the Two-Case and $m_3/3$ in the Three-Case, which masses are different due to $m_2 = m_3$. The wholes then differ because they have different constituents. No problem. No need in possibility (b) for Dimirli's primitive $n$-arby distance relation either.

Thus no need for a primitive $n$-ary distance relation, but an appeal to other features of the qualitative arrangement, e.g. mass, which obviously is permitted (Step 2). Demirli's invocation of an $n$-ary distance relation is otiose.

**C. Collocated Objects.** Della Rocca (2005, p. 485 ff.) has argued that if we were to reject PII and accept there are two indiscernible spheres as a primitive and unexplained fact, we land in the predicament of having no *principled* way to deny that there are 10 indiscernible collocated

[17]When massless particles are considered, like photons of the same frequency ($\nu$) propagating rectilinearly, one can consider their energy rather than their mass, by using Einstein's formula: $E_\nu = h\nu$, where $h$ is Planck's constant.

spheres where sphere Castor is located, and similarly for Pollux, so that we have 20 spheres. Or a zillion spheres. One can judge this as sufficiently absurd and consequently accept PII.[18] A zillion collocated spheres are a logical *possibility* but arguably a nomic *impossibility*. Whether they constitute a *metaphysical* impossibility is hard to decide.

Della Rocca (2005, p. 484) holds it as circular to discern the spheres by their locations because these locations can only be discerned by an appeal to the spheres — whether they are occupied by a sphere or not is the only thing that can discern the locations, Della Rocca holds. Our diagnosis here is that Della Rocca has skipped Step 2a and has overlooked the possibility of discerning the spheres *relationally*.

**D. Geometry.** Our distance relation relies on the real numbers and thereby on the identity relation between real numbers. Can we do without them? Yes we can. In his landmark axiomatisation of geometry, D. Hilbert (1902: 6) employs Pasch's primitive *ternary* ordering relation of 'point $p$ lying between points $q$ and $r$'. Let us abbreviate this Betweenness-relation by: $B(p, q, r)$. Then an identity-criterion for points reads that two points are identical iff there is no point in between:

$$\neg\exists\, q : \; B(p, q, r) \;\longleftrightarrow\; p = q \,. \tag{18}$$

According to Hilbert's Axiom I.1 of connexion, two distinct points determine a straight line; and his Theorem 3 says that between every two distinct points on a straight line, there lies an unlimited number of points.[19] From this, identity-criterion (18) follows. The negation of the left-hand-side of (18) discerns points weakly:

$$\exists\, q : \; B(p, q, r) \;\longleftrightarrow\; p \neq r \,. \tag{19}$$

Again, half of (18) is PII for spatial points. This goes to show that the points in every space meeting Hilbert's axioms of connexion and of order, of which $\mathbb{E}^3$ is but one example, can be discerned even without appealing to a distance relation and thereby relying on the real numbers.[20]

**E. Hacking's cylindrical space.** Inspired by Hacking (1975), Adams (1979, p. 15) addresses the claim that cases like Black's Spheres and Kant's Droplets are *inconclusive* when it comes to PII because every case that violates PII can be re-described into a case that obeys PII:

> The most that God could create of the world imagined by Black is a globe of iron, having internal qualities $Q$, which can be reached by traveling two diameters in a straight line from a globe of iron having qualities $Q$. This possible reality can be described as two globes in Euclidean space, or as a single globe in a non-Euclidean space so tightly curved that the globe can be reached by traveling two diameters in a straight line from itself. But the difference between these descriptions represents no difference in the way things could really be.

Three issues must be kept apart here.

*First*, Adams is raising *epistemic* question *Q2* (p. 7): how can an inhabitant of Black's world who travels 2 miles from Castor to Pollux *find out* that he has arrived at Pollux (as he would have

[18] Then this is a *reductio ad absurdum* argument for PII, not a *reductio ad contradictionem* argument — contradictions are absurd but not every absurdity is a contradiction, although standard terminology in logic and mathematics is to make these two categories coincide.

[19] Hilbert (1902, p. 4, 7).

[20] Euclid's Axiom of Parallels, Archimedes' Axiom of Continuity and the Congruence Axioms are all not needed to prove Theorem 3; see Hilbert (1902: Ch. I).

if space were Euclidean), and not returned to Castor (as he would have if space were cylindrical with a circumference of 2 miles)? Clearly the structure of the ambient space is relevant and in the description of the qualitative arrangment its structure needs to be specified (as Step 1 demands). As Adams rightly remarks, we have here two different possible worlds with a different spatial structure and not, *contra* Hacking, two descriptions of a single possible world.

*Secondly*, from the point of view of our traveller, things do not look different: he can provide two descriptons but does not know which one is correct. This is his *epistemic* predicament. Now, travelling through some space from location $p$ to $q$ and keeping track of the distance travelled coincides with the metrical distance between $p$ and $q$ iff one travels along the straightest path in that space. (Thus our traveller needs to have the capacity to discern straight from curved paths in every space he finds himself. For if not, he could also have travelled in Euclidean space in a circle from Castor to Castor rather than in a straight line from Castor to Pollux. This makes his epistemic predicament even worse.) But the distance between any two points $p$ and $q$ is an intrinsic feature of the structure of space and does not rely on the presence of travellers, least of all the epistemically debilitated traveller which Adams advances. The metric in every space $\mathbb{S}$ leads to a distance function $d : \mathbb{S} \times \mathbb{S} \to \mathbb{R}^+$ that meets the Fréchet axioms, and from these axioms it follows that $d(p,q) > 0$ iff $p \neq q$. Thus also in cylindrical space, with an appropriate cylindrical distance-function, we have exactly the same four judgements as in Euclidean space, as displayed in (10). (Adams ponders how a traveller would answer question *Q2*, whilst we are interested in answering *Q3*, and *Q3* is answered in favour of PII on the basis of the relevant distance-relation in every metrical space, Euclidean or not. We have here another instance of confusing these questions.)

*Thirdly*, in the background looms Poincaré's Thesis that the structure of space-time is a *convention* rather than a *fact*: since the formulation of the laws of physics presupposes that space-time has a certain structure, we can re-formulate these laws when we change the structure of space-time such that at the level of the behaviour of material bodies nothing changes. Again, this is an epistemic thesis, which is relevant for question *Q2*, but irrelevant for metaphysical questions *Q1* and *Q3*.

Hacking (1975, p. 251) considered the bearing of being a substantivalist or a relationist on Black's case. Consider the proposition:

$$\textit{Every sphere is } 2 \textit{ miles from some sphere} \text{ — } \forall\, a, \exists\, b : \; d(a,b) = 2 \,. \tag{20}$$

For a substantivalist, (20) is true and implies there are (at least) two spheres, because the spheres occupy different spatial points, which exist independently of the presence of material objects occupying or not occupying them; spatial points then are metaphysically prior to (or better: ontologically independent from) material objects. (The spheres then are *prima facie* even absolutely discernible by the monadic predicate '$a$ occupies spatial point $p_0$', whenever '$p_0$' is the name of the centre of Castor or Pollux. On closer inspection however, the spatial points themselves are not absolutely discernible, so that no name like '$p_0$' can ever be introduced by providing a Russellian definite description of that point. For a substantivalist, points are relationals.) PII is safe. For a relationist however, Hacking (1975, p. 251–252) submits, the truth of (20) cannot be established, because now material objects are metaphysically prior to space (or spatial relations depend ontologically on material objects), they determine the spatial relations, so that when one uses spatial relations to determine whether there is quantitative diversity, one is begging the question. Leibnizian relationism endangers Leibniz's principle!

Wait a minute. Begging *which* question? Of whether there is quantitative diversity or not? This is question *Q1* and not the question whether PII stands or falls, which is question *Q3*, and which presupposes an answer to *Q1* in favour of quantitative diversity. We have stumbled on yet another instance of confusing these two questions, this time by Hacking. Moreover, for a relationist, matter is not metaphysically prior to space, but material objects come together with their spatial relations, just as with their masses and shapes. Furthermore, just as material objects do not 'determine' their mass or shape, but *have* masses and shapes, they do not 'determine' their spatial relations but *are* spatially related. For a relationist, material objects have spatial relations and these relations provide the means to discern them qualitatively. Thus like the substantivalist, the relationist can, *contra* Hacking, also establish the truth of (20).

**F. Tear off and tear up.** Van Fraassen and Peschard (2008, p. 19) have recently rejected relational discernibiliy for Black's case and for the case of points in space:

> The condition of weak discernibility certainly entails distinctness. But *first*, such a predicate as 'is one metre from some other point but not from itself' also applies to all points, and so does not express a difference between them. *Secondly*, the deduction that there are at least two $X$s if some $X$ bears an irreflexive relation to some $X$ does not require the PII. It assumes only its converse, that is, substitutivity of identity. So although we shall stay with Saunders' terminology, we find it thoroughly misleading, for this use of the word 'discernible' is misplaced.

*Non placet.* As to the *first* reason, the predicate 'is one metre from some other point but not from itself' is:

$$P(a) \ \text{iff} \ \exists\, b: \ d(a,b) = 1 \ \wedge \ d(a,a) \neq 1 \,. \tag{21}$$

Predicate $P$ applies to all points in (Euclidean) space equally and does, indeed, not discern any point from another *absolutely*. But we have neither advanced $P$, nor any other monadic predicate, as a discerning one. When all points are absolutely indiscernible, there is no such permitted monadic predicate. We have advanced a dyadic predicate expressing a relation, not a property, not even an extrinsic (or 'relational') one.

As to the *second* reason, to present a counter-example to PII is to present some $a$ and some $b$ such that:

$$\neg\text{Disc}(a,b) \ \wedge \ a \neq b \,. \tag{22}$$

Logically speaking, we do not have to deduce one conjunct ($a \neq b$) from the other ($\neg\text{Disc}(a,b)$) in order to defend PII; to reject the putative counter-example to PII (22), it logically suffices to show that $\text{Disc}(a,b)$ granted $a \neq b$. When we do deduce that $a \neq b$ from $\text{Disc}(a,b)$, we rely on Leibniz's Law and we do not rely on PII. Correct. But that is a good thing and therefore, *contra* Van Fraassen and Peschard, does not constitute an objection against arguing in favour of $\text{Disc}(a,b)$ by means of relations; it is a good thing because if a rejection of (22), which challenges PII, were to rely on PII, we would have been proposing a visciously circular rejection of (22).

Ultimately, it seems that Van Fraassen and Peschard cling to absolute discernibility as the one and only kind of discernibility, no-matter-what. This is precisely the traditional straightjacket we want to tear off and tear up.

# 4 Sounds, Chessboard Squares and Space-Time Points

## 4.1 Ayer's Sound-Tokens

Ayer's case can be rendered harmless for PII by also using the Discerning Defence. Ayer (1952, p. 32) considers the following infinite sequence of sound-tokens,

$$\ldots\ldots\ A\,B\,C\,D \quad A\,B\,C\,D \quad A\,B\,C\,D\ \ldots\ldots, \tag{23}$$

rather than the following finite sequence of sound-tokens of the same type:

$$A\ A\,, \tag{24}$$

or the following infinite sequence of sound-tokens of the same type:

$$\ldots\ A\ A\ A\ A\ A\ A\ A\ A\ A\ A\ A\ A\ A\ A\ A\ A\ \ldots \tag{25}$$

Ayer asserts somewhat puzzlingly that (23) is "a simpler example" than Black's case — and simpler than case (24) we may safely assume, because the pair of sound-tokens (24) seems the sound equivalent of Black's material objects, but not as (25). The reason is that the tokens in (24) seem absolutely discernible by the predicate 'being the first sound-token' ($*$), which is not the case for infinite sequences (23) and (25). In these infinite cases, the relation 'occurs later than' discerns the sound-tokens *relatively* rather than weakly, because this relation is anti-symmetric.

But classical as well as relativistic space-times are not time-orientable, so that time is isotropic: predicate ($*$), which contains the phrase 'first', fails to pick out a single sound-token. Similary the relation 'occurs later than' fails to discern the sound-tokens in (23) and (25) relatively, because 'later' has no meaning when time is isotropic. The following relation is however temporally isotropic and suited for classical space-times ('$s$' and '$r$' are sound-token variables of a single type of sound):

$$S(s,r) \ \text{ iff } \ s \text{ and } r \text{ occur Simultaneously}\,. \tag{26}$$

Relation $S$ (26), and therefore $\neg S$, is invariant under Galilei transformations. Consider now two of the same sound-tokens occurring simultaneously far away from each other. These tokens are not identical. This shows that the simultaneity-relation $S$ (26) is not an identity-criterion, but only a necessary condition for identity. Then $\neg S$ is sufficient for the sound-tokens to be distinct but not necessary. So we must look further for a relation that discerns sound-tokens in (24) and in (25).

The key is to use the finite *spatio-temporal regions* of the longitudinal vibrations of the air molecules of the sound-tokens. Let '$r(G)$' denote sound-token $r$ having $G$ as its finite space-time region.[21] Then we have the following identity-criterion for sound-tokens:[22]

$$I\big(r(G_1),\ s(G_2)\big) \ \text{ iff } \ G_1 = G_2\,. \tag{27}$$

[21] We take space-time *regions* to be subsets of the space-time manifold that are bounded and simply-connected.

[22] Realize that the sound in a region $G$ is what results from the superposition of all longitudinal vibrations in $G$, so that there cannot be more than one sound in one space-time region. There can be more than one tone in one region: every vibration can be written as a superposition of monotones (Fourier analysis). In our purified Ayerian case (24), we have two tokens of one *tone*, which is a finite wave train of a single frequency. For those who hold there are no sounds without ears connected to brains: Ayer's case becomes a case of vibrations rather than sounds.

Relation $\neg I$ (27) is invariant under the space-time transformations of classical and relativistic space-times, and discerns the sound-tokens weakly. Thus Ayer's sound-tokens now become *weak relationals*.

Due to its reliance on an identity-criterion for space-time regions, relation $I$ (27) seems to presuppose a *substantivalist* interpretation of space-time in Newtonian tradition. What if one has *relationist* sympathies in Leibnizian tradition? Then *space* is a collection of primitive spatial relations between material bodies, and, extended to space-time, *space-time* is a collection of primitive spatio-temporal relations between events. The nature of this relation will then determine the ontological category of the sounds, i.e. whether they are relative or weak relationals. But relationals they will be and that is all we need in order to discern them qualitatively. (See Subsection 4.3 for how to discern space-time points relationally by a 'lightcone relation'; this relation can also be used to discern space-time regions in relativistic space-times generally. A relation to discern space-time regions in classical space-times is easily constructed by combining the absolute simultaneity-relation (26) and the Euclidean spatial distance-relation $D$ (9) of Black's case. We leave it as an exercise to work out the details.)

With a slight variation on Hacking (1975, pp. 254–255), one may wonder whether the interval between two sound tokens is exactly equal to one cycle of a temporally cyclic world. Suppose an inhabitant keeps hearing the same sound token in one and the same space-time region over and again. But there is only *one* sound token in this cyclic universe, so that no threat for PII arises. If the inhabitant in this world can count the sound tokens she hears, as the description above suggests, and puts a stroke on a piece of paper, then for every cycle there is a *different* number of strokes on the piece of paper. But then this world is not really cyclic, because in a truly cyclic world, the world is identical after every cycle. Hence there cannot be such an inhabitant. Our supposition was wrong. If an inhabitant hears the token, she hears it 'every cycle' for the first and for the last time in her life: she will affirm the existence of a single sound token and no more. PII stands tall by the Identity Defence.

### 4.2 Strawson's Chessboard

When discussing Leibniz's monads, Strawson (1959, p. 122) considers a universe consisting of a chessboard and claims that some white squares (f3 and c6) cannot be differentiated from others; it holds for all pairs of squares that are each other's mirror image under rotating the board 180°. Strawson (1959, p. 123) — who sees this as a case where one cannot identify without demonstratives —, considers these squares to constitute examples of quantitatively distinguishable but unidentifiable and qualitatively indistinguishable particulars, *contra* PII.

We disagree. The most simple language to describe Strawson's chessboard universe presumably proceeds by having 64 names (a1–f8), monadic predicates 'Black' and 'White' (of which one can be defined as the negation of the other), and a dyadic predicate expressing an irreflexive, symmetric and non-transitive adjacency-relation: $\text{Adj}(s,q)$; we then lay down a list of axioms that entails which square is adjacent to which other squares, so as to obtain a chessboard as we know it. Consider then the following *common-adjacency relation*:

$$A(s,q) \quad \text{iff} \quad \forall\, r:\ \text{Adj}(r,s) \longleftrightarrow \text{Adj}(r,q)\ . \tag{28}$$

The common-adjacency relation $A$ is an identity-criterion, because all and only identical squares share all their adjacent squares:

$$A(s,q) \longleftrightarrow s = q \; . \tag{29}$$

Then its negation, $\neg A$, discerns squares of the same colour weakly. Relation $A$ (28), and therefore $\neg A$, is invariant under the geometric symmetry-transformations of the chessboard universe: mirroring in the two diagonals, or which comes down to the same: rotating 180° around an axis perpendicular to the centre of the board. *Contra* Strawson, we conclude that the chessboard squares can be discerned qualitatively; by overlooking Step 2, he took it for granted that the only way to uphold PII is to be able *to identify* the squares, for to identify is to discern absolutely, and we have seen that this is not the only way to discern qualitatively. The squares are relationals.

D. Wiggins (2012, p. 7) submits that a relation like 'is to the left of' discerns some white squares relatively. We reject this relation because it breaks the symmetry of the chessboard: right becomes left after a rotation of 180°. Wiggins' relation is forbidden.

### 4.3 Wüthrich's Space-Time Points

Wüthrich (2010) argued, in the context of the General Theory of Relativity (GTR), that in symmetric space-times such as the one of our physical universe (globally speaking), all space-time points that belong to one 3-dimensional spatial hypersurface share all their physico-geometrical properties, because these properties all derive from the metrical tensor $g_{ab}$ and $g_{ab}$ is the same at every point in such a hypersurface. Therefore adherents of PII must conclude, absurdly, that in such 3-dimensional hypersurfaces, interpreted as 'global snapshots' of the universe, all points are identical and thus there is only a single point. In the case of the most symmetric and flat space-time of the Special Theory of Relativity (STR), the metrical tensor is the same at every point in space-time, so that adherents of PII must conclude, ridiculously, that Minkowski space-time consists of a single point. PII refuted?

*Quod non.* Saunders (2003b) had already proposed to discern the space-time points by means of the following relation, which relates points iff they belong to disjoint open sets:

$$H(p,q) \quad \text{iff} \quad \exists\, O, O' \in T(\mathcal{M}) : \; p \in O \;\wedge\; q \in O' \;\wedge\; O \cap O' = \varnothing \, , \tag{30}$$

where $T(\mathcal{M})$ is a topological subset-family of the space-time manifold $\mathcal{M}$, and $p, q \in \mathcal{M}$. Only with the requirement that $\mathcal{M}$ is 'Hausdorff' do we have a guarantee that relation $H$ (30) is a criterion for distinctness, so that $\neg H$ becomes an identity-criterion.[23] Relation $H$ (30) discerns the points weakly and is homeomorphic, which is to say that it is topologically invariant; this implies that it is also invariant under all physically significant space-time transformations, which codify the symmetries of space-time.

One may plausibly object that the physical significance of relation $H$ (30) is moot: no *physical* grounding of discerning relation $H$ has been provided at all, only a mathematical one. The physical significance of the following relation is however glaringly obvious. Muller (2011) discerns the space-time points relationally by a so-called *lightcone-relation* $L$. Let $LC(p) \subset \mathcal{M}$ be the lightcone

[23]Definition: space-time manifold $\mathcal{M}$ is *Hausdorff* iff for every two distinct points in $\mathcal{M}$, there are two disjoint open sets such that each open set contains one of these points and not the other, that is, relation $H$ (30) holds for every pair of distinct points.

of $p$, the set of points that can be reached from $p \in \mathcal{M}$ by travelling not faster than light.[24] Then

$$L(p,q) \quad \text{iff} \quad \big(\exists\, r \in \mathcal{M}:\ r \in LC(p)\backslash LC(q)\big) \vee \big(\exists\, t \in \mathcal{M}:\ t \in LC(q)\backslash LC(p)\big)\,, \tag{31}$$

where one disjunct would be enough because they imply each other. Lightcone-relation $L$ (31) discerns every two space-time points weakly. Relation $L$ is demonstrably invariant under all physically significant space-time symmetries of GTR and STR. Wüthrich (2010) has overlooked Step 2, jumped to identity from absolute indiscernibility, and thereby has ignored relational discernibility.

A circularity worry about relations $H$ (30) and $L$ (31) arises as follows. Both relations rely on sets of space-time points (open sets and lightcones); *their* identity relies on the identity between their members, which are space-time points. But relations $H$ and $L$ were supposed to provide us with a qualitative distinctness relation between space-time points. Vicious circle? Do relations $H$ (30) and $L$ (31) ultimately rely on the identity-relation and are they therefore forbidden?

The error in this worry is that the identity between sets relies on the identity-relation between their members. This is incorrect. The Axiom of Extensionality in set-theory is PII for sets, and provides, in conjunction with its converse, an identity-criterion for sets that relies only on the primitive membership-relation. For the lightcones, it becomes:

$$LC(q) = LC(p) \quad \text{iff} \quad \forall\, r \in \mathcal{M}:\ r \in LC(q) \ \longleftrightarrow\ r \in LC(p)\,, \tag{32}$$

and the same for the open sets in (30) and the regions in Ayer's sound-tokens (27). No circularity is involved.

### 4.4 Permissibility Conditions

Are there *general* conditions for what in a qualitative arrangement is permitted and what is forbidden to discern? Perhaps. We give it a try and thereby address Steps 2b and 2c in full generality (**Section 1**). *Forbidden* to discern are:

- (F0) predicates expressing trivialising properties (like containing '='; see footnote 13);
- (F1) predicates in which names occur;
- (F2) predicates expressing properties or relations that break a symmetry of the qualitative arrangement (as described in Step 1).

*Permitted* to discern are properties and relations that are not forbidden. Thus the conjunction of the negations of (F0)–(F2) is a *criterion* for permissibility. We want to emphasize that banning names (F1) is old hat, and presumably can be subsumed under trivialising (F0). We endorse (F1) somewhat reluctantly, because it is not the mere presence of names that is forbidden, but rather the way in which names are employed in putatively discerning predicates. The symmetry requirement (F2) can be dug up from Saunders (2003b); it is comparatively novel and the focus of our attention.

Summing up, in Black's case the two spheres are absolute indiscernibles, they have no identity, but they are relational discernibles, i.e. relationals; their distinctness comes from a spatial relation $D$ (9), which is permitted by our conditions (F0)–(F2), and is grounded in the structure of space, and is therefore not ungrounded. And *mutatis mutandis* for Kant's droplets, Ayer's sound-tokens,

[24] Despite appearances, this relation does not rely on the presence of travellers.

Strawson's chessboard squares and Wüthrich's space-time points, where the relational discernibility of the objects is grounded in the relevant structures; and the discerning relations respect the relevant symmetries (F2). The circularity charge misfires, because it quantitative diversity as what is to be demonstrated (question *Q1*), whereas what is to be demonstrated is that there is *qualitative* diversity (by means of permitted properties or relations, or both) in a given situation of *quantitative* diversity, so as to save PII (question *Q2*). For again, without being given quantitative diversity, no threat against PII can materialize: if there is no quantitative diversity, i.e. if there is a single object, there is not and cannot be a challenge for PII. In all threats against PII we have analysed, where we have accepted the quantitative diversity involved of the challengers, we have found qualitative diversity too. The Discerning Defence has been victorious; it thus seems to provide a uniform defence of PII against all putative counter-examples we have treated.

We finally turn to the remaining case on our list, Weyl's case of elementary particles.

# 5 The Quantum-Physical Universe

## 5.1 Quantum Mechanics

Consider two fermions of spin-1/2, *1* and *2*, in their quantum-mechanical spin-state:

$$|\Psi\rangle \;=\; \big(|1:\uparrow\rangle \otimes |2:\downarrow\rangle \;-\; |1:\downarrow\rangle \otimes |2:\uparrow\rangle\big)/\sqrt{2} \;\in \mathbb{C}^2 \otimes \mathbb{C}^2\,. \tag{33}$$

Saunders (2006) argues that since two similar *fermions* are weakly discernible in state (33) by the relation 'has opposite spin to', they still can be considered as material objects of sorts. Muller and Saunders (2008) define this relation rigorously (in a way that allays the suspicion that that the particles posses spin properties in an entangled state like $|\Psi\rangle$) and generalize this case of two spin-1/2 fermions to composite physical systems composed of an arbitrary number of absolutely indiscernible fermions of arbitrary spin. PII stands in the face of fermions. Although elementary *bosons* may not be discerned in this fashion in all their quantum-mechanically permitted states (e.g. in $|\Phi\rangle$ (34) below), Saunders does not conclude that PII goes down after all, but that elementary bosons are not material objects of the same general kind that the fermions belong to ('quantum-mechanical particles', say). There is only a single object in this case, Saunders (2006, p. 60) concludes, a quantum field, and the alleged bosons are excitations of this field; in some energy regimes the modes of the field can be considered as objects. The integer numbers that appear in the description of the quantum field (Fock-space) are not cardinal numbers of bosons, but excitation levels of quantum field modes; the excitations, then, are features of the modes of the quantum field regarded as a single physical system. In brief, Saunders answers the question in Step 2a for bosons in the negative and therefore mounts the Summing Defence. (In Black's case, the Summing Defence of PII would deny there are two black spheres: there is a single poly-located and partless object with two sphere-like features.)

Hawley (2009, p. 114) now charges Saunders with *ad hoc* discrimination: when it comes to fermions, Saunders chooses the Discerning Defence for PII, and when it comes to elementary bosons, he chooses the Summing Defence for PII. Hawley favours a *uniform* approach: the Summing Defence for PII in both cases. If uniformity of defence were the only virtue, Hawley would win the day.

Another advantage of the Summing Defence, Hawley (2009, p. 114) submits — and says she reached for in Hawley (2006) — is there is nothing about the qualitative arrangement of Weyl's case that presses us to acknowledge that there are *parts*. Ontological Parsimony (Occam's razor) thus favours the Summing Defence, because this Defence needs to posit only one object, whereas the Discerning Defence posits three objects (in both cases, let us not forget, to *explain* certain phenomena): the two particles and their composite system.[25] But then, as Hawley presumably knows all too well (but does not raise here), there are *other* virtues to consider besides Uniformity and Ontological Parsimony, and these other virtues ought to be considered because when we realize that the afore-mentioned virtues may derive from the metaphysical prejudice that the world is uniform and parsimonious, they lose their virtuous character, or else must live on in the borrowed robes of pragmatism. Other if not preferable virtues are Coherence and *Conservativeness*, i.e. continuity with gathered knowledge. Let us take a look at how Hawley's Summing Defence in QM fares on closer inspection.

*First*, talk of *composite* physical systems (wholes) and *subsystems* (parts) is mathematically ingrained into the standard language of QM, by means of direct-product-spaces and subspaces of Hilbert-spaces, and by means of state-operators and partial traces. Hawley now needs to propose a different interpretation of that part of the language of QM, in line with her Summing Defence, or else re-formulate QM in such a manner that direct-product-spaces and subspaces, state-operators and partial traces disappear from QM. This is a nasty dilemma, which does not arise for proponents of the Discerning Defence. Since there is nothing about the qualitative arrangement of Weyl's case that presses us to engage in revisionary activities with regard to the theory of QM, the Discerning Defence has the upper hand when it comes to Quine's virtue of Conservativeness.

*Secondly*, measurements with spatially separated pieces of measurement apparatus can be performed. When spin is simultaneously measured on the two particles in an entangled state like $|\Psi\rangle$ (33) at spatially distant locations, then each measurement on a particle is *a joint measurement of spin and position*, and then, just after the moment of measurement, one particle is located (with quantum-mechanical certainty) in the volume of one piece of measurement apparatus ($\mu_1$), and the other particle in that of the other piece ($\mu_2$).[26] At that moment we have a situation very similar to Black's case: two disjoint, spatial regions occupied by two objects, $\mu_1$ and $\mu_2$. In our world, $\mu_1$ and $\mu_2$ will always be absolutely discernible, and hence intrinsically discernible, or extrinsically discernible by relations to other objects in the world. But let us assume, for the sake of argument, we are in a world where $\mu_1$ and $\mu_2$ are absolutely indiscernible and we have two *bosons* having spin-1, such as two photons, call them again *1* and *2*, in a pure symmetric entangled spin-state:

$$|\Phi\rangle \;=\; \big(|1:\uparrow\rangle \otimes |2:\uparrow\rangle \;+\; |1:\downarrow\rangle \otimes |2:\downarrow\rangle\big)/\sqrt{2} \;\in \mathbb{C}^3 \otimes \mathbb{C}^3 = \mathbb{C}^9 \,. \tag{34}$$

Then both $\mu_1$ and $\mu_2$ register the same measurement outcome for spin when jointly measured (perfect correlation): both spin up (↑) or both spin down (↓). When it is odd *not* to conclude in Black's case that we have two objects, Coherence demands that we should also *not* conclude it in this elementary-particle case: $\mu_1$ and $\mu_2$ are weakly discerned by the distance-relation $D$ (9) and so are the measured bosons.[27] For the Discerning Defence there is no problem here, because

[25]We gloss over the dubious token-interpretation of Occam's razor at work here, rather than a type-interpretation, which arguably is the proper interpretation.

[26]To the best of my knowledge, this was first expounded by Muller (1997, p. 244). Without or before measurement, there is no certainty, so that the celebrated argument of Einstein, Podolsky and Rosen against the completeness of QM falters (Muller 1997, pp. 244–245).

[27]Hawley (2009, p. 113): "But the problem with scattered simples is that it is hard to see what more could be required for the existence of an object than existence of a maximally connected portion of matter; that is, it is hard to see what prevents each of the spherical regions from exactly containing one object." Hear hear.

according to it there were *two* particles all along. But the Summing Defence is in trouble, for it has to bite the bullet of oddity or else admit there are genuine parts *just after* the moment of measurement, but *not before*. Also, *after* the measurement process has ended, the parts pop out of existence again, because their position probability distribution unavoidably diverges all over space, so that the 'particle-like' features of the wave-function have disappeared and there are no spatial facts any longer that ground their distinctness in the structure of the ambient space; yet the quantum-mechanical description, however, remains one of *two* particles. More revisionary labour in store for the Summing Defence. The virtues of Coherence and Conservativeness speak *uno tenore* in favour of the Discerning Defence.

To conclude, the Discerning Defence trumps the Summing Defence. Recent arguments in the philosophy of physics conclude that in the context of QM, fermions and bosons alike are weakly discernible by physically significant relations that are invariant under the relevant symmetry-transformations of QM.[28] This means that the Discerning Defence is available in *both* cases, fermions and bosons. The Summing Defence now loses not only its edge over the Discerning Defence with respect to being the only *uniform* defence of PII in Weyl's case (in QM), but it furthermore evokes problems that the Discerning Defence does not evoke.

### 5.2 Quantum Field Theory

When we consider *Quantum Field Theory* (QFT), rather than QM, the plot thickens. The proper QFT-description of what is called in the language of QM 'composite systems of similar fermions' and 'composite systems of similar bosons' proceeds by means of a fermionic and a bosonic quantum field, respectively. This quantum field on space-time is a *single* object.[29]

If we, with Saunders, consider PII applicable to modes of this field, which modes we take as the putative 'objects' for PII to apply to, we are answering the question in Step 2a in the affirmative. The Discerning Defence enters again.

If we, against Saunders, reject his particle view of the quantum field, and strengthen this move by an appeal to Malament's Theorem (1996) concerning the impossibility of a relativistic quantum theory of localisable particles, and to Halvorson and Clifton's Corollary (2002) that a particle-interpretation of QFT is impossible, then, in this qualitative arrangement, PII is only applicable to (the space-time points and to) the single quantum field. There is, then, in QFT not even an apparent conflict with PII (as there is in QM); here the Summing Defence wards off the putative conflict. In QFT, PII thus is manifestly safe, and both fermionic and bosonic fields are treated *uniformly*.

In spite of these impossibility results, working physicists keep on talking about *particles*, in QM as well as in QFT. Then either working physicists have a different concept in mind than the one that is proved impossible to maintain in QFT, or there are lots of cases, notably in the universe we inhabit, where the concept of a particle, notably as a *probabilistically localised object with mass-energy*, remains applicable, as Saunders (1994) maintains. In those cases, the Discerning Defence is apt; in cases where the particle-concept does not apply, the Summing Defence will keep PII safe. Thus a non-uniform defence of PII transpires.

[28]Muller and Seevinck (2009). There are pockets of resistance: Dieks and Lubberding (2011) operate with a different 'particle conception'. They believe that particles are features of particular wave-functions, in which case the Summing Defence applies. PII is not in danger in such an interpretation and we therefore gloss over it.

[29]In the logical, metaphysically thin sense of 'object' to be sure: it is not a material object in any familiar sense of this word. For philosophical struggles with the quantum field, see Kuhlman *et al.* (2002, pp. 127–132, 145–161, 181–206).

## 6 Exitum — Relationals

Let us recapitulate the quantum case. When we compare the Discerning Defence with the Summing Defence of PII, uniformity of response seemed to favour the Summing Defence, according to Hawley. The ground for one opponent of the Discerning Defence and proponent of the Summing Defence (Saunders) was the difference between fermions and bosons: in the case of two fermions, the Discerning Defence was appropriate because the two fermions turned out to be weakly discernible; and in the case of bosons, it was denied that there are objects to begin with, because there was only a single quantum field with excitations. Saunders (2008) smoothly slides here from QM to QFT, which are two quantum theories having quite different *prima facie* ontologies. We have argued that as long as we remain in QM, fermions as well as bosons are weakly discernible objects, so that, *contra* Hawley, the Discerning Defence is a uniform defence of PII. We have further argued that the Summing Defence in QM is, in fact, severely problematic, in spite of its uniformity. Coherence and Conservatism speak in favour of the Discerning Defence, which thus has the upper hand in QM. Yet as soon as we leave QM and move to QFT, either (i) the Summing Defence is the uniform defence of PII, because then there is a single object in every case, namely a quantum field; or (ii) a mixed defence transpires: Discerning Defence whenever the concept of a particle applies and Summing Defence whenever it does not apply (Step 2a), where the application condition involves energy regimes. In the other cases on our list of challenges to PII, notably Kant's droplets, Black's iron spheres, Ayer's sound-tokens, Strawson's chessboard squares and Wüthrich's spacetime points, the Discerning Defence turns out to be triumphant across the board. All advocates of these alleged counter-examples to PII have leaped from Step 1 to Step 3 and have paid insufficient attention to Step 2, as a result of which they overlooked the discerning power of relations. And nearly all propounders of the circularity objection against relational discernment have confused questions *Q1* and *Q3* (p. 7).

We have attempted to establish that PII stands tall without exception as far as the physical universe is concerned. In all cases we have advanced relations to mount a Dicerning Defence of PII. Whence the title of this paper. We end with a few reflexions.

Does being a relational belong to the *essence* of elementary particles? If so, then they should be relationals in at least *all* quantum-mechanically possible worlds, rather than be indiscernibles or individuals in other worlds, for that is what it means to be a relational *essentially*. Well, elementary particles can be individuals. Think of a world with a single Hydrogen atom as its only material inhabitant: the proton (fermion) and the electron (fermion) are absolutely discernible by their different mass and therefore are individuals. In a world with only a single Helium atom, its two electrons are relationals again; the proton remains an individual. Are elementary particles, then, always either relationals or individuals *per world*? Perhaps. All we can then claim to have shown is that elementary particles necessarily are *discernibles*. If we assign a quantum state to an entire world, then due the permutation-symmetry of QM, all electrons in a world, say, are in the same physical state (partial trace), so that they share all their properties and their state, and thefore are not individuals but relationals, with the exception of worlds like the one with the lonely Hydrogen atom.

The second reflexion we alluded to at the end of **Section 2**: diachronic identity and persistence conditions (Wiggins 2010). Can relationals meet them? When we have a physical system composed of $N$ absolutely indiscernible particles at time $t$, then not a single particle can be re-identified at a later time $t' > t$ because it cannot be identified at all: they are not individuals but relationals. Elementary particles have no *genidentity*, as Reichenbach (1956, p. 226) put it. The relations we have employed to discern particles weakly use the quantum-mechanical state *at a time* and thefore discern them *synchronically*. Relations to discern particles *diachronically* are not forthcoming in QM. There are no persistence conditions for single particles. But there *are* such conditions when we consider the physical system they compose. When we still have $N$ particles at time $t > t'$, for example by determining the mass of the composite system at time $t$ and $t'$ and comparing those masses, and these masses turn out to be equal, we may safely conclude that all $N$ particles have *persisted*. Their persistence is guaranteed by their existence. As soon as we move to QFT, we loose this guaranatee, due the possibility of pair annihilation (one electron being wiped from the face of the cosmos by a positron) and pair creation (two photons creating an electron-positron pair) — bracketing the issue of wether we can speak about particles at all in QFT. Existence then no longer guarantees persistence.

The third and final reflexion concerns metaphysics generally. Prominent metaphysicians like Lowe (1998, p. 206; 2006, pp. 3–5) and Cocchiarella (2007, pp. xiii–xv) hold that one of the central aims of metaphysics is to erect a framework of metaphysical concepts — ontological categories included —, to deduce conceptual truths, and to classify the entities of experience and the posits of science. Concerning ontology, then metaphysics tells us *what there can be*, not *what there is* — which seems up to our experience and up to science. If this is correct, then all metaphysical knowledge is conceptual. We advance another possible aim of contemporary metaphysics besides the purely conceptual ones mentioned above: to find non-conceptual truths of utter generality about the nature of the universe. This paper presents an example of this: we have excellent reasons to believe that PII (5) is such a general truth about the universe, and those reasons are that PII emerges as a *theorem* in all fundamental theories of physics. This provides the basis for proclaiming PII to be a nomic necessity.[30] Thus PII is not some metaphysical principle we must *assume* in or *presuppose* by physical theories for some metaphysical reason or other; it is a piece *metaphysical knowledge*, justified deductively by modern physical theories. PII is a metaphysical crown on our most fundamental knowledge of the universe.[31]

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[30] One implication is that haecceities are nomic impossibilities; see Adams (1979).

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